5i. One of the statements that follow is true and the other is false. Prove the true statement and give a counter-example to disprove the false statement.

a. For all two-dimensional vectors a, b, c, $a.b = a.c \Rightarrow b = c$.

b. For all positive real numbers a, b $a + b \geq \sqrt{ab}$.

ii. Prove, by mathematical induction, that for all positive integers n,

$$\sum_{r=1}^{n} r(2r+1) = \frac{1}{6}(n+1)(4n+5).$$

6i. Solve the inequality $x(x^2 - 6) \geq x$.

ii. Find the set of real values of x for which the series $\sum_{r=0}^{\infty} r\left(\frac{3x}{2+x}\right)^r$ is convergent.

7i. Find the modulus and argument of each of the complex numbers z_1, z_2 and z_3, where

$z_1 = (-1 + i\sqrt{3})((1 - i\sqrt{3})$, $z_2 = \frac{-1+i\sqrt{3}}{1-i\sqrt{3}}$, $z_3 = z_1^3$.

Mark on an argand diagram the points representing z_1 and z_2.

ii. Prove that for any complex number z, if $|z| < 1$, then $\text{Re}(z + 1) > 0$.

8. Show that the equation $f(x) = 0$, where $f(x) = 12x^3 + 16x^2 - 3x - 4$, has a root in each of the intervals $-2 < x < -1$, $-1 < x < 0$, $0 < x < 1$. Use the Newton-Raphson method with initial value – 1.5 to find two further approximations to the root in the interval $-2 < x < -1$, giving your final answer to two decimal places.

9. A football team of 11 players is to be selected from a short list of 16 players. Find the number of ways in which the team can be chosen if:

a. at most one of the three oldest players is to be included.

b. At least two of the four youngest players are to be included.

10. The position vectors of the point A,B C are (I + j +2k), (-1 + 2k), (3i + 3j – k), respectively, and P is the midpoint of BC. Find

a. an equation of the plane ABC in the form r = a + sb + tc,

b. an equation of the line AP in the form r = a + tb,

c. the cosine of angle APB.

JUNE 2005

a. A function $f: \mathbb{I}^+ \to \mathbb{I}^+ - \{1\}$ is defined by $f(x) = \frac{x}{x+3}$, $x \neq -3$.

a. Show that f is bijective.

b. Obtain the composite function ff(x) and find its inverse.

2i. The line y = x is tangent to the circle S at the point (4, 4) on S. The line y = 3x passes through the center of the circle. Find an equation of the circle S.

ii. Sketch the curve $y = \frac{1}{(x+1)(x-3)}$, showing clearly the turning point(s) and the behaviour of the curve near its asymptotes.

3. Show that the lines l_1 and l_2 with equations

l_1: $\frac{x-2}{-3} = \frac{y-1}{3} = \frac{z-3}{2}$ l_2: $\frac{x+1}{-4} = \frac{y-4}{2} = \frac{z-5}{7}$,

Intersect, giving the coordinates of the point of intersection. Find

a. the cosine of the angle between l_1 and l_2 leaving your answer in surd form.