1. P, Q, R are points in a plane with position vectors $(i+2j)$m, $(2i-2k)$m, and $(2j-3k)$m respectively, relative to the origin O. Forces F_1, F_2, F_3 act along **PQ**, **QR**, **RP** respectively. Given that

$$|\mathbf{F}_1| = 3N, \quad |\mathbf{F}_2| = 6N, \quad |\mathbf{F}_3| = 3\sqrt{10}N,$$

find $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$ in terms of the unit vectors **i**, **j**, **k**.
Show that the system of three forces $\mathbf{F}_1, \mathbf{F}_2$, and $\mathbf{F}_3$ is equivalent to a single force **S** and find **S**. Find the vector equation of the line of action of **S**.

2. Two small smooth identical spheres A and B moving with constant velocity $\mathbf{U}_A$ and $\mathbf{U}_B$ pass simultaneously through two points whose position vectors are $\mathbf{a}_o$ and $\mathbf{b}_o$ respectively. Given that

$$\mathbf{U}_A = 4\mathbf{i}+\mathbf{j}+5\mathbf{k} \qquad \mathbf{a}_o = \mathbf{i}+4\mathbf{j}-26\mathbf{k}$$
$$\mathbf{U}_B = -\mathbf{i}+2\mathbf{j}-3\mathbf{k}, \qquad \mathbf{b}_o = 16\mathbf{i}+\mathbf{j}-2\mathbf{k}$$

and the coefficient of restitution between A and B is $\frac{1}{2}$,

(a) find the velocities of A and B after collision;
(b) show that if α is the deflection of B, then

$$\cos\alpha = \frac{25\sqrt{231}}{461}$$

3. (i) A particle P of unit mass moves along a straight line Ox such that the resistance to its motion is directly proportional to its displacement x from O. Show that its velocity , v, is given by
$$v^2 = k^2(a^2 - x^2),$$
where a is the maximum displacement and k is a constant.

(ii) A particle A lies 2 metres from a fixed point O in a smooth horizontal plane. It is subjected to a force of magnitude $\frac{1}{(2x-3)^3}$ per unit mass, which is directed away from O, and x is the distance of A from O. Show that in the ensuing motion

$$\frac{dx}{dt} = \frac{\sqrt{(2x-3)^2-1}}{\sqrt{2}(2x-3)}.$$

Find x in terms of t and prove that after 2 seconds, the speed of A is $\frac{2}{3}$ ms^{-1}.

4. (i) A particle of mass m is threaded to one end of an inelastic string of length $2l$. The other end of the string is attached to a fixed point A which is more than $2l$ above the floor. The particle is released from rest, with the string taut, from a point which is horizontal to A. When the particle is vertically below A, the string is held midway at O. Find

(a) the height of the particle above O, where the string slackens;

(b) the tension in the string when the particle is at a depth of $\frac{l}{2}$ below O.

(ii) A particle moves with constant angular velocity ω round the curve whose polar equation is $r = e^{k\theta}$, where k is a positive constant. Find in terms of θ and ω, the radial and transverse components of the acceleration of the particle when it is at the point (r,θ).

5. (i) Given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} + 20\frac{dy}{dx} - y^2 = x,$$

use the approximations

$$\left(\frac{d^2y}{dx^2}\right)_n \approx \frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} \quad \text{and} \quad \left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_{n-1}}{2h},$$

to show that

$$(10h+1)y_{n+1} \approx 2y_n + (10h-1)y_{n-1} + h^2(y_n^2 + x_n).$$

Deduce that when h = 0.1,

$$y_{n+1} = y_n + 0.005(y_n^2 + x_n).$$

Find the value of y when $x = 0.3$, given that y = 1 when $x = 0$. (work throughout to 3 decimal places)

(ii) Given that $\frac{dy}{dx} - \frac{y}{2x} = y^3$, and that $y = 1$ when $x = 1$, find y as a series of ascending powers of x up to and including the term in x^2. Use your result to find y, to 4 decimal places, when $x = 0.98$.

6. (i) A farmer has three tractors on hire on a daily basis. Assume that no demand is rejected when a tractor is available and that the daily demand for the tractors has a Poisson distribution with mean 2. Find, to 4 decimal places

(a) the probability that the farmer receives at least 2 demands for tractors on a given day,

(b) the probability that exactly 5 demands for tractors are made in a period of three days,

(c) the mean number of tractors hired a day.

(ii) A continuous variable X has the probability density function f given by

$$f(x) = \begin{cases} kx(3-x), & 0 \le x \le 3, \\ 0, & \textit{elsewhere.} \end{cases}$$

Find the value of the constant k. Find also, the mean and the variance of the distribution.

7. (i) Some study indicates that in 1981 when HIV/AIDS was discovered, approximately 1 out of 100 persons on earth was infected with the virus. It is further estimated that the spread of this virus is directly proportional to the number x of infected persons. In the year 2001, it was estimated that 1 out of every 14 persons is infected. Treating x as a continuous variable and assuming that the factors contributing to the spread of the virus are unchanged, show that the entire human population will be infected before the year 2028. (Assume the human population remains constant).

(ii) The velocity **v** of a particle P satisfies the vector differential equation

$$\frac{d\mathbf{v}}{dt} + 2\mathbf{v} = \mathbf{0}.$$

At time t = 0, the position vector and the velocity of P are (**i** +**j**) and −4(**i** − **j**), respectively. Find an expression for the position vector **r**, of P, at time t. Deduce that when $t = \frac{1}{2}\ln 2$, **r** = 2**j** and **v** = -2(**i** − **j**).

8. A uniform rod of length $3a$ and mass m carries a particle P of mass $2m$ at one end and the other end is smoothly pivoted on a horizontal axis. The rod is held horizontally and is then released. Prove that when the rod makes an acute angle θ, with the horizontal

$$7a\left(\frac{d\theta}{dt}\right)^2 = 5g\sin\theta$$

identical of mass m

When in a vertical position, P collides and coalesces with an identified particle, which is at rest. Find the maximum value of ϕ, the angle which the rod makes with the downward vertical. Further show that

$$39a\left(\frac{d\phi}{dt}\right)^2 = 27g\cos\phi$$