JUNE 2000

1i. Find an equation of the tangent to the curve $y = \ln(4 + x^2)$ at the point where $x = 1$.

ii. Solve the differential equation $\frac{dy}{dx} - xy - x = 0$, given that $y = 0$ when $x = 0$.

2. Given that f is a periodic function of period 5 and that $f(x) = \begin{cases} -x^2 - x + 6, 0 \le x < 2, \\ ax - 4, 2 \le x \le 5 \end{cases}$

a. find the value of a for which f is continuous.

b. find the value of f(-1).

(c) sketch the graph of y = f(x) in the interval $-3 \le x \le 7$

3i. Find n given that $\sum_{r=1}^{n}(5-2r) = -165$.

ii. The sum of the first n terms of a series is $9\left(1-\frac{1}{3^n}\right)$

a. Find the nth term of the series.

b. Show that the series is a geometric series and find its sum to infinity.

4. Sketch the curve $y = 3x^2 - 12x$ and hence find the area of the region bounded by the curve and the lines $y = 0$, $x = 0$ and $x = 5$.

ii. Find the volume generated by rotating completely the area bounded by the curve $x = 3(y^2 - 1)$ and the lines $x = 0$ and $x = 6$, about the x-axis. Find also the x-coordinate of the centroid of the solid generated.

5. The equations of two circles C_1 and C_2 are $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x = 0$, respectively.

a. Show that the circles C_1 and C_2 touch each other internally and find an equation of the common tangent at the point of contact.

b. Find the equation of the tangents to the circle C_1 which are perpendicular to the common tangent.

c. Find the area between the two circles.

6a. Write the complex number $\frac{1+i\sqrt{3}}{2-2i}$ in the form $R(\cos\theta = \sin\theta)$, where θ is in radians.

b. P_1 and P_2 are the points representing the complex numbers $3 + i$ and $-1 + 3i$ respectively. Show that OP_1 is perpendicular to OP_2, where 0 is the origin.

7. Given that the position vectors of the points A,B,C,and D relative to the origin O are a=3k, b=2i + 4j + 2k, c = 4i + 3j + k and d= 3i + 7j + 5k respectively, find

a. a vector equation of the line AB.

b. Cartesian equations of the line l through A and D.

c. A Cartesian equation of the plane π and the line l_1 to two decimal places.

8i. Show that $\cos(2\tan^{-1} x) = \frac{1-x^2}{1+x^2}$.

ii. Find, in radians, the general solution of the equation, $\sin 5x + \sin 3x = 0$.

iii. Express $7\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \lambda)$, where $R > 0$ and $0 < \lambda < 90°$, giving R and λ correct to one decimal place. Hence, find the value of θ in the interval $0° < \theta < 360°$ for which $7\cos\theta - 5\sin\theta = 8.6$.

9. Find (a) $\int xe^{x^2+1}dx$ (b) $\int_{0}^{\frac{1}{2}}\sqrt{\frac{x}{1-x}}\,dx$, using the substitution $x = \cos^2 t$

10i. A person sends messages by using six flags to which one is blue, two are white and three are red. He sends messages by hoisting the flags in a row, the message being conveyed by the order in which the colours of the flags are arranged. How many different messages can be sent by using (a) five flags only?