SECTION A: OPTICS

Light is a form of energy that travels in a straight line.

Some objects produce light on their own and these are called **luminous objects** e.g sun, fire worms, fire fly .

Most objects we see don't produce light on their own but reflect it from luminous objects and these are called **non-luminous objects** e.g. moon, the stars, etc.

Some objects do not allow light to go through them and these are called **opaque objects** e.g. wood, wall, people, etc.

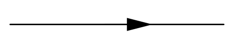
Some of them allow most of the light to go through them and these are called **transparent objects** e.g. glass, clear water, clear polythene.

Other objects allow some light to go through them and these are called **translucent objects** e.g. paper, bathroom glasses, tinted glass, etc.

RAYS AND BEAMS

A ray is the direction of the path taken by light.

It is indicated by a straight line with an arrow on it.

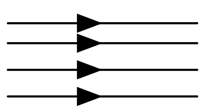


A beam is a collection of light rays.

OR: A beam is a stream of light energy.

Types of beams

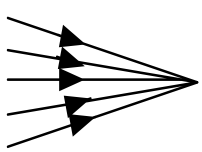
a) Parallel beam



Rays are parallel to each other.

This is obtained from light from a distant source and search lights.

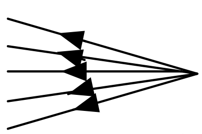
b) Convergent beam



Rays from different directions meet at a common point.

E.g. light behind a convex lens after passing through it.

c) Divergent beam



Rays start from a common point and separate into different directions.

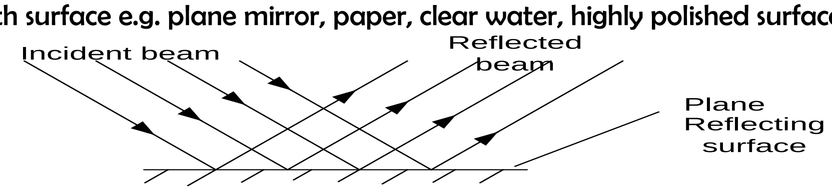
E.g. light from a torch and car lights.

REFLECTION OF LIGHT

Reflection is the bouncing of light as it strikes a reflecting surface.

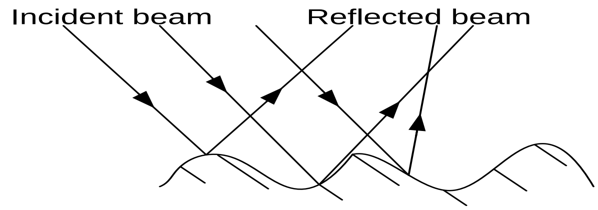
Types of reflection

a) **Regular reflection.** Here, an incident parallel beam is reflected as a parallel beam when light falls on a smooth surface e.g. plane mirror, paper, clear water, highly polished surfaces.

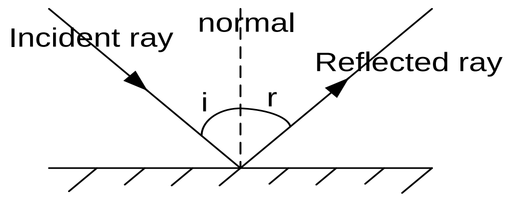


b) **Irregular (diffuse) reflection**

Here an incident parallel beam is reflected in different directions when light falls on a rough surface e.g. iron sheets, unclear water, etc.



Reflection from a plane mirror



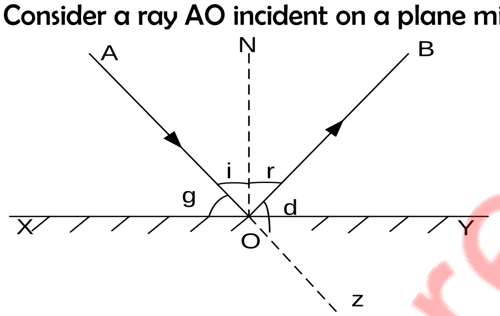
i = angle of incidence (angle between the normal and incident ray)
r = angle of reflection (angle between the normal and reflected ray)

LAWS OF REFLECTION

Law 1: The incident ray, the reflected ray and the normal at the point of incidence all lie on the same plane.
Law 2: The angle of incidence is equal to the angle of reflection.

