

**GCE ADVANCE LEVEL
FURTHER PURE MATHEMATICS
2017 PAPER 2**

Hence, show that $\int_0^1 \frac{5x^2}{(x^2-4)(x^2+1)} dx = \frac{\pi}{4} - \ln 3$

3) A sequence (U_n) is defined recursively by

$$U_0 = \frac{1}{2} \text{ and } U_{n+1} = \frac{2}{1+U_n}, \text{ for all } n \in \mathbb{N}$$

- Find U_1 and U_2
- Show by mathematical induction that all the terms of the sequence are positive.
- Given that the sequence (U_n) is convergent, show that its limit, l , is a solution of the equation $x^2 + x - 2 = 0$. Hence find l
- Given that (V_n) , a sequence of general term such that

$$V_n = \frac{U_n - 1}{U_n + 2}, \forall n \in \mathbb{N}$$

Show that (V_n) is convergence and determine its limit.

Hence, deduce the convergence of the sequence (U_n)

4) (i) Show that

$$\frac{d}{dx} (2 \tan^{-1} e^x) = \sec hx$$

(ii) A curve is given by the parametric equations

$$x = \sec ht, y = \tanh t, \quad 0 \leq t \leq \frac{1}{2} \ln 3.$$

Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \sec^2 ht$

(iii) Show that the length of the curve is $\frac{\pi}{6}$

(iv) The curve is rotated through 2π radians about the x -axis. Show that the area of the surface generated is $\pi(2 - \sqrt{3})$

5) (a) Using truth tables, or otherwise, determine which of the compound statements $(P \wedge \sim P) \Rightarrow Q$, and $(P \wedge \sim P) \vee Q$ is a tautology and which is a contingency

(b) Prove that an integer is odd if and only if n^2 is odd.

6) A transformation M is given by

$$M = \begin{pmatrix} 3 & 2 & 2 \\ -1 & 0 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

- Find the inverse of M
 - Find the values of λ for which $|M - \lambda I| = 0$, where I is the identity matrix.
 - Find the image of the plane $x + 3y = 0$ under the transformation M
- 7) Given that $z = \cos \theta + i \sin \theta$,
 Show that $\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta$ and find $\sin 5\theta$.
 Hence, or otherwise, show that

$$\cot 5\theta = \frac{\cot^5 \theta - 10\cot^3 \theta + 5\cot \theta}{5\cot^4 \theta - 10\cot^2 \theta + 1}$$

Solve completely the equation $\cot 5\theta = 0$, show that $\cot\left(\frac{\pi}{10}\right) = \sqrt{5 + \sqrt{20}}$

- 8) (a) (i) Given congruence $7^x \equiv k \pmod{9}$, find k for each of the values $x = 1, 2, 3, 4$
 (ii) Hence, or otherwise find the remainder when 7^{1200} is divisible by 9
 (iii) Find the general solution of the congruence $7^x \equiv 1 \pmod{9}$, $x \geq 0$

(b) A set F under function composition, \circ , has elements $\{f_1, f_2\}$, where $f_1 = x, f_2 = \frac{1}{x}$

Show that (F, \circ) is a group.

- 9) (a) Show that the vectors

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \text{ are linearly independent}$$

Express $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ as a linear combination of v_1, v_2 and v_3

(b) Find the Cartesian equation of an ellipse with foci at the points $(0,0)$ and $(8,0)$ and eccentricity $\frac{1}{2}$

10) Given that $f(x) = \ln(1 + \cosh 2x)$

- State the domain of f
- Show that as $x \rightarrow \infty$, $f(x) \rightarrow 2x + \ln 2$
- Find the asymptotes to the curve C , $y = f(x)$
- Investigate the variation of f and draw the table of variation.
- Sketch C

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