

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics

4030/2

Paper 2

Tuesday

15 NOVEMBER 2016

Additional Materials:

Answer Booklet;
Electronic calculators

Time: 2 hours 30 Minutes

Instructions to Candidates

Write your name, centre number and candidate number in the spaces on the separate answer booklet provided.

There are **twelve (12)** questions in this paper. Answer **all** questions.

Write your answers on the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

Check the formulae overleaf

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 SERIES

Arithmetic $S_n = \frac{1}{2}n \{2a + (n-1)d\}$

Geometric $S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$

$$S_{\infty} = \frac{a}{1-r} \quad \text{for } |r| < 1$$

3 TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2}bc \sin A$$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \text{SD} = \sqrt{\left\{ \frac{\sum (x - \bar{x})^2}{n} \right\}} = \sqrt{\left\{ \frac{\sum x^2}{n} - (\bar{x})^2 \right\}}$$

Grouped data

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \text{SD} = \sqrt{\left\{ \frac{\sum f(x - \bar{x})^2}{\sum f} \right\}} = \sqrt{\left\{ \frac{\sum fx^2}{\sum f} - (\bar{x})^2 \right\}}$$

1 Solve the following systems of equations

$$x + 3y - 2z = 5,$$

$$3x - y + z = 9,$$

$$x - 2y - 3z = -7.$$

[6]

2 (a) Find the range of values of x for which $2x^2 - 7x + 3 \geq 0$. [3]

(b) Express $3x^2 - 24x + 50$ in the form $a(x + b)^2 + c$, where a , b and c are constants.
Hence, find the coordinates of the turning point. [4]

3 Solve the equations

(a) $4^{x-1} - 2^x = 8,$ [4]

(b) $\log_3 (2x + 1) - 2 = \log_3 (3x - 11).$ [3]

4 (a) Find the value of q , given that the expression $2x^3 - 3x^2 - qx + 6$ is divisible by $2x - 1$. [3]

(b) The expression $7x^2 + 35$ and $44x - 2x^3$ leaves the same remainder when divided by $x - a$. find the three possible values of a . [4]

5 (a) In how many ways can 7 red marbles and 3 green marbles be put in a straight line if
(i) there are no restrictions, [2]

(ii) green marbles should not be next to each other? [3]

(b) A group of 6 pupils is to be chosen from 8 boys and 6 girls.
Find the number of ways of choosing at least 4 girls. [3]

6 (a) Solve the equation $2\sin 2x + 1 = 0$ for values of x in the range $0^\circ \leq x \leq 360^\circ$. [3]

(b) (i) Express $5\sin \theta - 12\cos \theta$ in the form $R \sin (\theta - \alpha)$ where $R > 0$ and $0 < \alpha < 90^\circ$. [3]

(ii) Hence solve the equation $5\sin \theta - 12\cos \theta = 6.5$, giving all solutions between 0° and 360° . [2]

- 7 (a) The third and sixth terms of a geometric progression are $2\frac{2}{3}$ and $\frac{8}{81}$ respectively.

Find

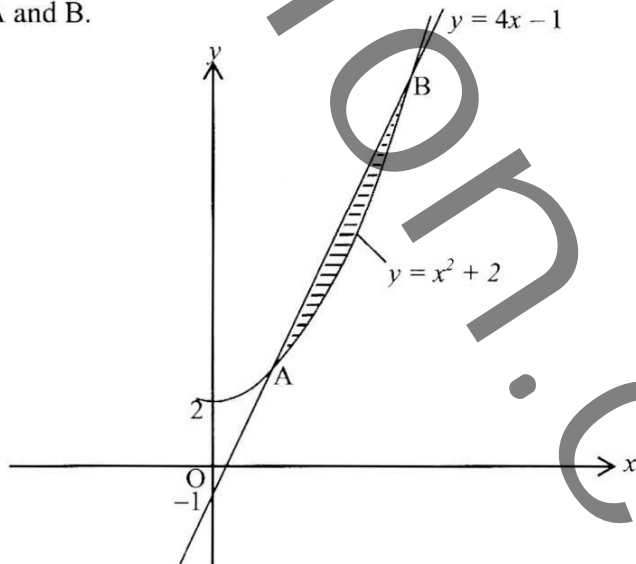
- (i) the common ratio and the first term, [3]
 (ii) the sum to infinity of the geometric progression. [2]
 (b) The n th term of an arithmetic progression is $4n + 1$. Find, in terms of n , the sum of the first n terms of the progression. [4]

- 8 A new drug for a certain disease was administered to 60 patients of different age groups. The table below shows the results obtained.

Age (x)	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35
Frequency	5	8	12	18	9	6	2

- (a) State the median class. [1]
 (b) Calculate
 (i) an estimate of the mean, [2]
 (ii) the standard deviation. [6]

- 9 The diagram below shows parts of the curve $y = x^2 + 2$ and the line $y = 4x - 1$ intersecting at the points A and B.



Find

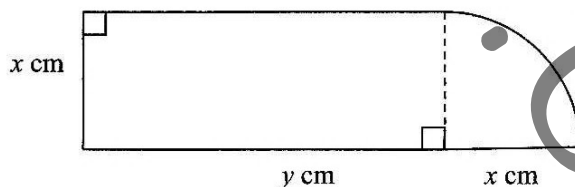
- (a) the coordinates of A and B, [4]
 (b) the volume obtained by rotating the shaded region through 360° about the x -axis. [5]

- 10 A small body moves in a straight line so that its velocity v m/s at time t seconds from the starting point O, is given by $v = 6t^2 - 18t + 12$.
- (a) Find an expression in terms of t for
- (i) its acceleration, [2]
 - (ii) its distance from O. [2]
- (b) Calculate the maximum velocity and the distance covered when the velocity is maximum. [6]
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- 11 (a) A curve has the equation $y = 2x^3 - 9x^2 + 12x + 8$.
- (i) Find the coordinates of the stationary points of the curve. [3]
 - (ii) Determine the nature of the stationary points. [3]
- (b) Find the coordinates of the point where the curves $y = e^{\frac{x}{2} - 3}$ and $y = e^{7+3x}$ meet and the gradient of each curve at the point. [4]
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12 Answer only one of the following alternatives:

Either

- (a) Given that $y = x + \sin 2x$, find the values of x for which $\frac{dy}{dx} = 0$, for the range $0 \leq x \leq \pi$ [3]
- (b) The diagram below shows a metal plate consisting of a rectangle of length y cm and width x cm, and a quarter-circle of radius x cm. The perimeter of the metal plate is 60 cm.



- (i) Show that the area of the plate, A cm², is given by $A = 30x - x^2$. [3]
- (ii) Given that x can vary, find the value of x at which A is stationary, [1]

- (iii) Hence, find the stationary value of A and determine whether it is a maximum or a minimum. [3]

Or

- (a) The eighth term of an arithmetic progression is four times the fifth term and the sum of the first eight terms is 20. Calculate the sum of the first ten terms. [5]

- (b) The second and the fifth terms of a geometric progression are $\frac{1}{18}$ and $\frac{4}{243}$ respectively.

Find

- (i) the common ratio and the first term, [3]
(ii) the sum to infinity of the progression. [2]
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