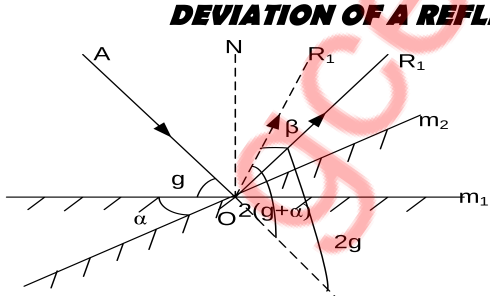
Deviation of light by a plane mirror

Consider a ray AO incident on a plane mirror M at a glancing angle g



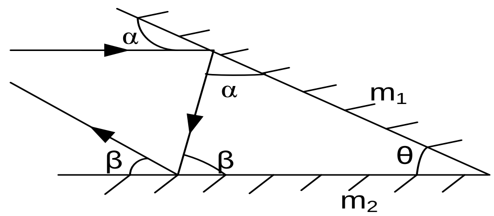
$$d = \angle BOY + \angle ZOY$$
$$\angle ZOY = g \text{ (vertically opposite)}$$
$$\angle BOY = 90 - r$$
$$\text{but } i = r \text{ (law of refelction)}$$
$$\angle BOY = 90 - i$$
$$\text{but } i = 90 - g$$
$$\therefore \angle BOY = 90 - (90 - g) = g$$
$$d = g + g$$
$$d = 2g$$

DEVIATION OF A REFLECTED RAY BY A ROTATED MIRROR



- A constant ray OA is incident onto a plane mirror in position m_1 and is reflected along OR_1
- The glancing angle= g and deviation d caused= $2g$
- When the mirror is turned through an angle α, the new glancing angle is $(g + \alpha)$ and new deviation $2(g + \alpha)$
- The angle β through whoich the ray is rotated is $\beta = 2(g + \alpha) - 2g$
  $$\beta = 2\alpha$$

DEVIATION BY SUCCESSIVE REFLECTION AT TWO INCLINED MIRRORS



- A ray is incident onto a plane mirror m_1 and is reflected
- The glancing angle= α and deviation d caused= 2α

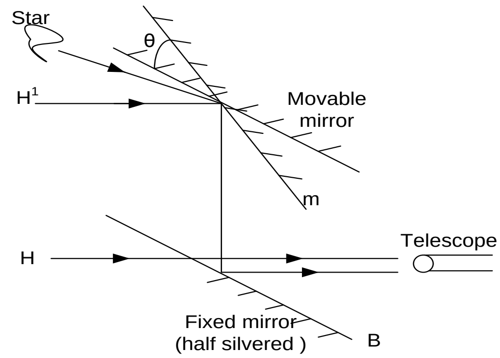
- When the ray is reflected at mirror m_2 , the new glancing angle is β and new deviation2β
- The net deviation, d $= 2(\alpha + \beta)$ clockwise

$$but\ \beta + \theta + \alpha = 180$$
$$\beta + \alpha = 180 - \theta$$
$$d = 2(180 - \theta)$$
$$d = (360 - 2\theta)\ clockwise$$

Or $360 - (360 - 2\theta) = 2\theta\ anticlockwise$

THE SEXTANT

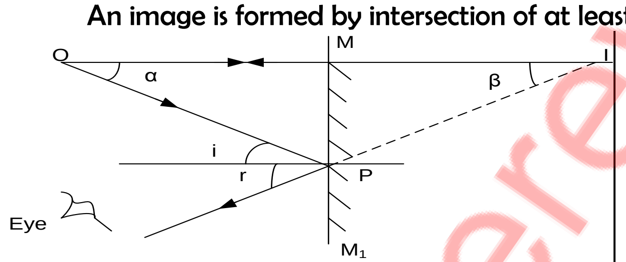
This is an instrument which is used for measuring the angle of elevation of heavenly bodies such as the star and the sun



- The setup is as above. B is half silvered fixed mirror while m can rotate
- First rotate m until the image of horizon H[1] is seen to coincide with horizon H
- At this point m and B are parallel, note position of mirror
- M is now rotated until the image of the star is seen to coincide with the horizon H
- The angle of rotation θ is measured. The angle of deviation is 2 θ

Formation of images by a plain mirror

An image is formed by intersection of at least two rays.



