1. A car of mass M starts from rest and moves along a straight level road, with its engine working at a constant rate P. The total resistance to the motion of the car is proportional to the speed v of the car. If the maximum speed of the ca[r] on this road is u, show that, when the car is traveling at a speed v, the total resistance to its motion is $\frac{Pv}{u^2}$.

   Show, also, that the equation of motion of the car is
   $$M\left(\frac{v^2}{u^2-v^2}\right)\frac{dv}{dx}=\frac{P}{u^2}.$$ **(4 marks)**

   Show, further, that in order to raise the speed from rest to $\frac{3}{4}u$, the car travels a distance
   $$\frac{Mu^3}{4P}(-3+2\ln 7).$$
   Find the time taken to cover this distance. **(16 marks)**

2. (a) Using Simpson's rule, with seven ordinates, estimate the value of
   $$\int_0^3\sqrt{(1+x^3)}dx,$$
   giving your answer to 3 decimal places. **(11 marks)**

   (b) For the differential equation
   $$\frac{d^2y}{dx^2}+2x^2y=0,$$
   use the approximation
   $$y_{n+1}\approx 2y_n-y_{n-1}+h^2\left(\frac{d^2y}{dx^2}\right)_n,$$
   and a step length of 0.1, to find the approximate value of its solution y, when $x = 0.3$, given that $y = 1$, when $x = 0$ and $y = 1.11$, when $x = 0.1$. (Give your answer to 4 decimal places). **(9 marks)**

3. (a) A force **F**, acting in the direction of the vector $2\mathbf{i}+\mathbf{j}-\mathbf{k}$, displaces a particle from the point with position vector $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$ to the point with position vector $9\mathbf{i}-4\mathbf{j}+7\mathbf{k}$.
   Given that the work done on the particle is 15 joules, find **F**. **(8 ma[rks])**

   (b) Forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body at points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, respectively, relative to a[n] origin O, where
   $$\mathbf{F}_1=(3\mathbf{i}-\mathbf{j}+2\mathbf{k})\text{ N},\quad \mathbf{r}_1=(3\mathbf{i}-\mathbf{k})\text{ m},$$
   $$\mathbf{F}_2=(-\mathbf{i}-4\mathbf{j}+\mathbf{k})\text{ N},\quad \mathbf{r}_2=(2\mathbf{i}-4\mathbf{j})\text{ m}.$$
   Given that the body is in equilibrium, find $\mathbf{F}_3$.
   Show that $\mathbf{F}_3$ acts along the line
   $$\frac{x-3}{-2}=\frac{y}{5}=\frac{z+1}{-3}.$$ **(12 ma[rks])**

4. (a) A continuous random variable, X, has probability density function f, given by
   $$f(x)=\begin{cases}kx(9-x^2), & 0\le x\le 2,\\ 0, & \textit{elsewhere.}\end{cases}$$
   Find,
   (i) the value of the constant k,
   (ii) the mode of X,
   (iii) the cumulative distribution function F,
   (iv) $P(X < 1)$. **(13 m[arks])**

   (b) A point P moves in a plane such that its position vector, at time $t > 0$, is
   $$\mathbf{r}=ct\mathbf{i}+\frac{c}{t}\mathbf{j}.$$
   (i) Show that P moves on the curve $xy = c^2$.
   (ii) Find the acceleration vector of P and hence show that this acceleration is equal to $k(\mathbf{r}\cdot\mathbf{j})^3\,\mathbf{j}$, where k is a constant.
   (iii) Express k in terms of c. **(7 m[arks])**

5. A particle P, of mass m, is attached to one end of a light inextensible string of length r, whose other end O is fixed. When at rest vertically below O, P is given a horizontal speed of $\sqrt{3rg}$. Show that whilst the string remains taut, the tension in it, when OP has rotated through an angle θ, is of magnitude

$$mg(1 + 3\cos\theta).$$

Find,
(a) the value of $\cos\theta$ for which the string goes slack, **(13 marks)**
(b) the speed and the value of θ just when the string goes slack, **(2 marks)**
(c) the greatest height reached by P above the point of projection. **(5 marks)**

6. Prove, by integration, that the moment of inertia of a uniform rod AB, of mass m and length $3a$, about an axis through its midpoint perpendicular to the rod, is

$$\frac{3}{4}ma^2.$$

(6 marks)

The rod is free to rotate in a vertical plane about a horizontal axis through a point distant a from the end A.
Find,
(a) the radius of gyration of the rod about this axis,
(b) the period of small oscillations about its position of stable equilibrium,
(c) the length of the equivalent simple pendulum. **(14 marks)**

7. (a) A smooth sphere A of mass $2m$ is projected on a smooth horizontal plane with speed $2u$, to collide obliquely with a stationary sphere B of mass m and same radius as A. Just before impact, A is moving in a direction, making an acute angle α with the line of centres. After impact, A moves in the direction, making an angle 2α with the line of centres.

Prove that $e > \frac{1}{2}$, where e is the coefficient of restitution between the two spheres. **(13 marks)**

(b) A particle P moves along a straight line Ox such that at time t, its displacement from O is x, where

$$4\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 13x = 0.$$

Show that P performs damped harmonic motion and find the period of the motion. **(7 marks)**

There are two mobile intervention units in a certain town: unit A and unit B. The number of calls received independently at units A and B are Poisson variables with mean 3 calls a day and 4 calls a day, respectively.

Find the probability that on a particular day,
(a) more than three calls are received by unit A,
(b) a total of five calls are received by the two units,
(c) each unit receives exactly two calls.
(Give each answer to 4 d.p.) **(10 marks)**
(d) use a normal approximation to find the probability that in a given 5-day period, the total number of calls, received by the two units, lies between 28 and 40 inclusive.
(Give your answer to 2 d.p.) **(10 marks)**