GCE ADVANCE LEVEL

FURTHER PURE MATHEMATICS

2007 PAPER 2

Show that as x increases, y lies approximately between $-\sqrt{5}$ and $\sqrt{5}$

3. (a) Find the arc length of the curve $y = \ln(\sin x)$ for $\frac{\pi}{3} \le x \le \frac{\pi}{2}$

(b) On the same diagram, sketch the parabola $y^2 = 4 + x$ and the line x = 2.
The region bounded by these curves is rotated completely about the x – axis. Show that the area of the surface generated is $\frac{62\pi}{3}$

4. (a) Show that the set of all even integers under ordinary addition forms a commutative group.
(b) The operation $*$ and $\circ$ are defined over the set S = {0, 1, 2, 3} as follows.
$a * b = (a + b + 1)\bmod 4$
$a \circ b = (a + b + ab)\bmod 4$
Construct two tables, one for $*$ and one for $\circ$, on the set S. Prove that $(S, *, \circ)$ is a commutative ring.

5. Show that $\sinh^{-1} x = \ln\left(x + \sqrt{1 + x^2}\right)$

(a) Sketch the curve $y = \sinh^{-1} x$

(b) Solve the equation $\sinh^{-1}(2x) = \ln(3x + 1)$

(c) Obtain a series expansion of $\sinh^{-1} x$ in ascending powers of x, up to and including the term in x^3. Hence, obtain to 3 decimal places, an estimate for $\int_0^1 \sinh^{-1} x\,dx$.

6. (a) Express $f(x)$, where $f(x) = \frac{x^3 + 1}{(x-1)^2(x^2+1)}$, in partial fractions.

Hence, or, otherwise, find $\int f(x)dx$.

(b) A curve C is given by the polar equation $r = a(1 + 2\cos\theta), a > 0, -\pi < \theta \le \pi$. Find the equation of the tangents at the pole and sketch C.

7. Given that the position vector of the points A, B, C, D are
$(i + 2j + 3k), (-2i + 8j + 9k), (5i + 7j)$ and $(3i + 4j + 2k)$ respectively.
Find
(a) the angle BAC.
(b) The area of triangle ABC
(c) The Cartesian equation of the plane ABC.
(d) The volume of the tetrahedron ABCD.

8. (a)Expand $\ln(1 + \sinh x)$ as a series in ascending powers of x, up to and including the term in x^2.

Hence, show that $(1 + \sinh x)^{\frac{2}{x}} = e^2\left(1 - x + \frac{x^2}{2}\right)$.

(b) Solve for real values of x, the equation
$$10\cosh\left(\ln\frac{x}{5}\right) + 8\sinh(\ln x) = 38.$$

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9. (a) Given that $z = \cos\theta + i\sin\theta$, show that $z^n + z^{-n} = 2\cos n\theta$ and $z^n - z^{-n} = 2i\sin n\theta$.

Hence, show that $\sin^4\theta = \frac{1}{4}\left[4\sin^2\theta - \sin^2 2\theta\right]$

(b) Given that $z - 1 = (z - 2)e^{i\alpha}$, where α is real, prove that $z = \frac{1}{2}\left[3 - i\cot\left(\frac{\alpha}{2}\right)\right]$