Consider an object O placed infront of a plane mirror. Rays of light form O are reflected form the mirror and appear to come form I. I is the virtual image of O

$$< i =< r\ \ (laws\ of\ reflection\ of\ light\ )$$
$$< i =< \alpha\ \ (alternate\ angles\ )$$
$$< r =< \beta\ \ (corresponding\ angles\ )$$
$$\Rightarrow\ \ \ < \alpha =< \beta$$

Since side MP is common to both triangles ΔOMP and ΔIMP, the triangles are congreguent

Hence $OM = MI$

The image is as far behind the mirror as the object is infront

PROPERTIES OF IMAGES FORMED BY A PLANE MIRROR.

1. The images are the same distance behind the mirrors as the distance of the object in front of the mirror.
2. The images have the same size as the object.
3. The images are erect (upright)
4. The images are laterally inverted (rotated through 180° in the mirror).
5. The images are virtual (they cannot be formed on the screen).

THE REAL AND VIRTUAL IMAGES

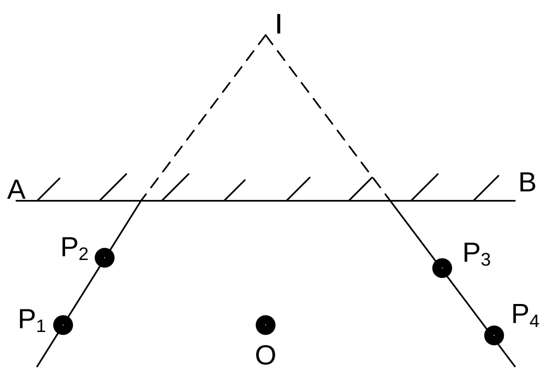
A real image

It is one which can be formed on the screen and is formed by the actual intersection of light rays e.g. images formed by concave mirror and convex lenses.

A virtual image

It is one that cannot be formed on a screen and is formed by the apparent intersection of light rays e.g. images formed by plane mirrors, concave lenses and convex mirrors.

LOCATION OF AN IMAGE ON PLANE MIRRORS

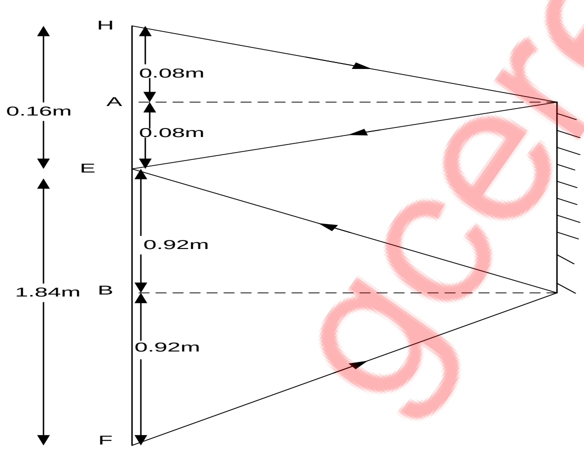


- An object pin O is placed in front of a plane mirror AB, on a white sheet of paper.
- Looking from side A of the mirror, two pins P_1 and P_2 are placed so that they look to be in line with the image of the pin O.
- The experiment is repeated with pins P_3 and P_4 on side B.
- The pins and the mirror are removed and lines drawn through the pin marks P_1P_2 and P_3P_4 to meet at I. I is the position of the image.

Minimum vertical length of a plane mirror

A man 2m tall whose eye level is 1·84m above the ground looks at his image in a Vertical mirror. What must be the minimum vertical length of the mirror so that the man can see the whole of himself **completely** in the mirror ?

