$x = t^2 y = 2ct$. This tangent meets the x-axis in a point Q and the line through P parallel to the x-axis cuts the y-axis at a point R. Show that the area of triangle QOR, where O is the origin is a constant.

8. Express $\frac{2}{(1+x)(1+3x)}$ in partial fractions. Hence solve the differential equation $\frac{dy}{dx} = \frac{2(y+2)}{(1+x)(1+3x)}$ given that y = -1 when x = 0.

9. Two lines are given by the equations $r = 17i + 9j + 9k + \lambda(3i + j + 5k)$, $r = 15i - 8j - k + \mu(4i + 3j)$, where λ and μ are scalar parameters.
Find (a) the position vector of their point of intersection.
b. The cosine of the acute angle contained by the lines.
c. A vector parametric equation of the plane containing the lines.

10i. Using the substitution $t^2 = x^2 + 3$, evaluate $\int_0^1 \frac{x\,dx}{\sqrt{x^2+3}}$.

ii. Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x\,dx$.

JUNE 2003

1. Express 4sin x – 3 cos x in the form R sin (x – a), where a is an acute angle and R is a positive real number. Find
a. All solutions of the equation 4sinx – 3cosx = 3, in the interval $0^\circ \le x \le 360^\circ$, given your answers to the nearest degree.
b. The greatest and least values of $\frac{1}{4\sin x - 3\cos x + 6}$.

2i. Differentiate sinx with respect to x from first principles.
ii. Find $\frac{dy}{dx}$ if (a) $y = (x \sin x)^2$ (b) $y = \ln\left(\frac{1+3x^2}{1-\tan x}\right)$.

3. With respect to the origin o, the points A,B C and D have position vectors (2i – j +k)m, (3i + j + k)m, (2j – k)m and (I – 2j – k)m respectively. Find
a. a vector equation of the line AB.
b. An equation of the plane BCD in the form r = a + sb + tc, where s and t are parameters. Show that the point A does not lie on the plane BCD.

4.

X	0.50	0.25	0.17	0.13	0.10	0.83	0.7
y	0.38	0.25	0.19	0.15	0.13	0.11	0.09

The table above shows corresponding values of x and y obtained experimentally. It is given that x and y are related by an equation of the form $\frac{1}{y} = \frac{1}{a} - \frac{b}{x}$, where a and b are constants. By drawing a suitable linear graph relating 1/y and 1/x, estimate the value of a and b correct to two decimal places.

5i. Show that the equation $x^5 - x^3 - 1 = 0$ has a root in the interval $1 \le x \le 2$. Use two iterations of the Newton-Raphson method to find an approximation to this root correct to 2 places of decimal.

ii. Car numbers in the ten provinces of Cameroon are in the form AD1323H, where the two letters at the beginning show the province in which the car is registered, the 4 digits following the two letters are taken from the digits 0 to 9 inclusive and the letter at the end is one of the 26 letters of the alphabet. Given that the 4 digits must not all be zero, find the maximum number of cars, which can be registered in Cameroon using this system.