CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2001 ADVANCED LEVEL

Subject Title	**Mathematics With Mechanics**
	Mathematics With Statistics
Paper No.	**Paper 2**
Subject Code No.	**765/770**

Two and a half hours.

Full marks may be obtained for answers to **ALL** questions.

Mathematical formulae and tables are provided.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

2000/765/770/2/B

1. Relations R_1 and R_2 are defined on the set of integers by

$x\,R_1\,y$ if and only if $x+y$ is an even number,

$x\,R_2\,y$ if and only if $x+y$ is an odd number.

Show that R_1 is an equivalence relation and that R_2 is not.

The relation R_3 is defined on the set S, where

$S=\{3,6,12,24\}$,

by $x\,R_3\,y$ if and only if x is a factor of y. Show that R_3 is a strict order relation.

(8 marks)

2.

x	2	3	4	5	6
y	56.6	350.7	1280.0	3493.9	7963.3

The table shows corresponding values of x and y obtained experimentally. By drawing a suitable graph relating $\log_{10} x$ and $\log_{10} y$, show that these values support the hypothesis that x and y are connected by a relationship of the form

$$y=px^q,$$

where p and q are consants. Use your graph to estimate the values of p and q to one decimal place.

Hence estimate the value of $\int_2^6 y\,dx$. Use the trapezium rule to obtain another estimate for $\int_2^6 y\,dx$.

(12 marks)

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3. (i) The angle between two vetors $\mathbf{a}$ and $\mathbf{b}$ is $\pi/3$. Given that $|\mathbf{a}|=3$ and $|\mathbf{b}|=4$, find

(*a*) $|2\mathbf{a}+3\mathbf{b}|$, Sine or cosine rules

(*b*) $|2\mathbf{a}-3\mathbf{b}|$. Draw vector diagram

(ii) Show that the lines

$\mathbf{r}=(1-\lambda)\mathbf{i}+3\lambda\mathbf{j}+(2\lambda+5)\mathbf{k}$,

$\mathbf{r}=2\mu\mathbf{i}+(\mu-4)\mathbf{j}+3\mathbf{k}$,

where μ and λ are parameters, intersect, giving the point of intersection.

Write down the vector parametric equation of the plane which contains these lines.

(11 marks)

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4. (i) Show that

$$\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}\equiv\tan\theta.$$

(ii) Find the general solution of the equation

$$\sin 4x-\cos 2x=0.$$

(9 marks)

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5. (i) Differentiate

$$\frac{1}{x-3} + 2\ln\left(\frac{x-4}{x-3}\right)$$

with respect to x and express your answer as a single fraction.

either (ii) Solve the differential equation OR Q9 (a)(i)

$$xy\frac{dy}{dx} = 1 + y^2,$$

given that when $x = 2$, $y = 0$.

(9 marks)

6. Express $f(x)$, where

$$f(x) \equiv \frac{x + x^2}{(x^2+1)(x-2)}$$

in partial fractions and hence show that

$$\int_0^1 f(x)\,dx = \frac{3\pi}{20} - \frac{13}{10}\ln 2.$$

(9 marks)

7. Find, in the form $x + iy$, where $x, y \in \Re$, the square roots of $\frac{169}{5-12i}$.

Find, also, in degrees, to one decimal place, the principal value of the argument of each of the square roots.

(11 marks)

8. Given that n is a positive integer, prove, by induction, that $7^n + 5$ is divisible by 6. Guess a common factor of $5^n + 3$ for all positive integers n and prove your guess by induction.

(9 marks)

9. Find the set of real values of x for which

(*a*) $\left|\frac{3x+4}{2x-3}\right| < 1$,

(*b*) $\frac{3x+4}{2x-3} < 1$.

(8 marks)

10. The function f is defined by

$$f(x) = \frac{3x-1}{(x-1)^2}, \qquad x \neq 1.$$

(*a*) Sketch the graph of $f(x)$, showing clearly the coordinates of the truning point(s) and its behaviour as it approaches its asymptotes.

(*b*) Show that the area of the finite region bounded by the curve and the lines $x = 4$ and $y = 2$ is $\frac{5}{3} + 3\ln\frac{2}{3}$.

(14 marks)