A particle is projected with speed 2 ms⁻¹ along a smooth horizontal table in a medium whose resistance to motion is of magnitude $(4v^2+1)$ N per unit mass, where v is the speed. Find the speed of the particle when it has travelled a distance of $\left(\frac{1}{8}\ln 2\right)$m.

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Given that $\sinh x=\frac{3}{4}$, find $\cosh 2x$.

Shade the region in an Argand diagram, the complex plane for which $|z-1+2i|\le 2$ and $\arg z>-\pi/4$.

Find the general solution of the differential equation $x\frac{dy}{dx}-y=x^2$.

Express $f(x)=\frac{x^2}{x^2-4}$ in partial fractions. Hence, find $\int f(x)dx$.

A particle is performing simple harmonic motion between two points A and A' which are 6 m apart. When the particle is at a distance $\sqrt{5}$ m from the midpoint of AA', its speed is 10 m s⁻¹.
Find
(a) the period of the motion,
(b) the maximum acceleration.

Evaluate $\lim_{x\to 0}\left(\frac{x-\sin x}{x-\tan x}\right)$.

Evaluate $\int_1^2 \frac{1}{\sqrt{x^2-2x+2}}dx$ giving your answer in terms of natural logarithms.

The moment of inertia of a uniform circular disc of mass m and radius $a\sqrt{2}$ about an axis through its centre perpendicular to the disc is ma^2. Find the moment of inertia of the disc about a tangent.

A function f is defined by

$$f(x)=\begin{cases}\frac{\sin 3x}{x}, x<0\\ \frac{a+2x^2}{3-x}, 0\le x<3\end{cases}$$

Find the value of a for which f is continuous at x = 0.

Shade in one diagram the two regions for which $(x^2+y^2-4)(y+x-2)(y-x-2)\ge 0$.

A particle of mass m falls from rest in a medium whose resistance to motion is of magnitude $\frac{1}{3}v$ per unit mass where v m/s is the speed of the particle. Find the time it takes for the particle to attain a speed of 2g, where g is the acceleration of free fall.

Solve the inequality $x^2+1<|x^2-2|$.

Find the derivative of $\sinh^{-1}\left(\frac{1}{x}\right), x>0$, with respect to x, simplifying your answer.

Find the first two terms in the Maclaurin series expansion of $\cosh\ln(1+x)$ in the ascending powers of x.

Find the general solution of the vector differential equation $\frac{d^2\mathbf{r}}{dt^2}+2k\frac{d\mathbf{r}}{dt}+(k^2+n^2)\mathbf{r}=0$, where k and n are positive constants.

Find all the asymptotes of the curve $y=\frac{3x^3+2x^2}{3x^2-x-4}$

Find the integral $\int \sinh^3 x dx$.

Find all the roots of the equation $(z-1)^3=1$.

The rate of destruction of a certain species of micro organism is λx where is λ a positive constant and x is the number of micro organism present at that instant. Initially, the number of micro organisms present is x_0. Find the time taken for the number of micro organisms to reduce to $\frac{1}{8}x_0$.

Evaluate $\int_0^3 x|x-2|dx$.

Derive, by means of the Taylor series method, the first three terms in the ascending powers of x, the solution of the differential equation $\frac{d^2y}{dx^2}+y\frac{dy}{dx}=1$ given that y = 1 and $\frac{dy}{dx}=2$, when x = 0.