ii. Given that $x = e^{3t}$ and $y = e^{2t+3}$, find $\frac{dy}{dx}$ in terms of x.

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1. Relations R_1 and R_2 are defined on the set of integers by
xR_1y if and only if x + y is an even number.
XR_2y if and only if x + y is an odd number.
Show that R_1 is an equivalence relation and that R_2 is not.
The relation R_3 is defined on the set S, where S = {3,6,12,24}
by xR_3 y if and only if x is a factor of y. Show that R_3 is a strict order relation.

2.

X	2	3	4	5	6
Y	56.6	350.7	1280.0	3493.9	7963.3

The table shows corresponding values of x and y obtained experimentally. By drawing a suitable graph relating $\log_{10}$ x and $\log_{10}$ y, show that these values support the hypothesis that x and y are connected by a relationship of the form $y = px^q$, where p and q are constants. Use your graph to estimate the values of p and q to one decimal place. Hence, estimate the value of $\int_2^6 y\,dx$. Use the trapezium rule to obtain another estimate for $\int_2^6 y\,dx$.

3i. The angle between two vectors a and b is $\frac{\pi}{3}$. Given that |a| = 3 and |b| = 4, Find (a) |2a + 3b| (b) |2a – 3b|,

ii. Show that the lines $r = (1-\lambda)i + 3\lambda j + (2\lambda+5)k$, $r = 2\mu i + (\mu-4)j + 3k$, where μ and λ are parameters, intersect, giving the point of intersection.
Write down the vector parametric equation for the plane, which obtains these lines.

4i. Show that $\frac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$

ii. Find the general solution of the equation sin4x-cos2x = 0

5i. Differentiate $\frac{1}{x-3} + 2\ln\left(\frac{x-4}{x-3}\right)$ with respect to x and express your answer as a single fraction.

ii. Solve the differential equation $x\frac{dy}{dx} = 1 + y^2$, given that when x = 2, y = 0.

6. Express f(x), where $f(x) = \frac{x+x^2}{(x^2+1)(x-2)}$ in partial fractions and hence show that $\int_0^1 f(x)dx = \frac{3\pi}{20} - \frac{13}{10}\ln 2$.

7. Find, in the form x + iy, where $x,y \in \mathbb{R}$, the square roots of $\frac{169}{5-12i}$. Find, also in degrees to one decimal place, the principal value of the argument of each of the square roots.

8. Given that n is a positive integer, prove by induction, that 7n + 5 is divisible by 6. Guess a common factor of 5n + 3 for all positive integers n and prove your guess by induction.