b. the equation of the plane containing l_1 and l_2, expressing your answer in the form r = a + tb + sc.

4. Show that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. Hence or otherwise

a. Find all solutions of the equation $\cos 3\theta = \frac{1}{2}$, in the interval $0 \le \theta \le \pi$.

b. Solve the equation $4x^3 - 3x = ½$, giving your answer correct to 4 decimal places.

5. Evaluate :

a. $\int_0^1 \frac{x}{(x+2)(x+3)}dx$, leaving your answer in terms of logarithms.

b. $\int_0^1 xe^{x+2}dx$, leaving your answer in terms of e.

c. $\int_0^{\frac{\pi}{3}} \sin^3 x dx$

6. Find $\frac{dy}{dx}$, given that

a. $y = \arctan\left(\frac{1-x}{1+x}\right)$, simplifying your answer as much as possible.

b. $y = \ln (x^3 - 3x^2 + 6)^2$.

c. $x = \frac{2t}{1+t}$, $y = 3t^2 + 1$, leaving your answer in terms of the parameter t.

7. Express the complex number z, where

$z = \frac{-5+10i}{1+2i}$ in the form a+ib, where a, b $\in$ i.

Hence or otherwise, find

a. The modulus and argument of z,

b. The square roots of z.

8i. Solve the differential equation $y(1+x^2)\frac{dy}{dx} - 2(1+y^2) = 0$, given that y = 0 when x = 0.

ii. Find the range of values of x for which $(x-3)(x+4)(x+6) \le 0$.

9a. Prove, by mathematical induction, that $\sum_{r=1}^{n}(4r+3) = 2n^2 + 5n$.

b. Prove by the method of contradiction that $\sqrt{2}$ is not a rational number. (You may assume that the square of an odd integer is always odd).

10. Show that x = -1 is a root of the equation f(x) $\equiv$ 0, where $f(x) \equiv x^4 + x^3 + 3x^2 - 9x - 12$.

Show also that another root of this equation lies in the interval $1 < x < 2$.

Starting with 1.5 as a first approximation, use the Newton-Raphson procedure twice to find another approximation to this root, giving your answer correct to three decimal places.