

1) Given that x is not an integer, then $[-x]$, the integer function, is equal to

- A) $[-x]$
- B) $-[x]+1$
- C) $[x]-1$
- D) $[-x]-1$

2) If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & x \neq 1 \\ \frac{1}{3}, & x = 1 \end{cases}$, then $f'(1) =$

- A) $-\frac{1}{9}$
- B) $-\frac{2}{9}$
- C) $-\frac{1}{3}$
- D) $\frac{1}{3}$

3) Let $g(x) = 1 + x - [x]$, where $[x]$ means the greatest integer less than or equal to x , and

$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ then for all x , $f[g(x)] =$

- A) x
- B) 1
- C) $f(x)$
- D) $g(x)$

4) The function $f(x) = x \sin \frac{1}{x}$, $x \neq 0$ is continuous at $x = 0$ provided $f(0)$ is defined as

- A) $f(0) = 0$
- B) $f(0) = \frac{1}{2}$
- C) $f(0) = 1$
- D) $f(0) = \frac{1}{\pi}$

5) If $f(x) = \cos x$, $0 \leq x \leq \frac{\pi}{2}$, then the real the mean value of $f(x)$ is

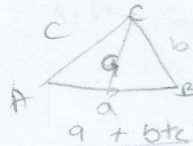
- A) $\frac{\pi}{6}$
- B) $\frac{\pi}{4}$
- C) $\sin^{-1}(\frac{2}{\pi})$
- D) $\cos^{-1}(\frac{2}{\pi})$

6) $\int \frac{1}{e^x-1} dx$

- A) $e^x - 1$
- B) $\ln(1 - e^x)$
- C) $\ln(e^x - 1)$
- D) $\tan^{-1}(e^x)$

- 7) If A, B and C are vertices of a triangle whose position vectors are a, b and c and G is the centroid of $\triangle ABC$, then $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} =$

- A) $\frac{a+b+c}{3}$
 B) $\frac{a+b+c}{2}$
 C) 0
 D) $a+b+c$



- 8) For non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}| |\vec{b}| |\vec{c}|$ holds if and only if

- A) $\vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0$
 B) $\vec{b} \cdot \vec{c} = 0, \vec{c} \cdot \vec{a} = 0$
 C) $\vec{c} \cdot \vec{a} = 0, \vec{a} \cdot \vec{b} = 0$
 D) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$

$$a \cdot b = c$$

$$abc = bca = cab$$

- 9) The volume of the parallelepiped represented by sides $i - 2j + 3k, 3i + j + 4k$ and $-2i + \mu j - k$ is 15, then the value of μ is

- A) 3
 B) 0
 C) -3
 D) 2

$$(i - 2j + 3k) \cdot (3i + j + 4k) \cdot (-2i + \mu j - k)$$

- 10) The differential equation of the family of curves $y = e^x(A \cos x + B \sin x)$ where A and B are arbitrary constants is

- A) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
 B) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 2y = 0$
 C) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = 0$
 D) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 2y = 0$

$$m^2 - 2 + 2$$

$$(m-1)(m-2)$$

$$m^2 + (m^2)$$

$$m^2 = 1 - m^2$$

$$m^2 + 2m - 2$$

$$m^2 + m^2 + 1$$

$$2m^2 = 1$$

$$m^2 = \frac{1}{2}$$

11) $\frac{x^2}{(x-1)^2(x^2+1)} =$

- A) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x^2+1}$
 B) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx}{x^2+1}$
 C) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$
 D) $\frac{Ax+B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$

$$x^2 \frac{A}{(x-1)(x-1)} + \frac{B}{(x-1)(x-1)} + \frac{Cx+D}{(x^2+1)}$$

12) $\sum_{n=1}^{\infty} \frac{1}{2n-1} x^{2n} =$

- A) $\frac{-n}{(1-\omega)^2}$
 B) $\frac{1}{2} \ln \left(\frac{1+x^2}{1-x^2} \right)$
 C) $x \ln \left(\frac{1+x}{1-x} \right)$
 D) $\frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$

13) If ω is the n^{th} root of unity, then $1+2\omega+3\omega^2+\dots$ to n terms is equal to

- A) $\frac{-n}{(1-\omega)^2}$
 B) $\frac{-n}{1-\omega}$
 C) $\frac{-2n}{1-\omega}$
 D) $\frac{-2n}{(1-\omega)^2}$

14) $\int \frac{\ln x^2}{x} dx =$

- A) $(\ln x)^2 + c$
 B) $(\ln x^2)^2 + c$
 C) $\ln x^2 + c$
 D) $\ln x + c$

15) The expansion $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ is valid for

