

P Maths With Mechs/ P Maths with Stats 2

765/770

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

Etuge's Library

JUNE 2003

ADVANCED LEVEL

Subject Title	Pure Mathematics With Mechanics
	Pure Mathematics With Statistics
Paper No.	Paper 2
Subject Code No.	765/770

Two and a half hours.

Full marks may be obtained for answers to **ALL** questions.

Mathematical formulae and tables are provided.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Turn over

1. Express $4 \sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where α is an acute angle and R is a positive real number.

Find

- (a) all solutions of the equation $4 \sin x - 3 \cos x = 3$,
in the interval $0^\circ \leq x \leq 360^\circ$, giving your answers to the nearest degree,
(b) the greatest and least values of

$$\frac{1}{4 \sin x - 3 \cos x + 6}.$$

(10 marks)

2. (i) Differentiate $\sin x$ with respect to x from first principles.

(ii) Find $\frac{dy}{dx}$ if

(a) $y = (x \sin x)^2$,

(b) $y = \ln \left(\frac{1 + 3x^2}{1 - \tan x} \right)$.

(9 marks)

3. With respect to the origin O , the points A , B , C and D have position vectors $(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ m, $(3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m, $(2\mathbf{j} - \mathbf{k})$ m and $(\mathbf{i} - 2\mathbf{j} - \mathbf{k})$ m respectively.

Find

- (a) a vector equation of the line AB ,
(b) an equation of the plane BCD in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$, where s and t are parameters.

Show that the point A does not lie on the plane BCD .

(12 marks)

4.

x	0.50	0.25	0.17	0.13	0.10	0.83	0.07
y	0.38	0.25	0.19	0.15	0.13	0.11	0.09

The table above shows corresponding values of x and y obtained experimentally. It is given that x and y are related by an equation of the form

$$\frac{1}{y} = \frac{1}{a} - \frac{b}{x},$$

where a and b are constants.

By drawing a suitable linear graph relating $\frac{1}{y}$ and $\frac{1}{x}$, estimate the values of a and b correct to two decimal places.

(10 marks)

5. (i) Show that the equation

$$x^5 - x^3 - 1 = 0,$$

has a root in the interval $1 \leq x \leq 2$.

Use two iterations of the Newton - Raphson method to find an approximation to this root correct to 2 places of decimal.

- (ii) Car numbers in the ten provinces of Cameroon are in the form AD 1323 H, where the two letters at the beginning show the province in which the car is registered, the 4 digits following the two letters are taken from the digits 0 to 9 inclusive and the letter at the end is one of the 26 letters of the alphabet. Given that the 4 digits must not all be zero, find the maximum number of cars which can be registered in Cameroon using this system.

(12 marks)

6. (a) Prove by mathematical induction that for all positive integers n ,

$$\sum_{r=1}^n 2r(r+1) = \frac{2}{3}n(n+1)(n+2).$$

- (b) Given that $x - 3$, $2x + 1$ and $7x - 2$ are the first three consecutive terms of an arithmetic progression, find the value of x and hence obtain the sum of the first ten terms of the progression.

(10 marks)

7. The graph of

$$y = \frac{a + bx}{(x-1)(x-4)}$$

has a turning point at $P(2, -1)$. Find the values of a and b and hence, sketch the curve $y = f(x)$ showing clearly the turning points, asymptotes and intercept(s) with the axes.

(12 marks)

8. (i) Given that $z_1 = \sqrt{3} + i$ and $z_2 = 1 - 3i$, find

- (a) $\frac{z_1}{z_2}$ in the form $p + iq$ where p and q are real,

- (b) $\arg z_1^5$ and $|z_2|^6$.

- (ii) Show that the roots of the equation $z^3 - 1 = 0$ lie at the vertices of an equilateral triangle.

(10 marks)

9. (a) Find

$$\int \frac{x+2}{(x+1)(x^2+1)} dx.$$

- (b) Evaluate $\int_0^4 |x-2| dx$.

(9 marks)

10. (i) Solve the inequality

$$(x - 3)(x + 2)(x + 4) > 0.$$

(ii) (a) Find the range of values of x for which the series

$$\sum_{r=1}^{\infty} \left(\frac{x-3}{x-1} \right)^r$$

converges.

(b) Determine the value of x for which

$$\sum_{r=1}^{\infty} \left(\frac{x-3}{x-1} \right)^r = \frac{1}{2}.$$

(9 marks)
