

JUNE 2009 *

1. The position of two particles A and B at time t seconds are $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively, where
 $\mathbf{r}_1 = [(5t^2 + 35t)\mathbf{i} + (t^2 - 2)\mathbf{j}] \text{ m}$,
 $\mathbf{r}_2 = [(t^3 + 3t^2)\mathbf{i} + (6t^2 + 5)\mathbf{j}] \text{ m}$.
 a) Show that the two particles will collide, stating the position vector of the point of collision.
 b) Obtain the velocity and acceleration vectors of each particle just before the collision and determine the particle which will be moving faster at that time.

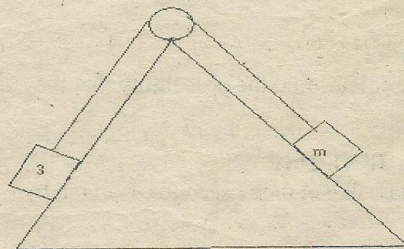
2. (i) Forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act at points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively where
 $\mathbf{F}_1 = (3\mathbf{i} + 5\mathbf{j})\text{N}$, $\mathbf{F}_2 = (2\mathbf{i} - 4\mathbf{j})$, $\mathbf{F}_3 = (-5\mathbf{i} - \mathbf{j})\text{N}$,
 $\mathbf{r}_1 = (\mathbf{i} - \mathbf{j})\text{m}$, $\mathbf{r}_2 = (2\mathbf{i} + \mathbf{j})\text{m}$, $\mathbf{r}_3 = (-6\mathbf{i} + \mathbf{j})\text{m}$,
 Show that this system of forces forms a couple, giving the magnitude of the couple.
 (ii) A uniform ladder of length 8m rests with its upper end against a smooth vertical wall and its lower end on a rough horizontal floor. The coefficient of friction between the ladder and the floor is $\frac{3}{4}$ and the angle between the ladder and the floor is 45° . A builder whose weight is twice that of the ladder slowly climbs up the ladder. Find how far up the ladder he will climb before it begins to slip.

3. A particle of mass 3kg, moving in a straight line with speed $v \text{ ms}^{-1}$ at time t seconds is subject to a retarding force of magnitude $(u + 3v)\text{N}$, where u is a positive constant. The particle starts from the origin with speed $4u \text{ ms}^{-1}$. Find in terms of u .
 a) the value of t when $v = 2u$,
 b) the distance x , of the particle from the origin, when $v = 2u$.

Two projectiles P and Q are fired from the same point O on the ground and at the same time, with their velocities in the same vertical plane. The horizontal and vertical components of the velocity of P are $2u$ and $3u \text{ ms}^{-1}$ respectively, while the horizontal and vertical components of the velocity of Q are $3u$ and $2u \text{ ms}^{-1}$ respectively.

- a) Find the ratio of the maximum height attained by P to the maximum height attained by Q
- b) Show that the two projectiles strike the ground at the same spot on the horizontal plane through O
- c) Find, the speed of the particle P , its horizontal distance from O , and its height above the horizontal plane through O at the time Q strikes the ground, giving your answers, in terms of u and g .
- d) Explain why P and Q cannot collide while moving freely under gravity.

5.



Two particles of mass 3kg and $M\text{kg}$ ($M < 3$), rest on the smooth incline faces of a fixed wedge as shown in the diagram. They are connected by a light inextensible string passing over a smooth pulley fixed at the top of the wedge. The faces of the wedge are inclined at an angle θ to the horizontal,

where $\sin \theta = 3/5$. The particles are released from rest when both parts of the string are taut. After traveling a distance $1\frac{1}{2}$ m, the speed of the particles is 2 m s^{-1} . Calculate

- the acceleration of the particles,
- the tension in the string,
- the value of M ,
- the magnitude and direction of the reaction of the pulley.

(ii) A particle of mass 2 kg is attached to one end B of a light inextensible string AB of length 0.5 m . The other end A of the string is fixed. The particle moves with a constant angular speed in a horizontal circle of radius 0.3 m whose centre O is vertically below A . Calculate

- the tension in the string,
 - the angular speed of the particle.
- (Take g as 10 m s^{-2})

6) Three smooth spheres A, B, C of mass $m, 2m$ and $3m$ respectively, lie on a smooth horizontal floor with their centres in a straight line. Sphere A is projected with speed $3u$ and strikes sphere B directly. Sphere B subsequently collides directly with sphere C . Given that the coefficient of restitution between any two spheres during impact is $\frac{1}{4}$, find

- the velocities of the spheres after the collisions,
- the impulse exerted on sphere A by sphere B due to the impact,
- the loss in kinetic energy due to the impact between spheres B and C

7. A car of mass 2000 kg is moving with a uniform speed on a level road against a constant friction resistance of 5000 N . Given that the engine of the car is working at a rate of 60 kW , find

- the speed of the car,
- the work done by the engine in 3s

The car starts to ascend a slope inclined at an angle θ to the horizontal, where $\sin \theta = 0.1$, with the engine working at the same rate and the frictional resistances remaining the same. When the car attains a steady speed $v \text{ ms}^{-1}$, the engine is shut off. Find

- the value of v ,
 - the further distance covered by the car before coming to rest.
- Take g as 10 ms^{-2}

8) i) Two independent events A and B , are such that $P(A) = 2/7$ and $P(A \cap B) = 1/5$. Find

- $P(B)$,
- $P(A \cup B)$.

ii) A bag contains 1 red ball and 2 white balls. A trial consists of drawing a ball at random from the bag, noting its colour and putting it back into the bag, together with an additional ball of the same colour. Given that two trials are made, draw a tree diagram illustrating all the possible outcomes and their corresponding probabilities. Hence or otherwise, find the probability that

- at least one red ball is drawn
- a red ball and a white ball are drawn
- two balls of the same colour are drawn

Given that the two balls drawn are of the same colour, find the probability that they are white.

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1. The position vector $\mathbf{r}$, of a rectangle P , of mass 2 kg at time t seconds, is given by

$$\mathbf{r} = [(t^3 - 2t)\mathbf{i} + t\mathbf{j}] \text{ m};$$

find

- the values of t for which the velocity of P is perpendicular to its acceleration;
- the force acting on P and the momentum of P when $t = 3$.
- the cosine of the angle between the force acting on P and the momentum of P when $t = 3$.