- A) all real x
 B) $-1 < x < 1$
 C) $-1 < x \leq 1$
 D) $-1 \leq x \leq 1$

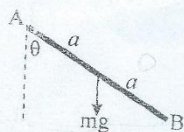
16) The solution of the equation $|z| - 3 = 1 + 2i$ is

- A) $\frac{3}{2} - 2i$
 B) $\frac{3}{2} + 2i$
 C) $2 - \frac{3}{2}i$
 D) does not exist

17) Given that $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$, then z^{20} in the form $a+ib$, where a and b are real numbers is

- A) $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$
 B) $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$
 C) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$
 D) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$

18)



The rod AB is pivoted at A and can rotate freely about a horizontal plane, is released from rest in the position shown in the diagram. The angular velocity of the rod AB when B is vertically below A is

- A) $\sqrt{\frac{4a}{3g}}$
- B) $\sqrt{\frac{3g}{4a}}$
- C) $\sqrt{\frac{g}{a}}$
- D) $\frac{3g}{4a}$

19) The range of values of x for which the expansion of $\ln(2-5x-3x^2)$ is valid is

- A) $-\frac{1}{3} \leq x \leq \frac{1}{3}$
- B) $-\frac{1}{3} \leq x < \frac{1}{3}$
- C) $-\frac{1}{3} < x \leq \frac{1}{3}$
- D) $-\frac{1}{3} < x < \frac{1}{3}$

20) Given that a coin is tossed 80 times, then the mean and variance of the number of heads respectively are

- A) 80, 40
- B) 40, 20
- C) 20, 40
- D) 20, 10

21) The unit vector which is perpendicular to each of the vectors $a=i-j+2k$ and $b=-i+2j+k$ is

- A) $-5i-3j+k$
- B) $\frac{1}{35}(5i+3j-k)$
- C) $\frac{1}{\sqrt{35}}(-5i-3j+k)$
- D) $-5i-3j+k$

$$\begin{aligned} & \mathbf{a} = i - j + 2k \\ & \mathbf{b} = -i + 2j + k \\ & \mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -1 & 2 & 1 \end{vmatrix} = i(-1-2) - j(1+2) + k(2-1) = -3i - 3j + k \end{aligned}$$

2) The cofactor of the entry 7 in the matrix $\begin{vmatrix} 2 & 3 & -1 \\ 7 & 4 & 0 \\ 6 & 9 & 8 \end{vmatrix}$ is

- A) $\begin{vmatrix} 3 & -1 \\ 9 & 8 \end{vmatrix}$
- B) $-\begin{vmatrix} 3 & -1 \\ 9 & 8 \end{vmatrix}$
- C) $\begin{pmatrix} 3 & -1 \\ 9 & 8 \end{pmatrix}$
- D) $-\begin{pmatrix} 3 & -1 \\ 9 & 8 \end{pmatrix}$

- 23) A particle travelling with retardation $(1+v^2)N$ in a straight line, where v m/s is its speed, has initial velocity u m/s. The particle comes to rest in the time

A) $\frac{u}{2}$ sec.
 B) $\frac{1}{2} \tan^{-1} u$ sec
 C) $\tan^{-1} u$ sec
 D) $\tan^{-1} \frac{u}{2}$ sec.

$$mvd \frac{dv}{dt} = 1+v^2$$

$$\int \frac{dv}{1+v^2} = \int \frac{dt}{u^2+v^2}$$

$$\tan^{-1} u = \frac{1}{u} \tan^{-1} \frac{v}{u}$$

$$\tan^{-1} \frac{v}{u} = u \tan^{-1} u$$

$$\tan^{-1} \frac{v}{u} = \tan^{-1} u$$

$$\frac{v}{u} = u$$

$$v = u^2$$

$$u^2 = 1$$

$$u = 1$$

- 24) The general solution, of the vector differential equation $\frac{d^2 \mathbf{r}}{dt^2} + 2 \frac{d\mathbf{r}}{dt} + 2\mathbf{r} = 0$, is

