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1. At time t seconds, the velocities v_1, v_2 of two particles P_1,P_2 respectively, are given by

   $V_1 = (2ti - 3t^{i\,1\,2}j)$ ms^{-1}

   v_2 - $[t^3i + (21 - 3)jj]$ ms^{-1}

   a) Find the non-zero value of t for which the accelerations of P_1 and P_2 are perpendicular.

   b) Obtain the velocity of P_1 relative to P_2 when their accelerations are perpendicular.
   Given that P_2 and P_2 are at the origin when t = 0, find

   c) The distance between P_1 and P_2 when their accelerations are perpendicular.

2.

i) Find the abscissa of the centroid of the finite region bounded by the curve y = In x and the lines y = 0 and $x = 2$, giving your answer correct to 2 decimal places..

ii) A uniform rod AB of weight W, rests in equilibrium with the end A against a smooth vertical wall and the end B on rough horizontal floor. Given that AB makes of 45° with the floor, find, in terms of W, the forces exerted on the rod at A and B. Hence show that if μ is the coefficient of friction between the rod and the floor, then ≥ ½

3.

i) A particle starts from rest at pint 0 and moves in a straight line with a constant acceleration of 2 ms^{-2}. Three seconds later, a second particle starts from rest at the same point 0, and moves after the first particle with constant acceleration 4 ms^{-2}. Find how much time elapses before the second particle overtakes the first.

ii) The magnitude of the retardation to the motion of a particle moving along the x — $axis$ is $1/2v^2$ ms^{-2}, where v ms^{-1} , is the speed of the particle. At time t =0, the particle passes through the origin at a speed u ms^{-1} in the direction of increasing x. Show that the speed of the particle is halved when t = 7u^2/12 and find, in terms of u, the value of x at this instant.

4. Referred to horizontal and vertical $axes\ OX$ and OY respectively, a particle is projected from the point 0 with a velocity of 65 ms^{-1} at an angle elevation of cos^{-1} (12/13)

   a) Show that the particle passes through the pint 4(60, 20) and $B($ 120, 30).

   b) Find the velocity of the particle B and the further horizontal distance covered before the particle is again at the level B.

(Take g as 10 ms^{-2})

5. Two particles *P and Q,* of mass 2*n and 5m* respectively, are connected by a light inextensible string. Particle *P* lies on a horizontal table and the string passes over a small smooth fixed pulley at the edge of the table, so that particle *Q* hangs freely at the other end of the string, at a distance *2l* above the ground.

The coefficient of friction between *P* and the table is ½ and the frictional force between the string and the table is negligible. The system is released from rest, with both parts of the string taut and the hanging part vertical. Find

a) The tension in the string and the acceleration of the system,

i) The magnitude and direction of the force exerted o the pulley.

Show that

c. The speed of *Q* just when it has descended a distance *l* is $\sqrt{\frac{8g}{7}}$ and that the time taken is $\sqrt{\frac{7l}{2g}}$

Just after *Q* has descended through a distance *l* the string is cut.

Find

d) The speed at Which Q eventually strikes the ground, giving your answer in trems of *g and l.*

[assuming that *P* does not reach the edge of the table.]

6. a) Three spheres S_1,S_2 *and* S_3 of each of mass m, lie in a straight line with S_3 *between* S_1 *and* S_3, S_1 impinges directly on S_2 with speed u *and* S_2 then impinges directly on S_3. The coefficient of restitution is *e* for all impacts involved. Calculate in terms of u *and e,* the speeds of

ii) S_1 *and* S_2 after their impact

iii) S_2 *and* S_3 after their impact

Show that the loss in kinetic energy during the impact between *and* S_2 is ¼ $mu^2(1-e)(1+e)$

7. The constant non-gravitational resistance to the motion of a car of mass 1500 kg is *R N*. The engine of the car works at a constant rate of 20 *kW.* Given that the maximum speed of the car up a plane of inclination $\sin^{-1}$ (1/10) to the horizontal is 10 mas^{-1}, find

a) The value of R ?

b) The maximum speed of the car on level ground.

Find also the speed of the car when it is accelerating at the rate of 2 ms^{-2}

c) Down the plane

d) Up the plane

e) On the plane

(Take *g as* 10 ms^{-2})

8.

i. Three balls are, to be drawn at random and without replacement from a bag containing 16 balls of which 4 are red. Find the probability that

a) All three balls will be red,

6. None of the three balls will be red,

b) At least one of the balls will be red,

c) Exactly one of the balls will be red.

ii. A die with faces numbered 1 to 6 is biased so that the probability of any even face is twice the probability of any odd face. The die is to be thrown twice. Find

d) The probability that the total score will exceed 10.