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Express $\frac{2x+1}{(x+1)^2}$ in partial fractions.

Given that the function *f*, where $f(x)=\begin{cases}\lambda\sin\frac{\pi x}{2}, 0\le x\le 2\\ \frac{x^2}{4}+\lambda, x>2\end{cases}$ is continuous at $x = 2$, find the real value of λ.

A particle oscillates with simple harmonic motion along Ox such that its displacement x from O at time t seconds is given by the equation $\frac{d^2x}{dt^2}+4x=0$. Find the period of oscillation.

Given that $1-2i$ is a root of the equation $f(z) = 0$, where $f(z)=z^3+z+10$, find the other roots.

Find the general solution of the vector differential equation $\frac{d^2\mathbf{r}}{dt^2}-2\frac{d\mathbf{r}}{dt}+5\mathbf{r}=0$.

Given that $\sinh x=\frac{4}{3}$, find, without using tables or calculator, the value of $\cosh\left(\frac{x}{2}\right)$.

Expand $\ln\left(\frac{1}{1+x^2}\right)$ as a series in ascending powers of x up to and including the tern in x^4.

Show, by integration, that the moment of inertia of a uniform rod of length $2l$ and mass m about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.

Find the radius of the circle, in the Argand diagram, represented by the equation $|z+2i|=|2iz-1|$.

Evaluate $\int_1^2\frac{1}{\sqrt{x^2+2x}}dx$, giving your answer in terms of natural logarithms.

A function *f* is defined by $f(x)=\sqrt{x+2}$. Show that *f* is continuous on the interval $-2\le x<2$.

Obtain the Taylor series expansion of the solution of the differential equation $\frac{dy}{dx}+ye^{x^2}=0$, in ascending powers of x up to and including the term in x^2 given that $y = 1$ when $x = 0$.

Using that same axes, sketch the curves $x^2-y^2=4$ and $xy=2$. Hence, shade the three regions for which $(x^2-y^2)(xy-2)\ge 0$.

A small bead is threaded onto a smooth circular wire of radius 1.2 m, which is fixed in a vertical plane. The bead is slightly disturbed from rest at the highest point of the wire. Find the speed of the bead when it reaches the lowest point of the wire. (Take g as 10 m s^{-2}).

Find all the asymptotes to the curve $y=\frac{x^3-2}{4-x^2}$.

A flywheel whose moment of inertia about its axis of rotation is 64 kg m^2 is rotated from rest by a constant torque of 16 Nm. Find the kinetic energy gained by the flywheel after 5 seconds of motion.

A particle of unit mass moves in space so that at time t seconds, its position vector $\mathbf{r}$ is given by $\mathbf{r}=(\mathbf{i}+t^2\mathbf{j}+\cos\pi t\mathbf{k})m$. Find its angular momentum about the origin when $t = 2$ seconds.

The following table relates the variables x and y for a continuous function $y = f(x)$.

x	1	2	3	4	5
y	2	3	2	1	2

Find, using Simpson's rule, an approximation, in terms of π, of the volume of the solid of revolution formed when the finite region bounded by the curve $y = f(x)$ and the ordinates at $x = 1$ and $x = 5$, is completely rotated about the x-axis.

The rate at which the temperature of a certain mixture is falling is $k(\theta-25)$, where θ^o C is the temperature of the mixture at time t seconds and k is a positive constant. Given that it takes 5 seconds for the temperature to drop from 60°C to 55°C, find the value of k.

Using the relation $y = vx$ where v is a function of x, transform the differential equation $x\frac{dy}{dx}=2y+x$ into a differential in v and x only. Hence, find the general solution in terms of v and x.

Find the area under the curve given by the parametric equations $x=1+t^2, y=t(2-t)$ in the interval $0\le t\le 1$.

A uniform rod AB of mass $3m$ and length $4l$ is free to rotate in a vertical plane about a smooth fixed horizontal axis through one end. Given that the moment of inertia of the rod about this axis is $16ml^2$, find the period of small oscillations about its position of stable equilibrium.