A) $\mathbf{r} = e^t (A \cos t + B \sin t)$
 B) $\mathbf{r} = -e^{-t} (A \cos t + B \sin t)$
 C) $\mathbf{r} = e^{-t} (A \cos t + B \sin t)$
 D) $\mathbf{r} = -e^t (A \cos t + B \sin t)$

$$m^2 + 2m + 2 = 0$$

$$(m+1)^2 + 1 = 0$$

$$m+1 = \pm i$$

$$m = -1 \pm i$$

$$\frac{d^2 \mathbf{r}}{dt^2} + 2 \frac{d\mathbf{r}}{dt} + 2\mathbf{r} = 0$$

$$\frac{d^2 \mathbf{r}}{dt^2} + 2 \frac{d\mathbf{r}}{dt} + 2\mathbf{r} = 0$$

$$\frac{d^2 \mathbf{r}}{dt^2} + 2 \frac{d\mathbf{r}}{dt} + 2\mathbf{r} = 0$$

$$\frac{d^2 \mathbf{r}}{dt^2} + 2 \frac{d\mathbf{r}}{dt} + 2\mathbf{r} = 0$$

- 25) Given that the moment of inertia about an axis through the centre of a uniform solid sphere of mass m and radius a is $\frac{2ma^2}{5}$, then the moment of inertia of the sphere about a tangent is

A) $\frac{7ma^2}{5}$
 B) $\frac{5ma^2}{7}$
 C) $\frac{7ma}{5}$
 D) $\frac{5ma}{7}$

$$\frac{2ma^2}{5}$$

$$\frac{7ma^2}{5}$$

$$\frac{5ma^2}{7}$$

$$\frac{7ma}{5}$$

$$\frac{5ma}{7}$$

- 26) The rate of decay of a substance is cx , where x is the amount of the substance remaining and c a constant. The time, t , in which the amount of the substance is halved is

A) $t = c \ln 2$
 B) $t = \frac{1}{2} \ln 2$
 C) $t = \ln 2c$
 D) $t = \frac{1}{2} \ln 2$

$$\frac{dx}{dt} = -cx$$

$$\frac{1}{x} \frac{dx}{dt} = -c$$

$$\ln x = -ct$$

$$\ln \frac{x}{x_0} = -ct$$

$$\ln \frac{1}{2} = -ct$$

$$t = \frac{\ln 2}{c}$$

- 27) The first two terms of the Maclaurin series expansion $e^x \cos x$ are

A) $x + \frac{1}{2}x^2$
 B) $1 + x$
 C) $x + \frac{1}{3}x^2$
 D) $1 + \frac{1}{2}x^2$

$$e^x \cos x = \sin x$$

$$f(x) = x f'(0)$$

$$1 + x$$

- 28) $\lim_{x \rightarrow 0} \frac{1-e^x}{\sin x}$ is

A) -1
 B) 0
 C) 1
 D) 2

$$\frac{1-e^x}{\sin x}$$

- 29) The table gives the values of x and then corresponding values of y of a continuous function $y=f(x)$

x	3	3.5	4	4.5	5.0
y	5.4	4.6	4.2	4.1	4.7

An approximate value of $\int_3^5 f(x)dx$ using the Simpson's rule is:

- A) 26.65
- B) 44.5
- C) 89.0
- D) 8.95

30) $e^{(2-2i)\theta} =$

- A) $e^{2\theta} + i \sin 2\theta$
- B) $\cos 2\theta - i \sin 2\theta$
- C) $e^{2\theta} (\cos 2\theta - i \sin 2\theta)$
- D) $e^{2\theta} (\cos 2\theta + i \sin 2\theta)$

31) The solution of the equation $\cosh x - \sinh x - 1 = 0$ is

- A) 2
- B) 1
- C) $\frac{1}{2}$
- D) 0

32) A force F acts at the point, with position vector r , where $F = (3i - 4j + k)N$ and $r = (2i - j - k)m$.

The vector moment of F about the origin is

- A) $-5i - 5j - 5k$
- B) $5i - 5j - 5k$
- C) $-5i + 5j - 5k$
- D) $5i - 5j + 5k$

33) The kinetic energy and moment of momentum of a flywheel whose moment of inertia is I and rotating at $\omega^2 \text{ rads}^{-1}$ are respectively

- A) $\frac{1}{2}I\omega^2, \frac{1}{2}I\omega^2$
- B) $\frac{1}{2}I\omega^4, I\omega^2$
- C) $\frac{1}{2}I\omega^2, I\omega$
- D) $\frac{1}{2}I\omega^4, I\omega$

4) Given the plane $2x - y + z$ and the line $r = i + 2j - k + \lambda(i - j + k)$, the sine of the angle between them is

- A) $\frac{\sqrt{2}}{3}$
- B) $\frac{\sqrt{5}}{2}$
- C) $\frac{\sqrt{6}}{5}$
- D) $\frac{\sqrt{3}}{4}$

5) Given the approximation $y_{n+1} = y_n + h\left(\frac{dy}{dx}\right)$ and a step of 0.1, given that

$\frac{dy}{dx} = xy$, and $y = 1$ when $x = 0$ the value of y when $x = 0.1$ is

- A) 1.01
- B) 1
- C) 0.2
- D) 1.1

end