

9. Find the set of real values of  $x$  for which (a)  $\frac{3x+4}{2x-3} < 1$  (b)  $\frac{3x+4}{2x-3} < 1$
10. The function  $f$  is defined by  $f(x) = \frac{3x-1}{(x-1)^2}, x \neq 1$ .
- a. Sketch the graph of  $f(x)$  showing clearly the coordinates of the turning point(s) and its behaviour as it approaches its asymptotes.
- b. Show that the area of the finite region bounded the curve and the lines  $x = 4$  and  $y = 2$  is  $\frac{5}{3} + 3 \ln \frac{2}{3}$ .

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- 1i. If the roots of the quadratic equation  $x^2 + bx + c = 0$  are  $\alpha$  and  $\beta$ . Find a quadratic equation whose roots are  $\alpha + 1/\beta$  and  $\beta + 1/\alpha$ .
- ii. Given that  $f(x) \equiv p^2 - 2x = 3p + 2$ , find the two values of  $p$  for which the equation  $f(x) = 0$  has real roots.
- iii. Show that for real values of  $x \neq 3$ ,  $\frac{x^2 + 2x - 11}{x - 3}$ , cannot take any value between 4 and 12.

- 2i. Find the maximum value of the function  $f$ , defined by  $f(x) = \frac{x}{1+x^2}, x \in \mathbb{R}$ .
- ii. Given that  $y = e^x \sin bx$ , where  $b$  is a constant, show that  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + (1+b^2)y = 0$ .
2. The function  $f$  is defined by  $f: x \rightarrow \frac{2x+1}{x-1}, x \in \mathbb{R}, x \neq 1$ . Find the function  $ff$ , stating its domain.

Another function  $g$  is defined by  $g(x) = x+2, 0 \leq x < 2$   
 $g(x) = x^2, 2 \leq x < 4$

$g$  is periodic of period 4. Sketch the graph of  $g$  for  $0 \leq x < 8$ .

Find  $g(-6)$ .

- 4i. At a party, 11 boys and 5 girls are to form 8 dancing pairs in such a way that each girls is paired with a boy. Find the number of dancing pairs that can be formed.

- ii. Find the set of real values of  $x$  for which  $\frac{x}{x-2} > \frac{1}{x+1}$ .

- 5i. Given that  $z = \frac{5}{2-i} - (3+2i)$ , express  $z$  in the form  $x + iy$ , where  $x$  and  $y$  are real numbers and find the values of  $|z^2|$  and  $\arg z^2$ .

- ii. Given that  $\frac{z}{1+2i} + \frac{z-1}{5} = \frac{1}{1-2i}$ , find real numbers  $p$  and  $q$  such that

$$(p + iq)z = 3 + 4i.$$

- 6i. Find the general solution of the equation  $\cos 2x + 1 = \sin 2x$ .

- ii. Given that  $f(\theta) \equiv \cos \theta - \sqrt{3} \sin \theta$ , express  $f(\theta)$  in the form  $R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ . Hence, find the minimum value of  $\frac{1}{1+f(\theta)}$ .

7. Show that the tangent at the point  $P$ , with parameter  $t$ , on the curve whose parametric equations are  $x = ct, y = c/t$ , where  $c$  is a constant, has equation