1. (i) Given that $(x-1)$ is a factor of the polynomial $f(x)$, where $f(x) = ax^4 + x^3 - 12x^2 - x + b$.
Find the value of the constant a and b such that $f(-1) = 0$.
(ii) Find the value of the constant k for which the equation
$$x^2 + (k-1)x + k = 0$$
has one root double the other.

(9 marks)

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2. (i) Show that $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$.

(ii) Find the general solution of the equation
$$\sin 4x + \cos 2x = 0.$$

(iii) Solve for x, where $0^\circ \leq x \leq 180^\circ$, the equation $\sin 3x +$ [illegible] $x = 0$

(10 marks)

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3. (i) The function $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x - \frac{x^3}{3}$.
Find the monotony of f, showing clearly its variation table.

(ii) Solve the differential equation $x\frac{dy}{dx} = y(2x^2 + 1)$.

(12 marks)

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4. (i) Given that $z = e^{i\theta}$, show that $z^n + z^{-n} = 2\cos n\theta$.
Use this result to express $\cos^5\theta$ in terms of cosines of multiples of θ.
(ii) Given that $z_1 = 1 + i\sqrt{3}$ and $z_2 = -1 + i$, evaluate
(a) $|z_1 z_2|$, [illegible]
(b) $\arg(z^4)$.

(11 marks)

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5. The coordinates of the points A, B and C are $(0, 1, 3)$, $(-1, 0, 1)$ and $(1, -1, 2)$ respectively.
Find
(a) $\overrightarrow{AB} \times \overrightarrow{BC}$,
(b) the sine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$,
(c) the value of the constant p for which the line $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3p\mathbf{i} - \mathbf{j} + 5\mathbf{k})$ is parallel to the plane containing A, B and C.

(10 marks)

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6. Given the matrix $\mathbf{A}$, where $\mathbf{A} = \begin{pmatrix} 2 & [illegible] & 2 \\ 3 & [illegible] & 2 \\ 2 & 2 & 1 \end{pmatrix}$,
Find
(a) $\det(\mathbf{A})$, the determinant of $\mathbf{A}$,
(b) $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

Hence, or otherwise, solve the system of equations,
$$2x - y + 2z = 3,$$
$$3x - y + 2z = 3,$$
$$2x - 2y + z = 2.$$

(8 marks)

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7. Express $\dfrac{1}{(3t+1)(t+3)}$ in partial fractions.

By using the substitution $t = \tan x$, or otherwise, show that

$$\int_0^{\frac{\pi}{4}} \frac{dx}{3+5\sin 2x} = \frac{1}{8}\ln 3.$$

(9 marks)

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8. (i) Find the set of real values of x for which $\left|\dfrac{3x+4}{2x-3}\right| < 1$.

(ii) Sketch the curve of $y = \dfrac{x+2}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$, showing clearly the intercepts with the coordinate axes and the behaviour of the curve as it approaches its asymptotes.

(11 marks)

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9. (i) The functions f and gof are defined by

$g: x \mapsto x+5, x \in \mathbb{R}$, $\text{gof}: x \mapsto \dfrac{5(x-3)}{x-4}, x \in \mathbb{R}, x \neq 4$,

find f and show that f is injective.

(ii) If p is the statement : *Eric plays golf* and q the statement; *Oscar plays tennis*.

Write down the statement represented by each of the following:

(a) $p \Rightarrow q$.

(b) $\sim q \Rightarrow p$.

(c) $\sim(p \vee q)$.

(11 marks)

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10. (i) The first three terms in the series expansion of $\sqrt{\dfrac{1-x}{1+kx}}$ are $1, -2x$ and $4x^2$ respectively. Determine the value of k and state the range of values of x for which the expansion is valid.

(ii) Five cards are to be dealt out to a player from a standard pack of 52 playing cards. How many different possibilities are there if,

(a) there is no ace

(b) there are at least two aces.

(11 marks)
