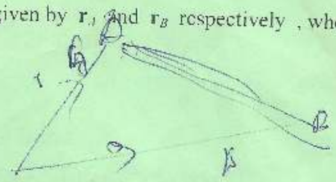
$\vec{AB} = \vec{OA} - \vec{OB}$

1. The position vectors of two particles A and B at time t seconds are given by $\mathbf{r}_A$ and $\mathbf{r}_B$ respectively , where

$$\mathbf{r}_A = [\, t^2\mathbf{i} + 2t^3\mathbf{j}\,]\ \text{m},$$
$$\mathbf{r}_B = [\, 3t^2\mathbf{i} - 4t^2\mathbf{j}\,]\ \text{m}.$$



(a) Calculate the distance between A and B when $t = 2$.

(b) Find the velocity of A relative to B when $t = 2$.

(c) Find the value(s) of t for which the velocities of A and B are perpendicular. $(V_A \perp V_B)$ $V_A \cdot V_B = 0$

(d) Find the value of t for which the accelerations of A and B are parallel.

2. (i)

$a_A = \frac{dv_A}{dt}$

$a_B = \frac{dv_B}{dt}$

$\underline{a_A \cdot a_B = |a_A||a_B|}$

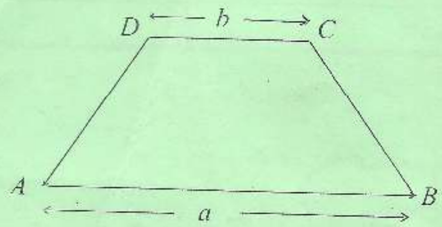


Figure 1

Figure 1 above shows a uniform lamina in the form of a trapezium in which AB and CD are parallel and of lengths a and b respectively. $(a > b)$. Prove that the distance of the centre of mass from AB is

$$\frac{1}{3}h\left[\frac{a+2b}{a+b}\right],$$

where h is the distance between AB and CD.

(ii) Forces $\mathbf{F}_1$, $\mathbf{F}_2$ act at the point with position vector $\mathbf{r}$, where

$$\mathbf{F}_1 = (4\mathbf{i} + 6\mathbf{j})\ \text{N},$$
$$\mathbf{F}_2 = (6\mathbf{i} - 8\mathbf{j})\ \text{N},$$
$$\mathbf{r} = (2\mathbf{i} - 3\mathbf{j})\ \text{m}.$$

Find the sum of the moments of $\mathbf{F}_1$ and $\mathbf{F}_2$ about the origin.

3. The forces $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$N and $\mathbf{F}_2 = (2\mathbf{i} + \mathbf{k})$N act at points whose position vectors are $(2\mathbf{i} + 5\mathbf{j} + c\mathbf{k})$ m and $(5\mathbf{i} + c\mathbf{j} + 2\mathbf{k})$ m respectively. Given that their lines of action intersect, find the value of c and the position vector of the point of intersection. Determine the vector equation of the line of action of the resultant of these forces and show that this line passes through the point with position vector $(7\mathbf{i} + 9\mathbf{j} + 8\mathbf{k})$ m.

4. A particle, P , of mass 4m kg is initially at rest on a smooth plane inclined at an angle α to the horizontal. It is supported by a light inextensible string which passes over a smooth light pulley A at the top edge of the plane. The other end of the string supports a particle Q , of mass 2m kg, which hangs freely. Given that the system is in equilibrium, find α and the magnitude and direction of the force exerted on the pulley by the string.

A further particle of mass M kg is now attached to Q and the system is released. Given that the acceleration of the system is $\frac{10}{3}$ m s^{-2}, find in terms of m , the value of M , the tension in the string and the magnitude and direction of the force exerted on the string by the pulley.

(Take g as 10 m s^{-2}.)

5. (i) A car of mass 1000 kg is moving on a smooth level road at a steady speed of $\frac{250}{9}$ m s^{-1} against an air resistance R. The engine works at a rate of 60 kW. Calculate the air resistance, R.
When the car starts to ascend a rough plane inclined at an angle $\sin^{-1}\left(\frac{1}{5}\right)$ to the horizontal with this speed, it runs out of fuel. Assuming that the air resistance remains the same and that the driver does not apply the brakes, find, to one decimal place, how far up the plane the car will climb before coming to rest, given that the coefficient of friction between the plane and the tyres of the car is 0.1175.

(ii) A particle moves in a horizontal circular path of radius 2 m with uniform angular acceleration. It is observed to make 2 revolutions in the first 4 seconds of motion and 4 revolutions in the next 4 seconds. Find
(a) the initial angular velocity of the particle giving your answer in rad s^{-1},
(b) the angular acceleration of the particle, giving your answer in rad s^{-2},
(c) the total distance, in metres, covered in the 8 seconds.

6. A particle is projected with speed u at an angle of elevation θ from a point O on a horizontal plane. Show that the equation of the trajectory referred to horizontal and vertical axes O_x and O_y respectively, is

$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}.$$

Given that $u = 20\sqrt{5}$ m s^{-1} and that the particle passes through a point which is 100 m from O horizontally and 50 m vertically above the level of O, show that there are two possible angles of projection.
If the angles are α and β, show that

$$\tan(\alpha + \beta) = 2.$$

For the smaller angle, calculate
(a) the range of the particle on the horizontal plane through the point of projection,
(b) the time of flight of the particle.

7. Two small smooth spheres A and B of masses 3m kg and m kg respectively, are moving towards each other on a smooth horizontal floor. A is moving with a speed of 2u m s^{-1} in a direction perpendicular to a smooth vertical wall and B is moving with a speed of u m s^{-1} away from the wall.
A strikes B directly and at a distance of 6 metres from the wall. Given that the coefficient of restitution between A and B, and between B and the wall is $\frac{1}{4}$, find
(a) the velocities of A and B after the first impact,
(b) the kinetic energy, in terms of m and u, lost in this impact,
(c) the distance, from the wall, of the point at which the second impact of the spheres takes place,
(d) the time, in terms of u, between the two impacts of the spheres.

8. A box contains only four red balls and five white balls. Three balls are drawn at random, successively and without replacement from the box. Draw a tree diagram illustrating the various possibilities.
Hence, or otherwise, find the probability that
(a) exactly one white ball is drawn,
(b) no white ball is drawn,
(c) at least one white ball is drawn,
(d) at least one red ball is drawn, given that at least one white ball is drawn.