1. Given the curve C such that

$$y = \sqrt{\left(1 - x^{\frac{2}{3}}\right)^3} \quad , \text{for } 0 \le x \le 1.$$

show that

(i) the length of the curve C is $\frac{3}{2}$.

(5 marks)

(ii) the point $\left(\sqrt{t^3}, \sqrt{(1-t)^3}\right)$ lies on the curve C.

(3 marks)

The curve C is rotated completely about the x – axis,
Find

(iii) the area of the surface of revolution obtained. **(7marks)**

(iv) using a theorem of Pappus, the y – coordinate of the centroid of the arc C. **(2marks)**

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2. (a) Using the substitution $y = vx$, where v is a function of x, show that the differential equation

$$x^2 \frac{dy}{dx} = xy - y^2$$

can be transformed into the form

$$x\frac{dv}{dx} + v^2 = 0.$$

Hence, find the general solution of the differential equation

$$x^2 \frac{dy}{dx} = xy - y^2$$

in the form $y = f(x)$

(6 marks)

(b) Given that $y = Ae^x\cos 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = e^x\left(\cos 5x - 10\sin 5x\right)$$

Find the value of the constant A.

Hence solve completely the differential equation given that y=3, $\frac{dy}{dx} = -4$ when $x = 0$. **(11 marks)**

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3. The points A, B and C have cartesian coordinates (2, -1, 4) (10, 7, 2) and (0, 0, 6) relative to the origin O.
Find

(i) **AB** × **AC** **(5marks)**

(ii) the area of triangle ABC **(2marks)**

(iii) the length of the perpendicular line from the point B to the line AC. **(2marks)**

(iv) the cartesian equation of the plane ABC. **(3marks)**

(v) the volume of the tetrahedron ABCD, given that the plane ABC cuts the x – axis at D. **(5marks)**

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4. (a) The polar curve C, has equation given by $C: r = \sqrt{3} + 2\cos\theta$
Find the tangents to the curve at the pole.
Sketch the curve C.

(4marks)

(b) Given that

$$f(x) = \frac{2x^2}{3-2x}, \quad x \neq \frac{3}{2}$$

(i) Show that $f(x)$ cannot lie between 0 and −6, for real values of x.
(ii) Sketch the curve $y = f(x)$, showing clearly the intercepts, turning points and the behaviour of the curve near its asymptotes.

(13marks)

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5. (a) Test whether or not the following series are convergent.

(i) $\sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}}$

(ii) $\sum_{n=1}^{\infty} \frac{3n}{\sqrt{2n^3+1}}$

(4 marks)

(b) Express $17\cosh x + 15\sinh x$ in the form $R\cosh(x + \ln a)$, where R and a are positive integers. **(4 marks)**
Hence, or otherwise
(i) Solve the equation
$17\cosh x + 15\sinh x - 40 = 0$.
(ii) Find the maximum value of
$$\frac{56}{34\cosh x + 30\sinh x + 12}$$

(9 marks)

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6. (i) Given that $x^2 - y^2 \neq 0$, find the inverse of the matrix A, where

$$A = \begin{pmatrix} x & 0 & y \\ 0 & 1 & 0 \\ y & 0 & x \end{pmatrix}$$

5/a

(ii) Show that the set S, of all matrices of the form **(5marks)**
$$\begin{pmatrix} a & 0 & b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}, a, b \in \mathbb{R}, a^2 \neq b^2$$
forms a group under matrix multiplication.

(iii) Given the sets M and N of matrices of the forms **(5marks)**
$$\begin{pmatrix} 0 & 0 & x \\ 0 & 1 & 0 \\ x & 0 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} y & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & y \end{pmatrix}, \text{ respectively, } x, y \in \mathbb{R} \text{ and } x \neq 0, y \neq 0$$

Determine with reasons, whether or not M and N are subgroups of S.
(Assume associativity of matrix multiplication) **(7marks)**

9 6

6

7. (a) Express $f(x)$, where

$$f(x) = \frac{2x^2+3}{(x^2+1)^2(x+2)^2}$$

in partial fractions.
Hence, or otherwise, show that

$$\int_1^3 f(x)dx = \frac{7}{60}$$

(8 marks)

(a) Given that

$$I_n = \int_{-1}^{1}(1+x)^n\sqrt{1-x}\,dx, n \geq 0$$

Show that

$$(2n+3)I_n = 4nI_{n-1},\ n \geq 1.$$

Hence find I_3.

(9 marks)

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8. (a) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $e^{2i\theta}$ and $e^{-2i\theta}$.
Hence, or otherwise, show that

$$16\sin^2 2\theta\cos^3 2\theta = 2\cos 2\theta - \cos 6\theta - \cos 10\theta$$

Evaluate

$$\int_0^{\frac{\pi}{3}} 16\sin^2 2\theta\cos^3 2\theta d\theta$$

(10 marks)

(b) A complex transformation T from the z – plane to the w – plane is given by

$$T: w = \frac{z-2}{z-i}$$

Show that the image of the circle $|z| = 2$ is also a circle in the w – plane and sketch it. **(7 marks)**

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9. (i) Prove that the equation of the tangent and normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point P($a\cos\theta$, $b\sin\theta$) are respectively

$$bx\cos\theta - ay\sin\theta = ab$$

and

$$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta.$$

(7 marks)

(ii) Given that the tangent cuts the x-axis at A and the y-axis at B and that the normal cuts the x –axis at C and the y – axis at D. Show that as θ varies the locus of the midpoint of CD is

$$4a^2x^2 + 4b^2y^2 = (a^2 - b^2)^2,$$

Given that $a = 5$ and $b = 4$, sketch this locus. **(10 marks)**

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