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Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act at points $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively, where

$\mathbf{F}_1 = (-8\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$N, $\quad \mathbf{r}_1 = (5\mathbf{i} - 6\mathbf{j} - \mathbf{k})$m

$\mathbf{F}_2 = (-4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$N, $\quad \mathbf{r}_2 = (\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$m

Show that $\mathbf{F}_1$ and $\mathbf{F}_2$ are concurrent. When a third force $\mathbf{F}_3$ acting through $\mathbf{r}_3$ where $\mathbf{r}_3 = (-\mathbf{i} - 4\mathbf{j} - 4\mathbf{k})$m, is added to the system, it reduces it to a single force $\mathbf{R} = (6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$N.

Find $\mathbf{F}_3$.

By finding the sum of vector moments of the system of forces about the origin, O, show that $\mathbf{R}$ acts along the line with equation

$$\frac{x+29}{6} = \frac{-y}{2} = \frac{z+18}{3}.$$

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2. Two smooth spheres A and B, of equal radii and masses 3m and m, respectively, lie at rest on a smooth horizontal floor. Sphere A is given a speed u and it impinges obliquely with B. At impact the direction of motion of sphere A makes an angle θ with the line of centers of the spheres. Show that the speed of B after impact is

$$\frac{3}{4}(1+e)u\cos\theta,$$

where e is the coefficient of restitution between the spheres.

Given that the direction of motion of A turns through an angle ϕ due to the collision, show that

$$\tan\phi = \frac{(1+e)\tan\theta}{3-e+4\tan^2\theta}.$$

Deduce that the maximum value of the angle of deviation of A occurs when

$$\tan^2\theta = \frac{3-e}{4}.$$

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3. A particle, P, of unit mass, falls from rest under gravity in a medium which exerts a resisting force of magnitude (k^2gv^2)N, where v ms^{-1} is the speed of the particle and k is a constant. When $t = T$, $v = \frac{1}{4k}$ and the particle has fallen a distance X.

Show that

(a) $\quad 2gX = \frac{1}{k^2}\ln(\frac{16}{15})$,

(b) $\quad 2gT = \frac{1}{k}\ln(\frac{5}{3})$.

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4. A particle P, moves in a vertical circle along the smooth inner surface of a fixed hollow sphere of internal radius 2 metres and centre, O. The plane of the circle passes through O. The particle is projected horizontally with speed u from the lowest point of the sphere. Show that for P to make a complete revolution,

$$u^2 \geq 10g.$$

Given that $u = 2\sqrt{2g}$, show that P loses contact with the sphere when OP is inclined at an angle α to the upward vertical, where $\cos\alpha = \frac{2}{3}$.

Find the maximum height reached by P above the point of projection.

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5. (i) A river is 50 m wide and depth soundings are taken at 5 m intervals across the width of the river. The results are shown in the following table.

Distance from the banks (m)	0	5	10	15	20	25	30	35	40	45	50
Depth of water (m)	0	2	5	7	8	6	7	5	3	1	0

Use Simpson's rule with 11 ordinates to estimate to the nearest m^2, the area of the cross-section of the water in the river.

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(ii) Given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} = x^2 - y^2,$$

use the approximation

$$y_{n+1} \approx 2y_n - y_{n-1} + h^2\left(\frac{d^2y}{dx^2}\right)_n$$

and a step length of 0.5 to find y when $x = 1.5$ given that $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 0$

(work throughout to 4 decimal places.)

6. (i) In a mass production of a certain device, it is found that 5% are defective. Devices are selected at random and packed in boxes of 10. If one box is selected at random, calculate to 3 decimal places the probability that there will be

(a) exactly 2 defectives,

(b) more than one defective.

If 3 boxes are selected, calculate the probability that exactly one box will contain just 2 defectives.

(ii) A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} k(x+2)^2 & ,-2 \le x \le 0, \\ \frac{1}{2} & ,0 < x \le \frac{4}{3} \end{cases}.$$

Find

(a) the value of k,

(b) the cumulative distribution function F(x),

(c) $P(-1 \le X \le 1)$.

7. (i) The polar coordinates of a particle P moving in a plane are such that at time t

$$\frac{dr}{d\theta} = 16\cos\theta \text{ and } \frac{d\theta}{dt} = (1 - 4\sin\theta)^2.$$

If $r = 4$ when $\theta = 0$, show that the polar equation of the curve is given by

$$r = \frac{4}{1 - 4\sin\theta}.$$

Find th transverse component of the acceleration of P when $\theta = 0$.

(ii) Solve the vector differential equation

$$\frac{d^2\mathbf{r}}{dt^2} = -n^2\mathbf{r},$$

Where n is a constant, given that $\mathbf{r} = a$ and $\frac{d\mathbf{r}}{dt} = nb$ when $t = 0$.

8. A uniform rod AB, of length $2a$ and mass m, rotates in a vertical plane about a smooth horizontal axis through the end A. A particle of mass ,m is attached at the end B vertically below A. At time $t = 0$, the end B is given a horizontal speed $\sqrt{(6ag)}$. Given that the rod turns through an angle θ in time t, show that

$$8a\left(\frac{d\theta}{dt}\right)^2 = 3(1 + 3\cos\theta)g.$$

Find the angle the rod has turned through when it reaches its highest position in the motion.

Find the horizontal and vertical components of the force exerted by the axis on the rod when it is horizontal.