1. (i) Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle moving it from a point with position vector **a** to a point with position vector **b**.
Given that $\mathbf{F}_1 = (9\mathbf{i} - \mathbf{j} + 2\mathbf{k})$N, $\mathbf{F}_2 = (-\mathbf{i} + 15\mathbf{j})$N, $\mathbf{a} = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k})$m, $\mathbf{b} = (9\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$m, find the total work done on the particle.

(ii) The position vectors of points A, B, C, D of a rigid body, with respect to a fixed origin O, are $\mathbf{a} = (3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$m, $\mathbf{b} = (2\mathbf{i} + 4\mathbf{j} + \mathbf{k})$m, $\mathbf{c} = (-2\mathbf{i} + \mathbf{j} + \mathbf{k})$m and $\mathbf{d} = (-4\mathbf{i} - \mathbf{j} + 1\mathbf{k})$m, respectively. A force $\mathbf{F}_1$ of magnitude 20N acts along **CB**. A force $\mathbf{F}_2$ of magnitude 15N acts along **CD**. A third force $\mathbf{F}_3$ acts at A. This system reduces to a couple **G**.
Find,
(a) the magnitude of $\mathbf{F}_3$,
(b) the moment of G.
Show that the line of action of the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is given by

$$\frac{x+2}{6} = \frac{y-1}{2} = \frac{z-1}{5}.$$

2. (i) The table below gives the coordinates of points on a curve. Using Simpson's rule, estimate the volume generated by rotating the region under this curve in the range $2.4 \le x \le 4.8$ completely about the x-axis. (Give your answer to 3 significant figures).

x	2.4	2.8	3.2	3.6	4.0	4.4	4.8
y	5	5	3	6	7	7	4

(ii) Using the approximations

$$h^2\left(\frac{d^2y}{dx^2}\right)_n \approx y_{n+1} - 2y_n + y_{n-1}, \quad y_{n+1} \approx y_n + h\left(\frac{dy}{dx}\right)_n \text{ and } 2h\left(\frac{dy}{dx}\right)_n \approx y_{n+1} - y_{n-1}$$ and a step length of 0.2,

estimate the value of y, correct to 4 decimal places, when $x = 0.6$, given that $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$ and $y = 1$, $\frac{dy}{dx} = 0$, when $x = 0$.

3. A particle A of mass 5m is projected along a smooth horizontal plane with speed u. Another particle B of mass m moving with speed 3u impinges obliquely on A. At impact B is moving at an angle α to the line of centres and after impact moves at an angle 2α to the line of centres. Prove that the speed of B after impact is $\frac{3u\sec\alpha}{2}$.
Find the velocity of A and the value of e, the coefficient of restitution, in terms of α only.

4. The motion of a body of mass m is moving along a straight line against a constant resistance of magnitude λm and a variable resistance of magnitude $\frac{mv^2}{k}$, where vms^{-1} is the speed and λ and k are positive constants. Initially the body is moving with speed u.
Show that the body is brought to rest over a distance $\frac{k}{2}\ln\left(1 + \frac{u^2}{\lambda k}\right)$ and that the time taken is $\sqrt{\left(\frac{k}{\lambda}\right)}\tan^{-1}\left(\frac{u}{\sqrt{\lambda k}}\right)$.

5. A thin uniform rod of length $2a$ and mass m, is attached to a smooth fixed hinge at one end O and then allowed to fall from a horizontal position. Show that in the subsequent motion,

$$2a\left(\frac{d\theta}{dt}\right)^2 = 3g\sin\theta,$$

where θ is the angle the rod makes with the horizontal.
Find, in terms of m, g and θ, the force exerted at the hinge. Show that the horizontal force exerted on the hinge is greatest when $\theta = \frac{\pi}{4}$ and that the magnitude of the vertical force on the hinge then is $\frac{11}{8}$mg.

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6. (i) A continuous random variable X is normally distributed with mean 235 and standard deviation 20. Calculate, to 3 decimal places,
   (a) the probability that a randomly observed value of X will lie between 225 and 240,
   (b) the probability that the sum of five randomly observed values of X does not exceed 1100.

   (ii) A discrete random variable Y has a Poisson distribution with mean λ. Given that $P(Y = 2) = 3P(Y = 4)$, find
   (a) the values of λ,
   (b) $P(X \geq 2)$, giving your answer to 3 decimal places.

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7. Given that the path of a particle P moving on the xy-plane satisfies the vector differential equation.

$$\frac{d^2\mathbf{r}}{dt^2} + 9\mathbf{r} = 5\mathbf{j}\cos3t\,.$$

Find the value of the constant vector **a** given that **a** tsin3t is a solution of this differential equation. Find the solution of the differential equation given that initially P is at the point with position vector $\frac{1}{2}$j and moving with velocity 4i..

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8. A smooth circular hoop, centre O and radius a, is fixed with its plane vertical. A small smooth bead Q, of mass m, is threaded to the hoop. The bead is projected with speed $\sqrt{(ag)}$ vertically downwards from a point B on the hoop, where OB is horizontal. Show that if θ, $0 \leq \theta < \frac{\pi}{2}$, is the angle OQ has turned through in time t ,

$$a\left(\frac{d\theta}{dt}\right)^2 = g(1+2\sin\theta)\ .$$

When the bead reaches the lowest point A of the hoop, it collides and coalesces with a smooth small bead of mass 2m, which is also threaded on the hoop and is at rest. Find the height above A, to which the combined particles rise before coming to instantaneous rest. Find also the angle which OQ makes with OA at this instant. Fine the magnitude of the force exerted by the beads on the hoop when the beads first return to A.
