

where  $\sin \theta = 3/5$ . The particles are released from rest when both parts of the string are taut. After traveling a distance  $1\frac{1}{2}$  m, the speed of the particles is  $2 \text{ m s}^{-1}$ . Calculate

- the acceleration of the particles,
- the tension in the string,
- the value of  $M$ ,
- the magnitude and direction of the reaction of the pulley.

(ii) A particle of mass  $2 \text{ kg}$  is attached to one end  $B$  of a light inextensible string  $AB$  of length  $0.5 \text{ m}$ . The other end  $A$  of the string is fixed. The particle moves with a constant angular speed in a horizontal circle of radius  $0.3 \text{ m}$  whose centre  $O$  is vertically below  $A$ . Calculate

- the tension in the string,
- the angular speed of the particle.

(Take  $g$  as  $10 \text{ m s}^{-2}$ )

6) Three smooth spheres  $A, B, C$  of mass  $m, 2m$  and  $3m$  respectively, lie on a smooth horizontal floor with their centres in a straight line. Sphere  $A$  is projected with speed  $3u$  and strikes sphere  $B$  directly. Sphere  $B$  subsequently collides directly with sphere  $C$ . Given that the coefficient of restitution between any two spheres during impact is  $\frac{1}{4}$ , find

- the velocities of the spheres after the collisions,
- the impulse exerted on sphere  $A$  by sphere  $B$  due to the impact,
- the loss in kinetic energy due to the impact between spheres  $B$  and  $C$

7. A car of mass  $2000 \text{ kg}$  is moving with a uniform speed on a level road against a constant friction resistance of  $5000 \text{ N}$ . Given that the engine of the car is working at a rate of  $60 \text{ kW}$ , find

- the speed of the car,
- the work done by the engine in  $3 \text{ s}$

The car starts to ascend a slope inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = 0.1$ , with the engine working at the same rate and the frictional resistances remaining the same. When the car attains a steady speed  $v \text{ ms}^{-1}$ , the engine is shut off. Find

- the value of  $v$ ,
- the further distance covered by the car before coming to rest.

Take  $g$  as  $10 \text{ ms}^{-2}$

8) i) Two independent events,  $A$  and  $B$ , are such that  $P(A) = 2/7$  and  $P(A \cap B) = 1/5$ . Find

- $P(B)$ ,
- $P(A \cup B)$ .

ii) A bag contains 1 red ball and 2 white balls. A trial consists of drawing a ball at random from the bag, noting its colour and putting it back into the bag, together with an additional ball of the same colour. Given that two trials are made, draw a tree diagram illustrating all the possible outcomes and their corresponding probabilities. Hence or otherwise, find the probability that

- at least one red ball is drawn
- a red ball and a white ball are drawn
- two balls of the same colour are drawn

Given that the two balls drawn are of the same colour, find the probability that they are white.

#### JUNE 2010

1. The position vector  $\mathbf{r}$ , of a rectangle  $P$ , of mass  $2 \text{ kg}$  at time  $t$  seconds, is given by

$$\mathbf{r} = [(t^3 - 2t)\mathbf{i} + t\mathbf{j}] \text{ m};$$

find

- the values of  $t$  for which the velocity of  $P$  is perpendicular to its acceleration;
- the force acting on  $P$  and the momentum of  $P$  when  $t = 3$ .
- the cosine of the angle between the force acting on  $P$  and the momentum of  $P$  when  $t = 3$ .

2.

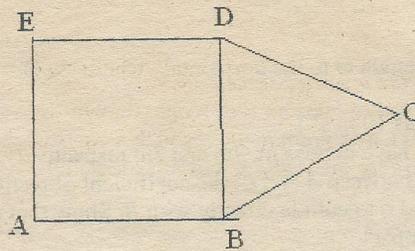


Figure 1

Figure 1 is a uniform density lamina  $ABCDE$ , made from a square of side  $2a$  and an equilateral triangle of side  $2a$ . Show that the distance of the centroid of the lamina from  $BD$  is

$$\frac{3a(4 - \sqrt{3})}{13}$$

The lamina is freely suspended at  $E$  so that it rests in equilibrium with  $ED$  inclined at an angle  $\theta$  to the vertical. Show that  $\tan \theta = \frac{14 - 3\sqrt{3}}{13}$

