

MATHEMATICS & MECHANICS 1
7066

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2015

ADVANCED LEVEL

Subject Title	MATHEMATICS & MECHANICS
Paper N°.	1 – Mathematics
Subject Code No	7066

THREE AND A HALF HOURS

Full marks may be obtained for answers to ALL questions in Section A and any FIVE question in section B.
(If you attempt more than five question in section B, only the best five answers will be taken into account).

You are advised to spend about ONE and a HALF HOURS on Section A.

Electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage

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SECTION A

ANSWER ALL THE QUESTIONS IN THIS SECTION

1. Solve in $\mathbb{R}^2$ the system $\begin{cases} 2(x+y) = 16 \\ x^2 + y^2 = 50 \end{cases}$ (4 marks)

2. Determine $\lim_{x \rightarrow \infty} \left(\frac{3e^x - 2}{5e^x + 3} \right)$ (3 marks)

3. Find the inverse, $g^{-1}(x)$ of $g(x) = \text{Log} \left(\frac{x + \sqrt{x^2 + 4x}}{2} \right)$ (4 marks)

4. Solve in $\mathbb{R}$, the equation $2x^2 - 3x + 1 = 0$
Hence determine the solution of the equation $2\cos^2 x - 3\cos x + 1 = 0$, $x \in [0, 2\pi]$ (5 marks)

5. Find the equation of the plane through $A(2, 2, -1)$ perpendicular to $\vec{OA}$. Which point on this plane is closest to the point $(2, -1, 2)$ (5 marks)

6. Consider the function $f(x) = x^2 e^x$
Given that $F(x) = (x^2 + Ax + B)e^x$ is the primitive function of f .
Find the values of A and B . (4 marks)

7. Solve the system of equation $\begin{cases} \frac{2}{\log x} + \frac{5}{\log y} = 29 \\ \frac{3}{\log x} - \frac{2}{\log y} = -4 \end{cases}$ (4 marks)

8. Given that $g(x) = -\frac{(1+x)}{e^x} - x$
Show that $g'(x) = \frac{x-e^x}{e^x}$ (4 marks)

9. Using De Moivre's theorem,
Find the exact value of $(1 + i\sqrt{3})^{100}$ (5 marks)

10. Solve the equation $\frac{e^x - 1}{e^{-x} - 1} = -3$ (4 marks)

11. Evaluate $(1-i)^2$
Hence calculate $(1-i)^9$ (3 marks)

12. Calculate $A = \int_2^3 x^2 |x| dx$ (4 marks)
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13. A numerical function g is defined on $\mathbb{R}$ by

$$g(x) = \frac{e^x - e^{-x}}{2}$$
 (a) Determine the parity of g
 (b) Evaluate the limit of g at $+\infty$ (4 marks)
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14. Find the equation of the ellipse (E) if two of its vertices are at the points $S(5, 0)$ and $S'(-5, 0)$ and the foci at the points $F(0, 3)$ and $F'(0, -3)$ (4 marks)
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15. The parametric equations of a circle is given by
 $x = 5 \cos \theta, y = -3 + 5 \sin \theta$.
 Find the Cartesian equation and express it in the general form. (3 marks)
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SECTION B
ANSWER ANY FIVE QUESTIONS FROM THIS SECTION

16. (a) The area of triangle ABC is 20cm^2 . Two of its sides are of lengths 8cm and 10cm . Find the possible lengths of the 3rd side to two decimal places.
 (b) The vertices of triangle ABC are given as $A(1, -1)$, $B(4, 2)$ and $C(0, 2)$. Find the equation of the circle circumscribed on ABC . (15 marks)
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17. (a) Solve in $\mathbb{R}^3$, the following system of equations

$$\begin{aligned} x + 3y - 2z &= 8 \\ 2x - y + z &= 1 \\ 3x + 2y - 3z &= 15 \end{aligned}$$
 (b) Find the integral value of the real number m for which the line
 $L: Mx - 4y + 14 = 0$ is a tangent to the circle
 $C: x^2 + y^2 + 4x + 6y - 3 = 0$ (15 marks)
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18. Consider the complex function

$$f(z) = z^3 - (1+i)z^2 - 9z + 9 + 9i$$
 Given that $Z^2 - 9$ is a factor,
 (a) Determine the roots of $f(z)$
 (b) Let Z_1, Z_2 , and Z_3 be the roots of $f(z)$ where $\text{Re}(Z_1) < \text{Re}(Z_2) < \text{Re}(Z_3)$. Represent Z_1, Z_2 and Z_3 on an argand diagram.
 (c) Given that A, B , and C are the points with affixes Z_1, Z_2 and Z_3 respectively, determine the equation of the circle (C) with AB as diameter.
 Hence find the equation of tangent to (C) at B (15 marks)
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19. The derived function $g'(x)$ of a function g is defined by $g'(x) = \frac{-5e^x}{(2e^x - 1)^2}$

- Determine the domain of definition Dg' of $g'(x)$
- Calculate the limits of $g'(x)$ at the bounds of the domain of definition, Dg' .
- Determine g , the primitive of $g'(x)$ given that the graph (Cg) of g passes through $(0, 3)$.

(15 marks)

20. The roots of a second degree equation are α and β .

- Write out this equation in terms of α and β and x .
From this equation α and β admits the following relations

$$\begin{cases} 2\alpha\beta - 3\alpha - 4 = 3\beta \\ \alpha + \beta - \frac{2(t+1)}{3} + \frac{2}{3}\alpha\beta(t+2) = 0 \end{cases}$$

Where t is a real parameter.

- Determine $\alpha + \beta$ and $\alpha\beta$ in terms of t
- Find the value of t such that one of the roots of the equation is -3

(15 marks)

21. Solve in $\mathbb{R}$ the following equations and system of equations

- $\log_3 x + \log_{\sqrt{3}} x + \log_{\frac{1}{3}} x = 6$

- $4^x - 2^{x+1} - 8 = 0$

- $\begin{cases} 3^x 3^y = 27 \\ 3^x + 3^y = 12 \end{cases}$

- Sketch the curve of the function f defined by $f(x) = \log_{\frac{1}{2}}(4 - 2x)$, indicating clearly its asymptote

(15 marks)

22. (a) Verify that the set H of Points $M(x, y)$ whose equation is given by

$$x^2 + 6y^2 + 2x - 36y + 37 = 0 \text{ in the } xOy \text{ plane is an ellipse}$$

- Determine the coordinates of its centre, foci, vertices and its eccentricity e
- Sketch the curve of this ellipse in an orthonormal reference system.

(15 marks)

23. (i) A polynomial G is defined as

$$G(x) = 2x^3 - 5x^2 + x + 2$$

- Calculate $G(2)$

- Determine a , b , and c such that

$$G(x) = (x - 2)(ax^2 + bx + c)$$

- Solve the equation $G(x) = 0$

- Solve, in $\mathbb{R}$ the equation $2\cos 2x = (1 - \sqrt{3})(\cos x - \sin x)$

(15 marks)