

GCE ADVANCE LEVEL
FURTHER PURE MATHEMATICS
2014 PAPER 3

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- a) A particle of mass 3kg moves under the action of a force F such that its position vector at time t seconds is given by $F = (2t + 1)\mathbf{i} + t^3\mathbf{j} + \frac{4}{3}t^3\mathbf{k}$.
Find, when $t = 3$,
- The kinetic energy of the particle,
 - The magnitude of the force F ,
 - The power developed by the particle. Find, also,
 - The work done by the force in the interval $1 < t < 4$,
 - The cosine of the angle between the velocity and acceleration vectors of the particle when $t = 1$.

4. A sphere, of mass $2m$, moving speed $2u$ on a smooth horizontal plane, collides directly with another sphere B of radius and of mass m which is moving with speed u in the opposite direction. Given that the coefficient of restitution between the spheres is $\frac{1}{2}$, find

- Their speeds after the collision,
- The magnitude of the instantaneous impulse,
- The loss in kinetic energy caused by the collision,

After a short interval, the sphere A is given a horizontal impulse of magnitude $7mu$ so that it collides again directly with sphere B . Find the speed of A and the speed of B after second impact.

3.

(a) A uniform ladder, of weight and length $2l$, rests with its upper end against a smooth vertical wall and its lower end on a rough horizontal ground. The coefficient of friction between the ladder and the ground is $\frac{1}{2}$. Given that the ladder is in limiting equilibrium, find the angle which the ladder makes with the horizontal.

7. A particle of mass m kg is projected vertically upwards with speed u ms^{-1} . The resistance to the motion of the particle is of magnitude mkv , where k is a positive constant and v is the speed at time t seconds. Find the velocity of the particle at time t seconds.

(a) i) Forces F_1 , F_2 and F_3 act at point vectors r_1 , r_2 and r_3 respectively, where $F_1 = (2\mathbf{i} - \mathbf{j})\text{N}$, $r_1 = (\mathbf{i} - 4\mathbf{j})\text{m}$, $F_2 = (-3\mathbf{i} + 5\mathbf{j})\text{N}$, $r_2 = (2\mathbf{i} - \mathbf{j})\text{m}$, $F_3 = (\mathbf{i} - 4\mathbf{j})\text{N}$, $r_3 = (3\mathbf{i} + 2\mathbf{j})\text{m}$. Show that this system of three forces forms a couple, and find the magnitude of the couple.

8. Two particles P and Q have velocities of $(3\mathbf{i} + 4\mathbf{j})$ ms^{-1} and $(-4\mathbf{i} + 2)\text{ms}^{-1}$ respectively. Initially, the position vectors of P and Q are $(13\mathbf{i} - 3\mathbf{j})\text{m}$ and $(12\mathbf{i} + 5\mathbf{j})\text{m}$ respectively. Find the distance between them at any time t . Hence find, to two decimal places, the least distance between them.

(a) A car of mass 9000kg pulls a carriage of mass 600kg. There is a total non gravitational resistance of 500N and this is divided between the car and the carriage in the ratio of their masses.

The engine of the car produces a constant pull and the carriage accelerates from a speed of 8ms^{-1} to a speed of 12ms^{-1} distance of 20m .

Find the magnitude of

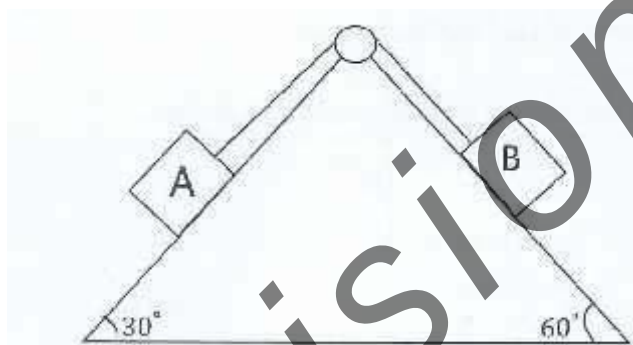
The tractive force of the engine of the car,
The tension in the tow-bar when the motion takes place on level ground.

(f) One end of light elastic string of natural length 6m is attached to a fixed point A and a mass 2kg is attached at the other end B. A horizontal force of magnitude F newtons is applied to the particle so that it is at rest with string taut and inclined at 30° to the horizontal. Given that the vertical distance from point A is 5m, find

- The value of F ,
- The modulus of elasticity of the string.

(Take g as 10ms^{-2})

6.



Two particles A and B , of masses $2m$ and $6m$, rest on the smooth and rough inclined faces respectively of a fixed wedge as in Fig. 1. They are connected by a light inextensible string passing over a smooth pulley fixed at the top of the wedge. The smooth face of the wedge is inclined at angle 30° to the horizontal while the rough face is inclined at angle 60° to the horizontal. The system is released from rest with the string taut. Given that the coefficient of friction between B and the plane is $2/3$ show that

- the acceleration of the particle is $\frac{g}{3}(\sqrt{3} - 1)\text{ms}^{-2}$
- the tension in the string is $\frac{20}{3}(3\sqrt{3} + 1)\text{N}$

iii) Find, in terms of g and m , the force exerted by the string on the pulley.

The particle B hits the ground 2 seconds after the system is released from rest and does not rebound from the ground.

- Show that the further distance which A travels before first coming to momentary rest is $g[4/3(\sqrt{3} - 1)]^2$

8. i) $ABCD$ is uniform rectangular plate of mass $6m$. The sides $AB = CD = 3a$ and $AD = BC = 4a$.

Particles of masses $3m$, $2m$ and $5m$ are attached at the vertices B , C and D respectively. Find the distance of the centre of mass of the loaded plate from

- a The side AD
- b The side AB .

The vertex D of the loaded plate is freely hinged to fixed point and the plate hangs at rest in equilibrium.

d) Find the angle between DC and the downward vertical,

- i A compact disc, spinning at a constant angular acceleration, spins 5 revolutions in the first second and 10 revolutions in the next second. Find the initial angular velocity, in rad s^{-1} , of the compact disc.

8.

- ii. Three random events A , B and C are such that $P(A) = 1/5$, $P(A \cup C) = 19/60$ and $P(B \cap C) = 1/24$. Events B and C are independent while events A and B are mutually exclusive.
 - a Find $P(B)$ and $P(A \cup B)$.
 - b Show that events A and C are independent.

- d) A research is carried on the existence of a disease in a certain population. It is assumed that 10% of the population has the disease. To verify this assumption, a test is conducted. It is found out that a person assumed to have the disease has 75% chance of the test being positive and a person assumed not to have the disease has a 5% chance that the test will be positive. Draw a tree diagram to illustrate this information. Hence find, the probability that

A person has the disease and test positive

The test is positive,

- d) A person has the disease, given that the test is positive.