6. (a) Write down the expansion of $\ln\left(\dfrac{1-2x}{(1+2x)^2}\right)$ in ascending powers of x up to and including the term in x^4. State the range of values of x for which the expansion is valid.
If x is so small that terms in x^2 and higher powers of x may be neglected, show that

$$\left[\frac{(1-2x)}{(1+2x)^2}\right]^{\frac{1}{2x}} \cong (1+x)\,e^{-3}$$

$(1-2x)^{\frac{1}{2x}}$

$1-2x\left(\frac{1}{2x}\right)$

(b) Using the substitution $x-1 = 2\sinh u$ or otherwise, evaluate

$1-1$ $(1+x)^{-\frac{1}{x}}$

$$\int_1^3 \sqrt{(x^2-2x+5)}\,dx,$$

$(x-3)$ 1

$5x$

giving your answer in terms of natural logarithms.

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7. Given the matrix M, where

$$M = \begin{pmatrix} 1 & 1 & 4 \\ 3 & 5 & 1 \\ 1 & 2 & 0 \end{pmatrix},$$

(a) find the image of the line

$t - \frac{t^3}{3}$

$$\frac{x-2}{1} = \frac{y-3}{2} = \frac{z+5}{-1},$$

2

$1-t$

under the transformation whose matrix is M.

(b) Find M^{-1} and hence,

(i) find the point whose image is (2, 4, -1) under the transformation whose matrix is M,

(ii) solve for x, y, z, the system of equations

$$x + y + 4z = 8,$$
$$3x + 5y + z = 0,$$
$$x + 2y = -1.$$

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8. The asymptotes of the rectangular hyperbola $x^2 - y^2 = p^2$, where p is a constant, cut the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at four points A, B, C and D. Fin the coordinates of the points A, B, C and D.
Show that the points A, B, C, D are the vertices of a square whose area is $\dfrac{4a^2b^2}{a^2+b^2}$.
The tangent to the ellipse at the point A meets the x-axis at the point F. Find the area of the triangle OAF, where O is the orgin.
