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# GCE ADVANCE LEVEL

# FURTHER PURE MATHEMATICS

# 2009 PAPER 2

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# CGCEB – Further Mathematics Paper 2, JUNE 2009

1.

i. Find the general solution to the differential equation

$$x\frac{dy}{dx} - y = x^2$$

.

ii. Find the value of the constant $a$ such that $y = axe^{-x}$ is a solution of the differential equation

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 2e^{-x}$$

.

Find the solution of this differential equation for which $y = 1$ and $\frac{dy}{dx} = 2$ when $x = 0$.

2.

a. Express into partial fractions

$$f(x) = \frac{1}{(x^2+1)(x-1)^2}$$

.

b. Hence, evaluate $\int_2^3 f(x)\,dx$.

c. Given that

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n\theta\,d\theta,$$

show that for $n \geq 2$, $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$. Hence, evaluate $I_5$.

3.

a. Using the definition of $\sinh x$ in terms $e^x$, show that $\sin^{-1} x = \ln(x + \sqrt{1+x^2})$.
Hence or otherwise, show that

$$\int_0^4 \frac{1}{\sqrt{9x^2+4}}\,dx = \frac{1}{3}\ln(6+\sqrt{37})$$

b. Show that

$$\int_0^{\frac{1}{4}} \tanh^{-1} 2x\,dx = \frac{1}{8}\ln\left(\frac{27}{16}\right).$$

4.

i. Determine whether or not the following series converge:

a. $\sum_{r=0}^{\infty} \frac{r^r}{r!}$ ; $\left(\text{Hint: } \lim_{r\to\infty}\left(1+\frac{1}{r}\right)^r = e\right)$

b. $\sum_{r=0}^{\infty} \left(\frac{3}{2}\right)^{r^2}$

c. $\sum_{r=1}^{\infty} \left(\frac{r}{2^r}\right)$

ii. Given that the terms in $x^5$ and higher powers of $x$ may be neglected, show that

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$$e^{\cos^2 x} \approx e\left(1 - x^2 + \frac{5}{6}x^4\right).$$

5.

i. Find graphically or otherwise, the set of values of $x$ foe which $|2x-5| + |x+2| > 7$.

ii. Given that

$$f(x) = \frac{x^2 - 5x + 6}{x - 1},$$

a. Find the equation of the two asymptotes to the curve $y = f(x)$.

b. Sketch the curve $y = f(x)$, showing clearly the intercepts, asymptotes and turning points.

6.

i. Show that $1 + i$ is a root of the equation $z^4 + 2z^3 - z^2 - 2z + 10 = 0$.
Hence or otherwise, find the remaining roots of the equation.

ii. If $z = \cos\theta + i\sin\theta$, show that:

a. $z + \frac{1}{z} = 2\cos\theta$.

b. $z^n + \frac{1}{z^n} = 2\cos n\theta$.

Hence, or otherwise, show that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$. Evaluate

$$\int_0^{\frac{\pi}{12}} (32\cos^6\theta - 15\cos 2\theta)\, d\theta.$$

7.

i. Use the theorems of Pappus to calculate:

a. The volume and,

b. The surface area, of the solid generated when the region for which $x^2 + (y-9)^2 \leq 9$ is rotated through $2\pi$ radians about the $x$ −axis.

ii. A curve is given parametrically $x = \theta - \sin\theta$, $y = 1 - \cos\theta$.

a. Show that the length of the curve, for $0 \leq \theta \leq 2\pi$, is 8.

b. If the arc in (a) is rotated through one complete revolution about the $x-$ axis, show that the surface generated is $\frac{64}{3}\pi$.

8.

a. Find the equation of the normal at the point $P(4, 1)$ to the rectangular hyperbola $xy = 4$.
This normal meets the hyperbola at the point $Q$. Find the length of $PQ$.

b. Write down the equations of the two asymptotes to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$.
The tangent to this hyperbola at the point $P(3\sec\theta, 4\tan\theta)$ meets the asymptotes at $S$ and $T$. Show that $P$ is the midpoint of $ST$.

9.

i. A binary operation $*$ is defined on $\mathbb{R}$, the set of real numbers, by $a * b = a + b + ab$. Determine whether or not $(\mathbb{R}, *)$ forms a group.

ii. Define a mapping from $(\mathbb{R}, +)$ to $(\mathbb{R}, +)$ by $f(x) = 3x$, where $\mathbb{R}$ is the set of real numbers.

a. Show that $f$ is a homomorphism.

b. Show, also, that $f$ is an isomorphism.

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