SECTION A: PURE MATHEMATICS

THIS SECTION IS COMPULSORY TO ALL CANDIDATES

(ANSWER FOUR QUESTIONS)

1. (i) Given that $(x-2)$ and $(x-1)$ are factors of $f(x)$, where $f(x) = 2x^3 + ax^2 + bx - 6$,
   (a) find the values of the constants a and b.
   Hence or otherwise,
   (b) solve the equation $f(x) = 0$.

   (ii) A quadratic equation $x^2 + x - 2 = 0$ has roots α and β. Without solving the equation,
   (a) write down the value of $\alpha^2 - \beta^2$.
   (b) find the quadratic equation, with integral coefficients, whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

---

2. (i) The sum, S_n, of the first n terms of a geometric progression is given by $S_n = 3\left[1 - \left(\frac{2}{3}\right)^n\right]$. Find:
   (a) the common ratio,
   (b) the fifth term,
   (c) the sum to infinity, of the progression.

   (ii) Given that $f(x) = \left(x - \frac{2}{x^2}\right)^7$, find:
   (a) the first three terms in the expansion of $f(x)$, in descending powers of x,
   (b) the numerical value of the term independent of x.

---

3. (i) Solve, for $0° < x < 360°$, the equations
   (a) $\sin 2x + \cos x = 0$,
   (b) $\tan^2 x - \sec x - 1 = 0$.

   (ii) Taking values of x at intervals of $\frac{\pi}{6}$ on the x-axis and 1cm to represent $\frac{\pi}{6}$ on the x-axis and 2 cm to represent 1 unit on the y-axis, draw the graph of $y = \cos 2x + \sin x$, for $0 \leq x \leq \pi$.
   Using your graph, estimate, to one decimal place, the maximum and minimum values of $y = f(x)$.

---

4. (i) Differentiate, with respect to x,
   (a) $\cos 3x$,
   (b) $\frac{x^2 + 1}{x - 2}$, $x \neq 1, x \in \mathbb{R}$.

   (ii) The derived function of the function f of the real variable x, is $3x^2 - 3x - 6$. Find the equation of the normal to the curve $y = f(x)$ at the point $(0, 5)$.

   (iii) The function g is defined as $g(x) = ax^2 + bx$, where a and b are constants, and $x \in \mathbb{R}$.
   Given that $g'(1) = 8$ and $\int_0^2 g(x)dx = 12$, find the values of a and b.

---

5. (i) The set $S = \{T_1, T_2, T_3, T_4\}$, where

$$T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and the operation $\otimes$ is defined as matrix multiplication on S, e.g. $T_2 \otimes T_3 = T_1$

(a) Copy and complete the following table:

$\otimes$	T_1	T_2	T_3	T_4
T_1	T_2			
T_2			T_1	
T_3				T_3
T_4		T_2		

(b) Show that the set S forms a group under the operation $\otimes$
(You may assume associativity).

(ii) Queen's foundation is to donate a maximum of 50 copies of English and Mathematics text books to a school library on the condition that.
Given that:

- at least 35 copies of the books are English text books, and
- that the number of mathematics text books must be least 40% and at most 70% of the English textbooks.

Taking x to represent the number of English text books and y to represent the number of Mathematics text books,

(a) write down four inequalities that represent the conditions above.

Taking 2cm to represent 10 units on both axes,

(b) shade, so as to leave unshaded, the region satisfied by the four inequalities.

(c) Given also that an English text book costs 1200FCFA and a Mathematics text book costs 1500FCFA, use your graph, or otherwise, to find the maximum amount of money to be spent by the foundation for the donation.