1. i) Two particles are moving with constant velocity. At a particular instant the particle with velocity vector $(\mathbf{i}-2\mathbf{j})\ ms^{-1}$ passes through a point with position vector $(6\mathbf{i}+8\mathbf{j})\ m$ and the particle with velocity vector $(3\mathbf{i}+2\mathbf{j})\ ms^{-1}$ passes through the point with position vector $2\mathbf{i}\ m$ Show that the two particles collide and find the position vector at the point of collision.

ii) The forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ act on a body with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ relative to a fixed origin, where

$\overrightarrow{F_1} = (2\mathbf{i}+3\mathbf{j}+4\mathbf{k})$ N $\mathbf{r}_1 = (-\mathbf{i}+2\mathbf{j}+\mathbf{k})$ m

$\overrightarrow{F_2} = (4\mathbf{i}-3\mathbf{j}+2\mathbf{k})$ N $\mathbf{r}_2 = (3\mathbf{i}-3\mathbf{j})$ m

$\overrightarrow{F_3} = (-3\mathbf{i}+2\mathbf{j}-\mathbf{k})$ N $\mathbf{r}_3 = (4\mathbf{i}+2\mathbf{k})$ m

This system can be reduced to a single force $\overrightarrow{F}$ acting at a point A with position sector $\mathbf{r}_A = \mathbf{i}+\mathbf{j}+\mathbf{k}$ together with a couple G.

a) Find the force $\overrightarrow{F_1}$ and the couple G.

b) Give, in vector form an equation for the line of action.

2. i) At time t seconds the position vector of a particle P relative to a fixed origin is $\mathbf{r}$ metres and satisfies the differential equation:

$$\frac{d\mathbf{r}}{dt}+5\mathbf{r}=(3\mathbf{i}+2\mathbf{j}+\mathbf{k})e^{t}$$

Given that P is at O when t = 0, show that

$$6\mathbf{r}=(3\mathbf{i}+2\mathbf{j}+\mathbf{k})(e^{t}-e^{-5t})$$

ii) A particle P describes the curve with polar equation $r=a\sin\theta$ in such a manner that the radius vector from the pole rotate with uniform angular speed ω. Find, in terms of ω, expressions for the radial and transverse components of the acceleration of the particle when $\theta=\frac{\pi}{4}$

3. A smooth sphere A, moving on a smooth horizontal table, impinges obliquely with an identical sphere B at rest on the table. At the moment of impart the line of centres makes an angle of 30° with the direction of A. given that the coefficient of restitution between the spheres is e, show that sphere A is deflected by the impact through an angle θ where

$$\tan\theta=\frac{\sqrt{3}(1+e)}{5-3e}$$

Given that $e=\frac{1}{3}$ Show that the kinetic energy after impact is $\frac{1}{3}mu^2$

4. Given that y satisfies the differential equation

$$\frac{d^2y}{dx^2}+10\frac{dy}{dx}-y^2=x^2,$$

Use the approximations

$$\frac{d^2y}{dx^2}\approx\frac{y_{n+1}-2y_n-y_{n-1}}{h^2}\ and\ \frac{dy}{dx}\approx\frac{y_{n+1}-y_{n-1}}{2h}$$ to show that

$$(5h+1)y_{n+1}=2y_n+(5h-1)y_{n-1}+h^2(y_n^2+x_n^2)$$

Deduce that when $h=0.2$,

$$y_{n+1}=y_n+0.02(y_n^2+x_n^2).$$

Find the value of y when x=0.4, given that y =1 when x = 0 (work to 4 decimal places)

ii) Evaluate $\int_0^{\frac{\pi}{3}}\sqrt{1-\frac{1}{4}\sin^2\theta}\,d\theta$ by Simpson's rule using seven ordinates (work up to 3 decimal places, take $\pi=\frac{22}{7}$)

5. A particle P of mass m is attached to one end of a light inextensible string of length a whose other end O is fixed. When at rest vertically below O, P is projected horizontally with speed $\sqrt{\lambda ga}$. Show that whilst the string remains taut, when OP has rotated through an angle θ, the tension is of magnitude

$$mg(\lambda-2+3\cos\theta).$$

Given that $\lambda = \frac{7}{2}$, find the value of θ for which the string slackens. Show that in this case, the greatest height of P above O is $\frac{11}{6}a$

6. A particle P, OF mass m, falls from rest under gravity in a medium which exerts a resisting force of magnitude $mkgv^2$, where vms^{-1} is the speed of the particle and is a constant.

When $t = T$, $v = \frac{1}{2\sqrt{k}}$ and the particle has fallen a distance x.

Show that

a) $2g\sqrt{k}\ T = \ln 3$

b) $2kg\ x = \ln\frac{4}{3}$

7. (i) The number of patients received by a specialised doctor on a weekday has the poising distribution with mean 1.5. Find the probability that on a particular day

a) no patients are received
b) exactly two patients are received
c) less than three patients are received
d) at least three patients are received

(ii) In a Further mathematics class of ten students the probability that a student passed in a particular test in $\frac{3}{5}$. What is the probability that

a) exactly 3 students passed
b) at least 8 students passed
c) from 2 to 5 students passed

8. Find the moment of inertia of a uniform circular disc of mass M and radius a about an axis passing through the centre O of the disc and perpendicular to the plane of the disc. Hence find the moment of inertia of the disc about an axis passing through a point A on the rim of the disc and perpendicular to the plane of the disc

The disc is free to rotate in its own plane about a smooth horizontal axis passing through A. It is released from rest in the position in which OA is horizontal and the disc vertical: Find the angular velocity of the disc when OA first becomes vertical.

REPUBLIQUE DU CAMEROUN
PAIX-TRAVAIL-PATRIE

MINISTERE DES ENSEIGNEMENTS SECONDAIRES

Cellule d'Appui à l'Action Pédagogique
Antenne Provinciale du Nord Ouest

REPUBLIC OF CAMEROON
PEACE-WORK-FATHERLAND

MINISTRY OF SECONDARY EDUCATION

Teachers' Resource Centre
Bamenda North West Province

The Teachers' Resource Centre and The Provincial Inspectorate of Pedagogy, in Collaboration with the Mathematics Teachers' Association (MTA)

Inter–Provincial Mock General Certificate of Education Examination

TUESDAY , 27/03/2006, MORNING	ADVANCED LEVEL
Subject title	FURTHER MATHEMATICS
Paper number	Paper 3
Subject code	775

Two And A Half Hours

Full marks may be obtained for answers to ANY SIX questions. ***(If you attempt more than six questions, only the best six answers will be taken into account).***

All questions carry equal marks.

You are reminded of the necessity for good English and orderly presentation in your answers.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Electronic calculators, mathematical formulae and tables are allowed.

A

TURN OVER