5. (i) Show that $\dfrac{\sin 6A}{\sin 6A} - \dfrac{\cos 2A}{\sin 2A} = \cot 4A$.

(ii) Given that

$$f(\theta) = 2\cos\theta + \sin\theta,$$

express $f(\theta)$ in the form $R\cos(\theta - \lambda)$, where $R > 0$ and $0 < \lambda < \frac{\pi}{2}$.

Hence obtain the general solution of the equation

$$f(\theta) = 1.$$

(10 marks)

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6. (i) Prove, by mathematical induction, that

$$4^n - 4n - 1$$

is divisible by 16,

for all positive integers n.

(ii) A and B are members of a group of 10 boys.

(a) Find the number of ways in which all the boys can be seated in a row if A and B must sit together.

(b) Find the number of ways in which a committee of 5 can be selected from the group if either A or B but not both must be a member of the committee.

(9 marks)

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7. (i) Factorise $f(x)$ completely, where

$$f(x) = -3x^3 + 5x^2 + 4x - 4.$$

(ii) Given that the roots of the quadratic equation

$$x^2 + (6 - \alpha)x + 2\alpha = 0$$

are real and equal, find the values of the real constant α. Using the smaller value of α, form a quadratic equation whose roots are $-\alpha - 2$ and $-\frac{1}{2}\alpha$, where α is the root of the equation

$$x^2 + (6 - \alpha)x + 2\alpha = 0.$$

(10 marks)

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8. Given that

$$g(x) = \frac{2}{(1 + x)(1 + 3x^2)},$$

(a) express $g(x)$ in partial fractions.

Hence,

(b) evaluate,

$$\int_0^1 g(x)\,dx,$$

(c) expand $g(x)$ as a series in ascending powers of x, up to and including the term in x^3.

(9 marks)

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1. (i) The functions f and g are defined by

$$f: x \to \frac{x+2}{x-1}, \quad x \in \mathbb{R}, \ x \neq 1$$

$$g: x \to 1-x, \quad x \in \mathbb{R}.$$

(a) Find $fg(x)$ and $gf(x)$.
(b) Show that the equation

$$fg(x) - gf(x) = 0,$$

has no real roots.
(c) Show that f is injective.

(ii) Find the coefficient of x^7 in the binomial expansion of $\left(\frac{x^2}{2} - \frac{2}{x^2}\right)^9$.

(12 marks)

2.

x	2	3	4	5	6
y	13.6	27.2	54.4	180.8	217.6

The table above shows corresponding values of x and y which approximately satisfy a relation of the form

$$y = ab^x,$$

where a and b are constants. One value of y is incorrect. By drawing a suitable linear graph, determine the incorrect value of y and estimate the values of a and b correct to one decimal place.

(11 marks)

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3. Given that

$$(x^2 - x)\frac{dy}{dx} = y,$$

and that $y = 1$ when $x = 2$, show that

$$y = \frac{2|x-1|}{x}.$$

Sketch the graph of

$$y = \frac{2|x-1|}{x}$$

showing clearly the point(s) where the curve meets the coordinate axes and the behaviour of the curve near its asymptotes.

(13 marks)

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4. Given the lines

$$r = (i + 2j - k) + s(i - 2j - 2k),$$
$$r = (6i + 6j + 6k) + t(5i - 4j + 3k).$$

find
(a) the point of intersection of the lines,
(b) the cosine of the acute angle contained by the lines,
(c) a vector parametric equation of a plane containing the lines.

(10 marks)

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9. (i) Given that $z = 2\sqrt{3}$, express

$$\frac{z}{z^*} \text{ and } z(\cos\tfrac{\pi}{4} - i\sin\tfrac{\pi}{4})$$

in the form $a + bi$, where a and b are real constants and z^* is the complex conjugate of z.

(ii) Find $\frac{d^2y}{dx^2}$ when $t = \frac{\pi}{3}$, where

$x = a(t - \sin t)$, $y = a(1 - \cos t)$

and a is a constant.

(9 marks)

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10. (i) Given that the line

$$x + 3y = 5$$

is normal to the curve

$$y = x^2 + 5x - 6$$

at the point C, find

(a) the coordinates of the point C,

(b) the equation of the tangent to the curve at the point C.

(ii) Find the set of real values of x for which

$$|3 - 2x| < |x + 4|$$

(9 marks)
