6. A particle, P, of mass m, is projected vertically upwards from a point O with speed u in a medium which exerts a resisting force of magnitude mk$v$, where v is the speed of the particle and k is a positive constant. Find the time T taken for P to reach the highest point H of its path.

Show that

$$\text{OH} = \frac{1}{k^2}\left[uk - g\ln(1 + \frac{ku}{g})\right].$$

Find, in terms of k, g, and t, the speed of P at time t, after it reaches H. Hence, or otherwise, show that this speed tends to a finite limit as t increases indefinitely. Find this limiting value.

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7. (i) The probability density function f of a continuous random variable is given by

$f(x) = kx(4 - x)$, for $0 \le x \le 4$,

$f(x) = 0$, elsewhere,

where k is a positive constant.

(a) Evaluate k.

(b) Draw a sketch of $f(x)$ and give the mode of X.

(c) Calculate $(p(1 \le x \le 3)$.

(ii) A large mixture of tomato seeds consists of two strains A and B in the ratio 4 : 1. Seeds are chosen at random from the mixture and planted in rows with 10 seeds in each row. Given that all the seeds germinate, find the mean and the variance of the number per row of plants of strain B. Find, also, to 3 decimal places, an approximate value for the probability that in a total of 50 rows, there will be more than 110 plants of strain B.

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8. A smooth sphere A, moving on a smooth horizontal table, impinges obliquely with an identical sphere B at rest on the table. At the moment of impact the line of centres makes an angle 60^0 with the direction of A. Given that the coefficient of restitution between the spheres is e, show that sphere A is deflected by the impact through an angle θ, where, $\tan\theta = \dfrac{\sqrt{3}(1+e)}{7-e}$.

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1. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$ N and $\mathbf{F}_2 = (2\mathbf{i} + 2\mathbf{j} - 12\mathbf{k})$ N, act respectively at points with position vectors $(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$m and $(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$m.
Show that the lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ intersect at the point with position vector $(2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ m.
A force $\mathbf{F}_3 = (\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ N and another force $\mathbf{F}_4$ act through the point with position vector $(\mathbf{i} + \mathbf{j} - \mathbf{k})$m so that this system of four forces reduces to a couple. Find the magnitude of $\mathbf{F}_4$ and a vector equation of its line of action. Find, also, the magnitude of the couple.

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2. A particle P of mass m whose polar coordinates referred to an origin O are (r, θ), moves along a plane curve under the action of a force F, directed along **OP**. The straight line OP rotates about O with angular speed $\dot{\theta}$.
Prove that $r^2\dot{\theta}$ is a constant. Taking this constant as h and $|\mathbf{F}| = \dfrac{mk}{r^2}$, where k is a constant, show that

$$\ddot{r} = \frac{h^2}{r^3} - \frac{k}{r^2}.$$

At time t = 0 the particle is at a distance a from O and moving in a direction perpendicular to **OP**. Prove that, at any later time, the component of the velocity of the particle along OP is $\dot{r}$, where

$$\dot{r}^2 = \left(\frac{1}{r} - \frac{1}{a}\right)\left[2k - h^2\left(\frac{1}{r} + \frac{1}{a}\right)\right]$$

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3. A particle moves along the x- axis so that at time t seconds, its displacement x metres from the origin O, satisfies the differential equation

$$2\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \cos t - 5\sin t.$$

Find, in the form $a\cos t + b\sin t$, where a and b are constants, a particular integral of this differential equation.
Find, also, the solution of this differential equation for which $x = 0$ and $\dfrac{dx}{dt} = 0$ when t = 0.
Show that, when t is large, the motion of the particle is approximately simple harmonic with amplitude $\sqrt{2}$. Find the period of the motion.

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4. Prove that for the Poisson probability distribution, the mean is equal to the variance.
In a certain football leaque, it is noted after a long period of observation that the number of goals scored per match has a Poisson distribution with mean 3.
Calculate, to 2 significant figures, the probability that
   (a) more than five goals will be scored in a particular match;
   (b) at least two out of three matches will be without any goals scored.

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5. A smooth sphere of internal radius a, and centre O is fixed. A particle P of mass m, is projected with speed $\sqrt{(10ga)}$ vertically downwards from a point A on the inside surface of the sphere, where OA is horizontal.
When P reaches the lowest point of the sphere, it collides with and adheres to a stationary particle Q, also of mass m. Find the height above O of the point at which the combined particle loses contact with the sphere. Find, also, the greatest height above O reached by the combined particle in the subsequent motion.
