GCE ADVANCE LEVEL

FURTHER PURE MATHEMATICS

2015 PAPER 2

1. Express $8\cosh(x+\ln 4)$ in the form $Ae^{x}+Be^{-x}$ where A and B are real numbers.
Hence, or otherwise,
(i) find the two solutions of the equation $8\cosh(x+\ln 4)=12-e^{-x}$
(ii) show that
$$\int_{0}^{\ln 2}\frac{1}{\cosh(x+\ln 4)}dx=2\tan^{-1}\left(\frac{4}{33}\right)$$

2. (a) Find the set of values of x for which
$$\sum_{r=1}^{\infty}(-1)^{r-1}\frac{x^{r}}{2^{r}(r+1)}$$ is convergent

(b) The function f, for each positive integer n, is given by $f(n)=15^{n}-8^{n-2}$.
(i) Express $f(n+1)-8f(n)$ in the form $k(15^{n}), k\in Z$
(ii) Hence, or otherwise, prove that $15^{n}-8^{n-2}$ is a multiple of 7 for all $n\geq 2$

3. (a) Solve completely the equation
$z^{6}-64=0$, giving your answers in the form $re^{i\theta}, r>0, -\pi<\theta\leq\pi$
(b) Given that $\omega=e^{i\theta}, \theta\neq n\pi, \pi\in$ □, show that

i. $$\frac{\omega^{2}-1}{\omega}=2i\sin\theta$$

ii. $$(1+\omega)^{2}=2^{n}\left(\frac{1}{2}\theta\right)e^{\frac{1}{2}(i-n\theta)}$$

(c) Given that one of the roots of the equation $z^{4}=a(1+i\sqrt{3})$ is $z=2e^{\frac{i\pi}{12}}$. Find the value of a

4. By using the substitution $\frac{dy}{dx}=v-x$, or otherwise, show that the differential equation
$$(x^{2}-1)\frac{d^{2}y}{dx^{2}}-2x\frac{dy}{dx}=x^{2}+1$$
can be transformed to the differential equation
$$\frac{dv}{dx}=\frac{2xv}{x^{2}-1}, \text{ where } y=f(x).$$
Hence find the solution, in the form $y=f(x)$, of the differential equation
$$(x^{2}-1)\frac{d^{2}y}{dx^{2}}-2x\frac{dy}{dx}=x^{2}+1,$$ for which $y=-2, \frac{dy}{dx}=1$, when $x=2$

5. (a) The parametric equations of a curve are

$$x=\frac{2}{3}(t-1)^{\frac{3}{2}} \text{ and } y=\frac{2\sqrt{2}}{3}(1+t)^{\frac{3}{2}},\ 2\le t\le 4$$

Show that the length of the arc is

$$\frac{4\sqrt{3}}{3}\left(4-\sqrt{2}\right)$$

(b) A function f is given by

$$f(x)=\frac{1}{4}\sqrt{4x-1},\ \text{for } 1\le x\le 9.$$

Show that the surface area obtained when the curve $y=f(x)$ is rotated completely about the x - axis is $\frac{104}{3}\pi$

6. Find the equation of the normal at the point $P(4\cos\theta, 3\sin\theta)$ on the ellipse

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

The normal at P crosses the x - and y - axes at A and B respectively.

(i) Find the coordinates od A and B.

(ii) Show that as θ varies, the locus of the point M, the midpoint of AB is another ellipse and find its equation.

7. (a) Investigate and sketch the curve $y=|x-2|+|3x+4|$.

Hence, or otherwise, solve the inequality $|x-2|+|3x+4|\le 6$

(b) Given that $r=2a\sin^2\theta, a>0$ is the polar equation of a curve.

(i) Find the equation of the tangents at the pole to the curve.

(ii) Show that $f(\theta)=2a\sin^2\theta$ is an even function.

(iii) Sketch the curve $r=2a\sin^2\theta$.

8. (a) Two plane Π_1 and Π_2 with equations $px+4y-2z=10$ and $5x+y+pz=13$ respectively are perpendicular, find the value of p.

(b) Two lines L_1 and L_2 are such that $L_1: r=2i+j-k+\lambda(i+j+k)$ and L_2 passes through the point $(3,1,-1)$ and is parallel to the line $r=j+t(2i+j+k)$.

Find

(i) the position vector of the intersection, r_0, of L_1 and L_2.

(ii) the Cartesian equation of the plane containing L_1 and L_2.

(iii) The area of the triangle with vertices at the point with position vector r_0 and the points when $\lambda=2$ on L_1 and $t=1$ on L_2

9. (a) Given that x, y are integers, find the general solution of the equati...

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$$969x-1683y=51$$

(b) Using Euclid's algorithm, or otherwise, find the greatest common divisor, h, of the numbers $a=1625$ and $b=858$ and express it in the form, $h=sa+tb$, where s and t are integers