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1. The position vector of two particles A and B at time t seconds are given by r_A and r_B respectively where

$$r_A = [t^2i + 2t^3j]m, \quad r_B = [3t^2i - 4t^2j]m$$

- Calculate the distance between A and B when $t = 2$.
- Find the velocity of A relative to B when $t=2$
- Find the values of t for which the velocities of A and B are perpendicular
- Find the values of t for which the accelerations of A and B are parallel.

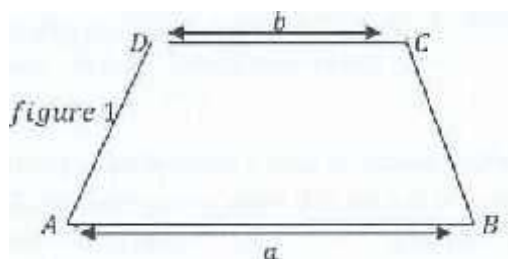


Figure 1 above shows a uniform lamina in the form of a trapezium in which AB & CD are parallel and of

lengths a and b respectively. ($a > b$). Prove that the distance of the centre of mass from AB is

$$\frac{1}{3}h\left[\frac{a+2b}{a+b}\right] \text{ where } h \text{ is the distance between AB \& CD.}$$

ii) Forces F_1 and F_2 act at the point with position vector r , where

$$F_1 = (4i + 6j)N, \quad F_2 = (6i - 8j)N, \quad r = (2i - 3j)m$$

Find the sum of moments of F_1 and F_2 about the origin.

3. The forces $F_1 = (i + 2j + 3k)N$ and $F_2 = (2i + k)N$ act at points whose position vectors are $(2i + 5j + ck)m$ and $(5i + cj + 2k)m$ respectively. Given that their lines of action intersect, find the value of c and the position vector of the point of intersection. Determine the vector equation of the line of action of these forces and show that this line passes through the point with position vector $(7i + 9j + 8k)m$.

4. A particle, P, of mass $4m$ kg is initially at rest on a smooth plane incline at an angle α to the horizontal. It is supported by a light inextensible string which passes over a smooth light pulley A at the top edge of the plane. The other end of the string support a particle Q, of mass $2m$ kg, which hangs freely. Given that the system is in equilibrium, find α and the magnitude and direction of the force exerted on the pulley by the string.

A further particle of mass M kg is now attached to Q and the system is released. Given that the

acceleration of the system is, $\frac{2}{5} \text{ ms}^{-1}$ find in terms of m , the value M , the tension in the string and the magnitude of and direction of the force exerted on the string by the pulley.

(Take g as 10 ms^{-2})

5. i) A car of mass 1000 kg is moving on a smooth level road at a steady speed of $^{\wedge} \text{ms}^{-1}$ against an air resistance R . The engine works at a rate of 60 kW .

Calculate the are resistance R .

When the car starts to ascend a rough plane inclined at an angle $\sin^{-1} (2/3)$ to the horizontal with this speed, it turns out of fuel. Assuming that the air resistance remains the same and that the driver does not

apply the brakes, find, to two decimal places, how far up the plane the car will climb before coming to rest, given that the coefficient of friction between the plane and the tires of the car is 0.1175.

ii) A particle moves in a horizontal circular path of radius 2m with uniform angular acceleration. It is observed to make two revolutions in the first 4 seconds of motion and 4 revolutions in the next 4 seconds. Find

- the initial angular velocity of the particle giving you answer in rads^{-1}
- the angular acceleration of the particle giving your answer in rads^{-1}
- the total distance in meters covered in 8 seconds

6. A particle is projected with speed u at an angle of elevation θ from a point O on a horizontal plane. Show that the equation of trajectory referred to horizontal and vertical axes O_x and O_y respectively, is

$$Y = x \tan \theta - \frac{gx^2 \sec^2 \theta}{2u^2}$$

Given that $u = 20\sqrt{5} \text{ms}^{-1}$ and that the particle passes through a point which is 100m from O horizontally and 50m vertically above the level of O , show that there are two possible angles of projection. If the angles are α and β show that $\tan(\alpha + \beta) = 2$.

For the smaller angle, calculate

- The range of the particle on the horizontal plane through the point of projection,
- The time of flight of the particle.

7. Two small smooth spheres A and B of masses $3m$ kg and m kg respectively, are moving towards each other on a smooth horizontal floor. A is moving with a speed of $2u \text{ms}^{-1}$ in a direction perpendicular to a smooth vertical wall and B is moving with a speed of $u \text{ms}^{-1}$ away from the wall.

A strikes B directly and at a distance of 6 meters from the wall. Given that the coefficient of restitution between A and B and between B and the wall is $\frac{1}{4}$,

Find,

- The velocity of A and B after the first impact
- The kinetic energy, in terms of m and u , lost in this impact
- The distance, from the wall, of the point at which the second impact of the spheres takes place,
- The time, in terms of u , between the two impacts of the spheres.

8. A box contains only 4 red balls and 5 white balls. Three balls are drawn at random, successfully and without replacement from the box. Draw a tree diagram illustrating the various possibilities.

Hence, or otherwise, find the possibility that

- Exactly one white ball is drawn,
- No one white ball is drawn,
- At least one white ball is drawn
- At least one red ball is drawn, given that at least one white ball is drawn.

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