Solution



Rays from the top of the man are reflected from the top of the mirror and are incident in the man's eyes (point E is the man's eye level)

Since HA = AE then,

$AE = \frac{1}{2}x \ \ 0{\cdot}16 \ = \ 0{\cdot}08m$

Similarly $EB \ = \ BF.$

Thus $EB = \frac{1}{2}x \ \ 1{\cdot}84 \ = \ 0{\cdot}92m$

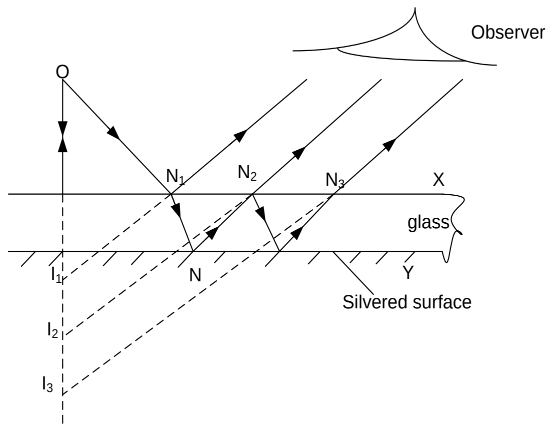
The minimum length of the mirror $= \ AE + EB$

$= \ 0{\cdot}08 \ + \ 0{\cdot}92$

$= \ 1m$

Hence the minimum length of the mirror is half the height of the object

FORMATION OF MULTIPLE IMAGES IN THICK PLANE MIRROR



- A thick plane has two plane surfaces say X and Y, reflection takes place at the two surfaces.
- The reflection at N_1 leads to the formation of image I_1
- The transmitted light is reflected at the silvered surface N, it ndergoes partial reflection and transmission at N_2
- The transmitted light appears to originate from I_2
- The successive internal reflections will lead to multiple images.

Note:

(i) Thick mirror forms multiple images and its distant images are faint. Multiple images is due to multiple images is due to successive reflections and faint images is due the energy absorbed at each reflection

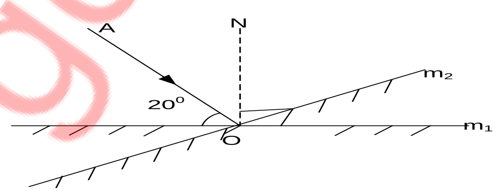
(ii) The disadvantages of using plane mirrors as reflectors of light in optical instruments such as submarine periscopes are overcome by using reflecting prisms.

COMPARISION OF PLANE MIRRORS AND REFLECTING PRISMS.

(i) Un like in prisms, plane mirrors produce multiple images
(ii) The silvering in plane mirrors wears out with time while no silvering is required in prisms
(iii) Unlike in prisms, plane mirrors exercise loss of brightness when reflection occurs at its surface.

EXERCISE: 1

1. What is meant by reflection of light?
2. State the laws of reflection of light
3. Distinguish between regular and diffuse reflection of light
4. Show with the aid of a ray diagram that when a ray of light is incident on a plane
5. mirror, the angle of deviation of a ray by the plane surface is twice the glancing angle.
6. Derive the relation between the angle of rotation of a plane mirror and the angle of deflection of a reflected ray, when the direction of the incident ray is constant.
7. An incident ray of light makes an angle of **20**° with the plane mirror in position m_1,as shown below



Calculate the angle of reflection, if the mirror is rotated through **6**° to position m_2 while the direction of the incident ray remains the same.

8. **(i)** Show that an incident ray of light reflected successively from two mirrors inclined at an angle θ to each other is deviated through an angle **2**θ.
(ii) Name one application of the result in **7(i)** above.

9. Describe how a sextant is used to determine the angle of elevation of a star.

10. Show that the image formed in a plane mirror is as far behind the mirror as the object is in front

11. State the characteristics of images formed by plane mirrors.

12. (i) What is meant by **No parallax method** as applied to location of an image**?**

(ii) Describe how the position of an image in a plane mirror can be located

13. Show that for a man of height, **H**, standing upright the minimum length of a vertical plane mirror in which he can see the whole of him self completely is $\frac{H}{2}$.

