June 2008

l. The position vector of a particle P, at time t seconds is r, where

$$r = (ti + j + t^2 k)m$$

a) Show that the acceleration of the particle is constant.

b) Find the cosine of the angle between the velocity vector and the position of the particle when $t = 1$

The velocity of another particle Q relative to P is $(i - j)ms^{-1}$. Given that when $t = 0$, $PQ = jm$, find the equation of the path of Q.

Find also the time at which Q is nearest to P.

2. A particle with speed $20\sqrt{5}$ ms^{-1} from the top of a cliff so that its hits a target which is 200 m horizontally from the foot of the cliff and 200m vertically below the horizontal line through the point of projection. Show that the two possible directions of projection are at right angles and find the corresponding times of flight.

Using the angle of projection whose value lies the first quadrant, find the maximum height above the point of projection attained by the particle, giving your answer correct to two decimal places.

(Takes g as $10ms^{-2}$)

3. Two forces, F_1 and F_2, where

$$F_1 = (i + 2j + 3k)N \text{ and } F_2 = (2i + k)N$$

Act at points with position vectors $(2i - 5j + ck)m$ and $(Si + cj + 2k)m$ respectively.

Given that the lines of action of F_1 and F_2 intersect at point T, find

a) The value of the constant c and the position vector of T,

b) The magnitude of the resultant of F_1 and F_2,

c) The equation of the line of action of the resultant of F_1 and F_2.

4.

i) A uniform bar AB of weight W newtons and length $2a$ meters rests with its upper end A against a rough vertical wall and the lower end B on rough horizontal ground. The coefficient of friction between the bar and the wall and between the bar and the ground is μ Given that the bar is just about to slip when it is inclined at 45° to the horizontal, show that $\mu = ½$

ii) A block of mass 3 kg rests on a smooth plane inclined at an angle θ to the horizontal, where $\sin\theta = 2/3$. The block is connected by a light inextensible string parallel to a line of greatest slope of the plane. The string passes over a smooth fixed pulley at the plane to another particle B of mass 5 kg hanging freely. The system is released from the rest.

Find

a) The velocity of the system when B has descended through a distance of 2 m, given that it does not hit the ground,
b) The magnitude of the force exerted on the pulley by the string.

(Take g as $10ms^{-2}$)

5. Three spheres A, B and C of equal radii but of masses 2m, 3m and 4m respectively, rest in that order, in a straight line, on a smooth horizontal floor. A is projected with speed u so as to strike B which subsequently impinges on C, both impacts being direct. Given that the coefficient of restitution between A and B is 2/3 and between B and C is ½ , find.

a) The impulse experienced by A in its collision with B,
b) The kinetic energy lost due to the collision of A with B,
c) The speed in terms of μ acquired by c .

6. A train is uniformly retarded from $25/3\ ms^{-1}$ to $25/9\ ms^{-1}$.The magnitude of the acceleration is half the magnitude of the retardation. The train takes 450 seconds to cover a total distance of 2 km. By sketching a velocity time graph or otherwise find the distance travelled at $25/9 ms^{-1}$.

i) A particle moves along a straight line so that at time t seconds its velocity $v\ ms^{-1}$ and its displacement, s metres, from a fixed on the line are given by

$$v = (1 + 2S^2/S) \text{ and } s = 3t + 2$$

Find the acceleration of the particle when s = 3.

7.

i) The non-gravitational resistance to the motion of a car of mass 1000 kg is proportional to the square of the speed of the car. When the engine is working at 60 kW, the maximum speed of the car on a level road is $25ms^{-1}$. When the car ascends a plane inclined at an angle θ to the

horizontal, with its engine still working at 60 kW, the maximum speed is 20ms^{-1}. Find, correct to 3 decimal places, the value of sin θ.

(Take g as 10ms^{-2})

i) A ship A travelling due North at 30 $Kmh^{\sim 1}$ observes another ship B on its radar screen. Ship B is 5 km due East and appears to be travelling 40kmh^{-1} on a bearing of 210°. Find the actual velocity of ship B and the shortest distance between the two ships, giving your answer correct to 2 decimal places.

i) A man usually travels to Yaounde from Douala by air (34), by bus (#) or by train (T). When he travels by air, bus or train, the probability that he will have an accident is 1/5, 3/5, and 1/10 respectively.

Given that the man is travelling to Yaounde, find the probability that

a) Fie will be involved in accident
b) Fie was travelling by air given that he is involved in an accident,
c) Fie will arrive safety, given that he is travelling by bus

ii) 10% of a large consignment of mangoes is known to be bad. Three mangoes are chosen at random from this consignment.

Calculate the probability that

d) All will be bad
e) None will be had
f) At least one will be