6a. Prove by mathematical induction that for all positive integers n,

$$\sum_{r=1}^{n} 2r(r+1) = \frac{2n(n+2)}{3}$$

b. Given that x – 3, 2x + 1 and 7x – 2 are the first consecutive terms of an arithmetic progressive, find the value of x and hence obtain the sum of the first ten terms of the progression.

7. The graph of $y = \frac{a+bx}{(x-1)(x-4)}$ has a turning point at P(2, -1). Find the values of a and b and hence, sketch the curve y = f(x) showing clearly the turning points, asymptotes and intercept(s) with the axes.

8i. Given that $Z_1 = \sqrt{3} + I$ and $Z_2 = 1 - 3i$, find

a. Z_1/Z_2 in the form p + q where p and q are real.

b. Arg Z_1^5 and $|Z_2|^6$.

ii. Show that the roots of the equation $Z^3 - 1 = 0$ lies at the vertices of an equilateral triangle.

9a. Find $\int \frac{(x+2)dx}{(x+1)(x^2+1)}$.

b. Evaluate $\int_0^4 |x-2|dx$.

10i. Solve the inequality (x – 3)(x+2)(x+4) > 0.

ii. Find the range of values of x for which the series $\sum_{r=1}^{\infty}\left(\frac{x-3}{x-1}\right)^r$ converges.

b. Determine the value of x for which $\sum_{r=1}^{\infty}\left(\frac{x-3}{x-1}\right)^r = \frac{1}{2}$.

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1. Two quantities x and y, are known to be connected by a law of the form $y = a(x + 2)^n$, Where a and n are constants. Using the given table of values, plot log y against log (x +2) and hence find, to one decimal place, approximate values for n and a.

X	0	1	2	3	4
Y	1.06	0.58	0.38	0.27	0.20

2. The graph of the function f, where $f(x) = \frac{ax+b}{x^2+2}$, has a stationery value at the point (-1, 1). Find the values of the constants a and b. Show that this stationery value is a minimum and find the value of x for which the function has a maximum value. Sketch the curve y = f(x).

✓3i. Evaluate $\int \frac{3x^2+1dx}{(x^2+1)(x-1)}$, leaving your answer in terms of n and natural logarithms.

ii. Find the area of the finite region bounded by the curve y = x ln x, the x-axis and the line x = 2.

4. Verify that $a^4 + b^4 \equiv \frac{1}{2}[a^2 + b^2)^2 + (a^2 - b^2)^2]$. Hence show that

$$\frac{\cos^4 A + \sin^4 A}{\cos^4 A - \sin^4 A} \equiv \frac{\cos 2A + \sec 2A}{2}.$$

Find the general solution of the equation $\frac{\cos^4 A + \sin^4 A}{\cos^4 A - \sin^4 A} = 2$, giving your answer in degrees correct to one decimal place.