

P Maths with Mechs/P Maths with Stats 2
765/770

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2000

ADVANCED LEVEL

Subject Title	Pure Mathematics With Mechanics Pure Mathematics With Statistics
Paper No.	Paper 2
Subject Code No.	765/770

Two and a half hours

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and tables are provided.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

1. (i) Find an equation of the tangent to the curve

$$y = \ln(4 + x^2)$$

at the point where $x = 1$.

- (ii) Solve the differential equation

$$\frac{dy}{dx} - xy - x = 0,$$

given that $y = 0$ when $x = 0$.

(10 marks)

2. Given that f is a periodic function of period 5 and that

$$f(x) = \begin{cases} -x^2 - x + 6, & 0 \leq x < 2, \\ ax - 4, & 2 \leq x \leq 5, \end{cases}$$

- (a) find the value of a for which f is continuous,
 (b) find the value of $f(-1)$,
 (c) sketch the graph of $y = f(x)$ in the interval $-3 \leq x \leq 7$.

(8 marks)

3. (i) Find n given that

$$\sum_{r=1}^n (5 - 2r) = -165.$$

- (ii) The sum of the first n terms of a series is $9\left(1 - \frac{1}{3^n}\right)$.

- (a) Find the n^{th} term of the series.
 (b) Show that the series is a geometric series and find its sum to infinity.

(9 marks)

4. (i) Sketch the curve $y = 3x^2 - 12x$ and hence find the area of the region bounded by the curve and the lines $y = 0$, $x = 0$ and $x = 5$.

- (ii) Find the volume generated by rotating completely the area bounded by the curve $x = 3(y^2 - 1)$ and the lines $x = 0$ and $x = 6$, about the x -axis. Find also the x -coordinate of the centroid of the solid generated.

(10 marks)

5. The equations of two circles C_1 and C_2 are $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x = 0$, respectively.

- (a) Show that the circles C_1 and C_2 touch each other internally and find an equation of the common tangent at the point of contact.
 (b) Find the equation of the tangents to the circle C_1 , which are perpendicular to the common tangent.
 (c) Find the area between the two circles.

(11 marks)

6. (a) Write the complex number $\frac{1 + \sqrt{3}i}{2 - 2i}$ in the form $r(\cos \theta + i \sin \theta)$, where θ is in radians.

- (b) P_1 and P_2 are the points representing the complex numbers $3 + i$ and $-1 + 3i$ respectively. Show that OP_1 is perpendicular to OP_2 , where O is the origin.

(10 marks)

3

7. Given that the position vectors of the points A , B , C and D relative to the origin O , are $\mathbf{a} = 3\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, $\mathbf{c} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{d} = 3\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$ respectively, find

- a vector equation of the line AB ,
- cartesian equations of the line, l_1 , through A and D ,
- a cartesian equation of the plane, π , containing A , B and C .
- the angle between the plane, π , and the line, l_1 , to two decimal places.

(13 marks)

8. (i) Show that $\cos(2 \tan^{-1} x) = \frac{1-x^2}{1+x^2}$.
- (ii) Find, in radians, the general solution of the equation, $\sin 5x + \sin 3x = 0$.
- (iii) Express $7 \cos \theta - 5 \sin \theta$ in the form $R \cos(\theta + \lambda)$, where $R > 0$ and $0 < \lambda < 90^\circ$, giving R and λ correct to one decimal place.
- Hence find the value of θ in the interval $0^\circ < \theta < 360^\circ$ for which $7 \cos \theta - 5 \sin \theta = 8.6$.

(11 marks)

9. Find

- $\int x e^{x^2+3} dx$,
- $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}} dx$, using the substitution $x = \cos^2 t$.

(8 marks)

10. (i) A person sends messages by using six flags of which one is blue, two are white and three are red. He sends messages by hoisting the flags in a row, the message being conveyed by the order in which the colours of the flags are arranged. How many different messages can be sent by using

- all the six flags,
- five flags only?

- (ii) Given that $x = e^{3t}$ and $y = e^{2t+3}$, find $\frac{dy}{dx}$ in terms of x .

(10 marks)