10. Given that α and β are the roots of the equation

$$2x^2 - x + 4 = 0,$$

find, without solving the equation, an equation whose roots are α^2 and β^2.

(4 marks)

11. Find the positive constants a and b such that $4, a, 9$ are in a geometric progression and $4, a, b$ are in an arithmetic progression.

(4 marks)

12. Given that $y^3 + 2xy + x^2 = 0$, find $\frac{dy}{dx}$ at the point $(-1, 1)$.

(3 marks)

13. Find all values of x in the interval $0 \leq x \leq \pi$ for which

$$\cos x = \sin 2x.$$

(4 marks)

14. Solve the equation

$$\ln (x^2 - 15) = 0.$$

(2 marks)

15. Find the x-coordinate of the centroid of the solid obtained by rotating completely about the x-axis the finite region bounded by the curve $y^2 = 8x$, the x-axis and the line $x = 1$.

(4 marks)

16. An equation of a curve is given parametrically by

$$x = 2\cos\theta, \quad y = 3\sin\theta.$$

Find the cartesian equaion of the curve.

(2 marks)

17. Use the trapezoidal rule to find, approximately, the value of

$$\int_0^4 y\,dx,$$

given the following table:

x	0	1	2	3	4
y	3	5	6	4	3

(3 marks)

18. Find the coordinates of the point of inflexion on the curve
$y = xe^x$.

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(4 marks)

19. Find the number of different permutations of the 8 letters of the word CAUCASUS.
Find the number of selections of 3 different letters which can be made from the letters of the word CAUCASUS.

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(4 marks)

20. (*a*) Simplify $\frac{(r+1)!}{(r-1)!}$, where r is a positive integer.

(*b*) Evaluate $\sum_{r=1}^{10} r(r+1)$.

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(4 marks)

21. Given that θ is constant, find

$$\lim_{x \to 0} \frac{\sin(x+\theta) - \sin(\theta - x)}{2x}.$$

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(3 marks)

22. Find the coefficient of x^2 in the series expansion of $(1 + 3x)^{-\frac{1}{2}}$ in ascending powers of x.
State the range of values of x for which the series converges.

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(3 marks)

23. Solve the differential equation

$$(y - 2x^2y)\frac{dy}{dx} = x,$$

given that $y = 0$ when $x = 0$.

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(5 marks)

1. Find the values of x for which
$$2^{2x+1} - 3(2^x) + 1 = 0.$$

(4 marks)

2. Find the set of real values of x for which
$$|x-1| > 2.$$

(3 marks)

3. A real-valued function f is defined by
$$f(x) = x^2 + 6.$$
Show that f is not injetive and find the range of f.

(4 marks)

4. Express $\dfrac{i}{(2-i)^2}$ in the form $a+bi$, where $a, b \in \Re$.

(3 marks)

5. Given that $y = 4^{\sin x}$, find $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{6}$.

(4 marks)

6. With respect to the origin O, the position vectors of the points L, M and N are $4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ respectively. Find $\cos \angle LMN$.

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(4 marks)

7. A relation R is defined on $\mathbf{Z}$, the set of integers, by $a\,R\,b$ if and only if $a - b$ is even. Show that R is transitive.

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(3 marks)

8. Evaluate

$$\int_1^2 \frac{x^3}{1+x^4}\,dx.$$

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(3 marks)

9. State the equations of the three asymptotes of the curve

$$y = \frac{(x-1)^2}{(x-2)(x-3)}.$$

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(3 marks)