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Find the general solution of the differential equation $\dfrac{d^2y}{dx^2}+9y=0$.

The period T, of a compound pendulum, is given by $T=2\pi\sqrt{\left(\dfrac{8a^2+5x^2}{5gx}\right)}$.

Find, (a) the length of the equivalent simple pendulum,

(b) the value of x for which T is maximum.

Find the general solution of the vector differential equation $\dfrac{d^2\mathbf{r}}{dt^2}-4\dfrac{d\mathbf{r}}{dt}+4\mathbf{r}=\mathbf{0}$.

Find the equations of the two asymptotes of the curve $y=f(x)$, where $f(x)=\dfrac{x^2+2x-1}{3(x+1)}$.

Given that $f(x)=\begin{cases} x^2 & \text{for } 0\le x<1, \\ 2x-1 & \text{for } 1\le x\le 3, \\ 7-x & \text{for } x>3 \end{cases}$

(a) State the value of x for which f is not continuous,

(b) Sketch the curve y= f(x) for $0\le x\le 7$.

A particle is moving in a straight line with simple harmonic motion whose period is 10 seconds. Find the maximum speed of the particle.

Given the complex numbers $z_1=e^{-i\pi/2}$ and $z_2=2e^{i\pi/6}$, find $|z_2/z_1|$

Evaluate $\lim_{x\to 0}\left(\dfrac{\sinh x-x}{x\cosh x-x}\right)$.

Given that $H=5\cosh x+3\sinh x$, find the minimum value of H.

The population of a certain town doubles in 50 years. In how many years will the population triple given that the rate of increase is proportional to the number of inhabitants?

Sketch the curves $y=\dfrac{3}{x}$ and $x^2+y^2=10$ on the same axes and shade the region(s) for which $(xy-3)(x^2+y^2-10)\ge 0$.

Write down an expression for $\cosh 2x$ in terms of e^{2x}.

Hence, find $\int e^{2x}\cosh 2x\,dx$.

Express $f(x)$ in partial fractions, where $f(x)=\dfrac{4x-5}{x(2x+1)^2}$.

Expand e^{x+x^2} as a series in ascending powers of x as far as the term in x^3.

Given that $f: x\mapsto\begin{cases}\dfrac{1-\cosh 2x}{x^2}, & x\ne 0 \\ a, & x=0\end{cases}$

is continuous at 0, find the value of a.

A particle of mass m is suspended from a fixed point by a string of length l. When the particle hangs vertically downwards, it is projected with speed $\sqrt{3gl}$. Find the height of the particle above the point of projection when the string first becomes slack.

Find the set of real values of x for which $2|x|>3-x$.

Solve the equation $z^3=8i$, giving the roots in the form of $re^{i\theta}$, where $r>0$ and $0<\theta\le 2\pi$.

Prove, by integration, that the moment of inertia of a uniform rod of mass m and length $5a$, about an axis through one end and perpendicular to the rod, is $\frac{25}{3}ma^2$.

Evaluate I, where $I=\int_1^{49}\dfrac{1}{3+\sqrt{x}}dx$ using Simpson's rule with three ordinates.

Find, as a series of ascending powers of x, up to and including the term in x^3, an approximate solution to the differential equation $\dfrac{d^2y}{dx^2}+20\dfrac{dy}{dx}-y^2=0$, where $y=1$ and $\dfrac{dy}{dx}=-1$, when $x=0$.

Find the equation of the curve, which passes through the point (1, 1) and satisfies the differential equation $x^2\dfrac{dy}{dx}+xy=x+1$.