June 2014

1. A particle of mass 3kg moves under the action of a force $F$ such that its position vector at time $t$ seconds is given by $F$ = (2t +1 )i + $t^3j + \frac{4}{3}t^3k.$

Find, when t = 3,

a) The kinetic energy of the particle,

b) The magnitude of the force $F$,

c) The power developed by the particle.

Find, also,

d) The work done by the force in the interval $1 < t < 4$,

e) The cosine of the angle between the velocity and acceleration vectors of the particle when $t = 1$.

2. A sphered, of mass 2m, moving speed 2$u$ on a smooth horizontal plane, collides directly with another sphere $B$ of radius and of mass $m$ which is moving with speed $u$ in the opposite direction. Given that the coefficient of restitution between the spheres is ½ , find

a) Their speeds after the collision,

b) The magnitude of the instantaneous impulse,

c) The loss in kinetic energy caused by the collision,

After a short interval, the sphere $A$ is given a horizontal impulse of magnitude 7$mu$ so that it collides again directly with sphere $B$. Find the speed of $A$ and the speed of $B$ after second impact.

3.

i) A uniform ladder, of weight and length $2l$, rests with is upper end against a smooth vertical wall and its lower end on a rough horizontal ground. The coefficient of friction between the ladder and the ground is ½ Given that the ladder is in limiting equilibrium, find the angle which the ladder makes with the horizontal.

ii) A particle of mass $m$ kg is projected vertically upwards with speed $u$ $ms^{-1}$. The resistance to the motion of the article is of magnitude $mkv$, where $k$ is apositive constant and $v$ is the speed at time $t$ seconds. Find the velocity of the particle at time $t$ seconds.

4. i) Forces $F_1$, $F_2$ and $F_3$ act at point vectors $r_1$, $r_2$ and $r_3$ respectively, where

$F_1$= (2$i$ -$j$)$N$, $r_1$ = ($i$ 4- 3$j$)$m$ ,$F_2$ = (-3$i$ + 5$j$)$N$, $r_2$ = (2$i$ - $j$)$m$, $F_3$ = ($i$- 4$j$)$N$, $r_3$ =(3$i$ + 2$]$)$m$

Show that this system of three forces forms a couple, and find the magnitude of the couple.

i) Two particles $P$ and $Q$ have velocities of (3$i$ + 4j) $ms^{-1}$ and (—4$i$ + 2)$ms^{-1}$ respectively. Initially, the position vectors of $P$ and $Q$ are (13i — 3$j$)$m$ and (12$i$ + 5$j$)$m$ respectively. Find the distance between them at any time t. Hence find, to two decimal places, the least distance between them.

5. A car of mass 9000kg pulls a carriage of mass 600kg. There is a total non gravitational resistance of 500N and this is divided between the car and the carriage in the ratio of their masses.

The engine of the car produces a constant pull and the carriage accelerates from a speed of 8$ms^{-1}$ to a speed of speed 12$ms^{-1}$ distance of 20m.

Find the magnitude of

(a) The tractive force of the engine of the car,

(b) The tension in the tow-bar when the motion takes place on level ground.

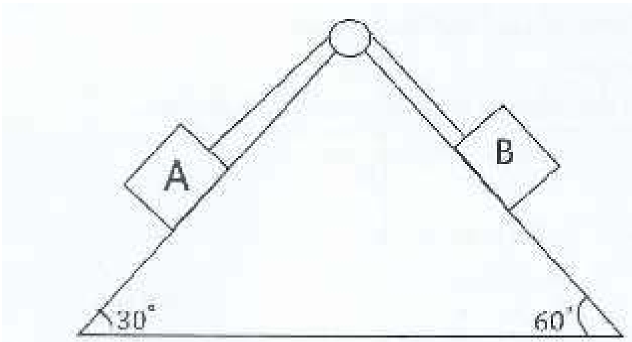
ii) One end of light elastic string of natural length $6$m is attached to a fixed point A and a mass 2kg is attached at the other end B. A horizontal force of magnitude $F$ *newtons* is applied to the particle so that it is at rest with string taut and inclined at 30° to the horizontal. Given that the vertical distance from point A is 5m, find

(a) The value of F,

(b) The modulus of elasticity of the string.

(*Take g as* $10$ms$^{-2}$)

6.




Two particles $A$ *and* $B$, of masses $2m$ *and* $6$m, rest on the smooth and rough inclined faces respectively of a fixed wedge as in Fig. 1. They are connected by a flight inextensible string passing over a smooth pulling fixed at the top of the wedge. The smooth face of the wedge is inclined at angle 30° to the horizontal while the rough face is inclined at angle 60° to the horizontal. The system is released from rest with the string taut. Given that the coefficient of friction between $B$ and the plane is 2/3 show that

a) the acceleration of the particle is [illegible]$(\sqrt{3} - 1)$ ms$^{-2}$

b) the tension in the string is $\frac{m}{4}(3\sqrt{3} + 1)$N

c) Find, in terms of $g$ and m, the force exerted by the string on the pulling.

The particle $B$ hits the ground 2 seconds after the system is released from rest and does not rebound from the ground.

d) Show that the further distance which $A$ travels before first coming to momentary rest is

$g[4/3 (\sqrt{3} -1 )]^2$

8. i) $ABCD$ is uniform rectangular plate of mass 6m. The sides $AB = CD = 3a$ and $AD = BC = 4a$.
   Particles of masses 3m, 2m *and* 5m are attached at the vertices $B$, $C$ and $D$ respectively. Find the distance of the centre of mass of the loaded plate from
   a) The side $AD$
   b) The side $AB$.

The vertex $D$ of the loaded plate is freely hinged to fixed point and the place hangs at rest in equilibrium.

   c) Find the angle between $DC$ and the downward vertical,

ii) A compact disc, spinning at a constant angular acceleration, spins 5 revolutions in the first second and 10 revolutions in the next second. Find the initial angular velocity, in *rad* $s^{-1}$, of the compact disc.

. 8.

i) Three random events $A$, $B$ and $C$ are such that $P(A) = 1/5$, $P(A \cup C) = 19/60$ *and* $P\{B \cap C) = 1/24$
   Events $B$ and $C$ are independent while events $A$ and $B$ are mutually exclusive.
   a) Find $P(B)$ *and* $P\{A \cup B)$ .
   b) Show that events $A$ *and* $C$ are independent.

ii) A research is carried on the existence of a disease in a certain population. It is assumed that 10% of the population has the disease. To verify this assumption, a test is conduced. It is found out that a person assumed to have the disease has 75% chance of the test being positive and a person assumed not to have the disease has a 5% chance that the test will be positive. Draw a tree diagram to illustrate this information. Hence find, the probability that
   c) A person has the disease and test positive
   d) The test is positive,
   d)A person has the disease, given that the test is positive.