3. (i) A block of mass  $5\text{ kg}$  lies at rest on a rough horizontal plane. The coefficient of friction between the block and the plane is  $0.6$ . A force of magnitude  $20\text{ N}$  is applied to the block at an angle of  $30^\circ$  to the horizontal. Find the frictional force between the block and the plane. Investigate whether the block has been set in motion by this force.
- (ii)

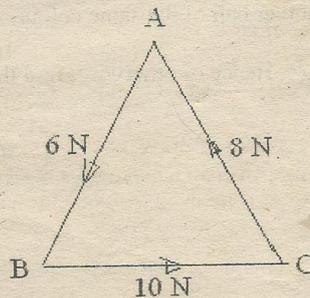


Figure 2

Forces of magnitude  $10\text{ N}$ ,  $8\text{ N}$  and  $6\text{ N}$  act along the sides  $BC$ ,  $CA$  and  $AB$  of an equilateral triangle  $ABC$  as in figure 2. Find the magnitude of the resultant of these forces and the angle which it makes with  $BC$ ;

4. A golf player projects a ball from a point on horizontal ground with a speed of  $50\text{ m s}^{-1}$  inclined at an angle  $\theta$  to the horizontal. The ball passes through a point  $P$  which is at a horizontal distance of  $200\text{ m}$  from the point of projection and at a vertical distance of  $25\text{ m}$  above the point of projection. Find the possible values of  $\tan \theta$ .

For the smaller value of  $\tan \theta$ , find the magnitude of the velocity of the ball at  $P$  and the range of the ball on the horizontal plane through the point of projection.  
(Take  $g$  as  $10\text{ m s}^{-2}$ )

\* 5. A car of mass 1500kg tows a carriage of mass 500kg. The frictional resistance to the motion of the car is 700 N and the frictional resistance to the motion of the carriage is 300 N. The engine of the car works at a constant rate of 20 KW. Find the acceleration of the car and the carriage and the tension in the tow rope when the speed of the car is  $5 \text{ m s}^{-1}$ ,

(a) on level ground,

(b) directly up a track inclined at an angle  $\Theta$  to the horizontal, where  $\sin \Theta = \frac{1}{20}$

(Take  $g$  as  $10 \text{ m s}^{-2}$ )

6. Three spheres  $A, B, C$  of equal radii have masses  $m, 2m$  and  $2m$  respectively. They lie in a straight line on a smooth horizontal plane with  $B$  between  $A$  and  $C$ . The coefficient of restitution between any pair of spheres is  $\frac{1}{2}$ .  $A$  is projected with speed  $u$  to collide directly with  $B$ . Show that the speed of  $B$  after impact is  $4u$  and find the speed of  $A$  after impact.

$B$  subsequently collides with  $C$ . Find

(a) the speeds of  $B$  and  $C$  after this impact,

(b) the magnitude of the impulse experienced by  $B$  due to the impact,

(c) the loss in kinetic energy due to the impact between  $B$  and  $C$ ;

7. A particle of mass 4 kg moves in a straight line under the action of a force of magnitude  $(16 - v^2) \text{ N}$ , where  $v \text{ m s}^{-1}$  is the speed of the particle. Initially the particle is at rest at the origin. Show that the speed of the particle at time  $t$  seconds can be expressed as

$$v = \frac{4(e^{2t} - 1)}{e^{2t} + 1}$$

and deduce that the particle cannot attain a speed of up to  $4 \text{ m s}^{-1}$

8: (i) Two events  $A$  and  $B$  are such that  $P(A) = \frac{2}{5}$  and  $P(A \cup B) = \frac{7}{10}$

Find  $P(B)$  if

(a)  $A$  and  $B$  are mutually exclusive,

(b)  $A$  and  $B$  are independent,

(c)  $P(A/B) = P(B/A)$

(ii) A box contains 4 red balls and 6 yellow balls. A ball is drawn at random from the box, its colour is noted and it is returned into the box together with another ball of the same colour. A second ball is drawn at random from the box.

Draw a tree diagram illustrating the possible outcomes. Hence or otherwise, find the probability that;

(d) two red balls are drawn,

(e) two balls of the same colour are drawn,

(f) at least one yellow ball is drawn