1. Solve the vector differential equations

   (a) $\frac{dr}{dt} = r$,

   given that when $t = 0$, $\mathbf{r} \cdot \mathbf{i} = 1$ and $\mathbf{r} \times \mathbf{i} = 2\mathbf{j} + 3\mathbf{k}$;

   (b) $\frac{d^2r}{dt^2} - 2\frac{dr}{dt} + r = 0$,

   given that $\mathbf{r} = \mathbf{i} - \mathbf{j}$ and $\frac{dr}{dt} = \mathbf{i} + \mathbf{j}$ when $t = 0$;

   (c) $\frac{d^2r}{dt^2} + 4r = 4\mathbf{j}\cos t$,

   giving the general solution, if the particular solution is of the form
   $a\cos t + b\sin t$.

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2. (i) Three forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ act at points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ respectively, where

   $\mathbf{F}_1 = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N, $\quad \mathbf{r}_1 = (\mathbf{j} - 3\mathbf{k})$ m,
   $\mathbf{F}_2 = (\mathbf{i} - 4\mathbf{j} + \mathbf{k})$ N, $\quad \mathbf{r}_2 = (3\mathbf{i} - 8\mathbf{j} + \mathbf{k})$ m,
   $\mathbf{F}_3 = (\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ N, $\quad \mathbf{r}_3 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

   When a fourth force $\mathbf{F}_4$ is added, the system is in equilibrium. Find $\mathbf{F}_4$ and a vector equation of its line of action. Find also the magnitude of the moment of $\mathbf{F}_4$ about the origin 0.

   (ii) A force $\mathbf{F}$ of magnitude 26 N acts along the direction of the vector $4\mathbf{i} - 3\mathbf{j} + 12\mathbf{k}$. A bead moves along a smooth wire from the point A to the point B, where

   $\mathbf{OA} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $\mathbf{OB} = 5\mathbf{i} - 24\mathbf{j} + 3\mathbf{k}$ under the influence of $\mathbf{F}$.

   Find the work done by $\mathbf{F}$.

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3. A smooth sphere A moving with speed u collides with a stationary identical sphere B. Just before impact the direction of motion of A is inclined at 45° to the line of centres of A and B. The coefficient of restitution between the two spheres is e.
   Find the speeds of A and B immediately after impact.
   Find also the loss of kinetic energy due to the impact.
   Show that the direction of motion of A after impact is inclined at an angle θ to the line of centers, where
   $(1 - e)\tan\theta = 2$.

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4. (i) A discrete random variable X has a binomial distribution with parameters 5 and 0.2. Find

   (a) $P(X = 3)$,
   (b) $P(X < 3)$, giving your answers to 4 decimal places.

   (ii) The marks Y in an examination are normally distributed with mean 20 and standard deviation 8.

   (a) Find the probability that a candidate scores a mark more than 30.
   (b) Given that $P(\alpha < Y < 24) = 0.6611$, find the value of α to the nearest whole number.

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5. Find the Taylor series expansion for the solution of the differential equation

$$\frac{dy}{dx} = x^2 - y,$$

in ascending powers of x up to and including the term in x^3, given that y = 1 when $x = 0$.
Find the value of y, to 3 decimal places, when $x = 0.1$.
Obtain another expansion for $y = f(x)$ in ascending powers of $(x - 0.1)$, giving the coefficients to 3 decimal places. Hence, find the value of y to 3 decimal places when $x = 0.2$.

6. A particle of mass m is projected vertically downwards in a medium in which resistance to motion is mkv, where k is a positive constant and v is the speed. Given that the initial speed is u, find the terminal speed of the particle. Show that the distance traveled by the particle in time T is

$$\left(\frac{u}{k} - \frac{g}{k^2}\right)(1 - e^{-kT}) + \frac{gT}{k}.$$

7. Show by integration that the moment of inertia of a uniform rod of mass M and length $2a$, about an axis perpendicular to its length and passing through one end, is $\frac{4}{3}Ma^2$. A uniform rod AB of mass m and length $2a$ has a particle of mass m attached to the end B. The rod can rotate in a vertical plane about a smooth axis through A. If the body is slightly displaced from the position in which B is vertically above A, show that, when the rod makes an angle θ with the upward vertical,

$$8a\left(\frac{d\theta}{dt}\right)^2 = 9g(1 - \cos\theta).$$

Find, the magnitude of the reaction at the axis when B is vertically below A.

8. (a) X is a random variable having a Poisson distribution given by

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \; x = 0, 1, 2, \ldots,$$

prove that the mean and variance of X has value λ.

(b) The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx(1-x), & 0 \le x \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find
- (i) the value of k,
- (ii) the mode of X,
- (iii) the mean and variance of X.