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1. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act at points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, where

$$\mathbf{F}_1 = (\mathbf{i}-3\mathbf{j}+3\mathbf{k})\text{N}, \quad \mathbf{r}_1 = (3\mathbf{i}+\mathbf{j}+a\mathbf{k})\text{m},$$
$$\mathbf{F}_2 = (2\mathbf{i}+2\mathbf{j}+6a\mathbf{k})\text{N}, \quad \mathbf{r}_2 = (\mathbf{i}+3\mathbf{j}+\mathbf{k})\text{m},$$

The lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ intersect.

a) Find the value of the constant a and the position vector of the point of intersection.

The forces $\mathbf{F}_3$ and $\mathbf{F}_4$ act through the point with position vector $(\mathbf{i}+\mathbf{j}-\mathbf{k})m$, so that the system of the four forces reduces to a couple $\mathbf{G}$.

b) Find the magnitude of $\mathbf{F}_4$ and a vector equation of its line of action.

c) Find, also, the magnitude of $\mathbf{G}$.

2. a) The table below gives the speed, v m s^{-1}, of a particle moving in a straight line between two points A and B. At time t = 0.2 s the particle is at A and at time t = 1.0 s, it is at the point B.

$v\,(m\,s^{-1})$	6.03	4.04	2.71	1.82	1.22
$t\,(s)$	0.2	0.4	0.6	0.8	1.0

Using Simpson's rule estimate, to 2 decimal places, the distance AB.

b) Given that $\frac{dy}{dx} - x^2 + xy = 0$, and that $y = 1$ when $x = 0$, show that for values of x close to 0,

$$y = 1 - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{8}x^4 + \ldots$$

Use the approximation $2h\frac{dy}{dx} \approx y_{n+1} - y_{n-1}$ and a step width of 0.2 to find the value of y when x = 0.4, giving your final answer to 4 decimal places.

3. a) Find the general solution of the vector differential equation $\frac{d^2\mathbf{r}}{dt^2} + 4\mathbf{r} = (6\sin t)\mathbf{i}$, given that $\mathbf{A}\sin t$ is a particular integral of this differential equation.

A particle P moves in the x-y plane so that its position vector $\mathbf{r}$ at time t satisfies the differential equation above. Initially P is at the point with position vector $2\mathbf{j}$ and moving with velocity $2\mathbf{i}$. Given that P moves on the parabola $y = \lambda + \mu x^2$, find the values of μ and λ.

b) A particle moves on a polar curve with constant angular velocity ω. At time t, the transverse component of its acceleration is $2r\omega^2$. Given that $r = a$ when $\theta = 0$, find the equation of the curve.

4. Two spheres A and B of equal radii and of masses m and λm lie on a smooth horizontal floor. Sphere A is projected towards B with speed u so that they collide obliquely. Just before impact, the direction of motion of A makes an angle θ with the line of centres of the two spheres. Given that the coefficient of restitution between the spheres is e, find the speed of B after impact.

After impact, the velocity of A makes an angle ϕ with the line of centres.

Show that $\tan\phi = \frac{1+\lambda}{1-\lambda e}\tan\theta$.

Given that the final velocity of A is perpendicular to its initial velocity, show that $\tan^2\theta = \frac{e\lambda - 1}{\lambda + 1}$.

Show, further that, $\lambda > 1$.

5. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus mg. The other end of the string is fixed to a point A on a smooth plane which is inclined at an angle θ to the horizontal. When the particle lies in equilibrium, find the extension of the string. The particle is now given a speed u down plane, parallel to the line of greatest slope of the plane. Given that x is the displacement of the particle from the equilibrium position at time t, show that the equation of motion of P is given by $x = u\sqrt{\left(\frac{l}{g}\right)}\sin\left(\sqrt{\frac{g}{l}}\right)t$, where g is the acceleration of free fall.

Show further that the particle executes complete simple harmonic motion provided that $u \le \sqrt{(gl)}\sin\theta$.

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