14. With the aid of a ray diagram, explain how a thick plane mirror forms multiple images of an object.

15. Give three reasons for using prisms rather than plane mirrors in reflecting optical instruments.

REFLECTION IN CURVED MIRRORS

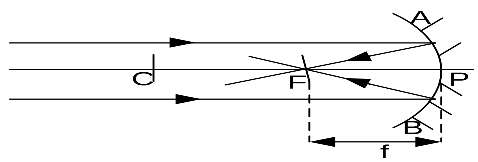
CURVED MIRRORS ((Spherical mirrors)

Curved mirrors are mirrors whose surfaces are obtained from a hollow transparent sphere.

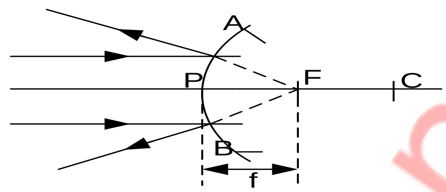
There are two types of curved mirrors;

i) Concave mirror (converging mirror) ii) Convex mirror (diverging mirror)

Concave (Converging) mirror: it is part of the sphere whose centre **C** is in front of its reflecting surface.



Convex (Diverging) mirror: it is part of the sphere whose centre **C** is behind its reflecting surface.



Where;

P is the pole of the mirror

F is the principal focus (focal point)

C is the centre of curvature

f is the focal length

r is the radius of curvature.

APB – Aperture

PFC – Principal axis

Terms used in Curved mirrors

Definitions

1. **Centre of curvature C:** it is the centre of the sphere of which the mirror forms part.
2. **Radius of curvature r:** it is the radius of the sphere of which the mirror forms part.
3. **Pole of the mirror:** it is the mid-point (centre) of the mirror surface.
4. **Princpal axis CP**: it is the line that passes through the centre of curvature and the pole of the mirror.
5. **Secondary axis:** line through the center of a thin lens or through the center of curvature of a concave or convex mirror other than the principal axis of the lens or mirror
6. **Paraxial rays:** These are rays close to the principal axis and make small angles with the mirror axis.
7. **Marginal rays:** These are rays furthest from the principal axis of the mirror.
8. **(i) Principal focus "F" of a concave mirror:** it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis converge after reflection by the mirror.

   A concave mirror has a real (in front) principal focus.

   (ii). Principal focus "F" of a convex mirror: it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis appear to diverge from after reflection by the mirror

   A convex mirror has a virtual (behind) principal focus.
9. **(i) Focal length "f" of a concave mirror:** it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the

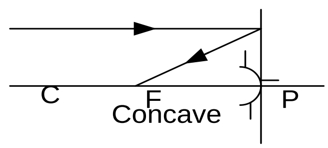
principal axis converge after reflection by the mirror.

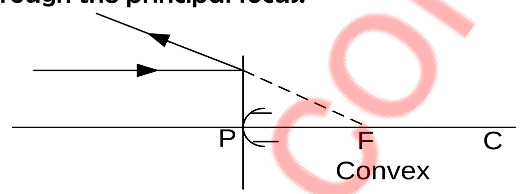
(ii) Focal length "f" of a convex mirror: it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after reflection by the mirror.

10. Aperture of the mirror: it is the length of the mirror surface.

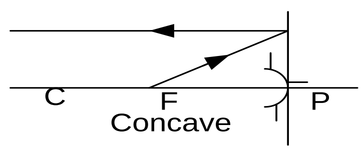
GEOMETRICAL RULES FOR THE CONSTRUCTION OF RAY DAIGRAMS(TO LOCATE IMAGE POSITIONS)

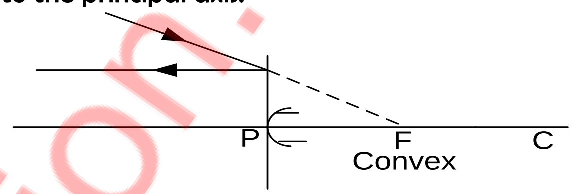
1. Rays are always drawn from the top of the object.
2. A ray parallel to the principal axis is reflected through the principal focus.



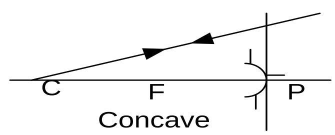


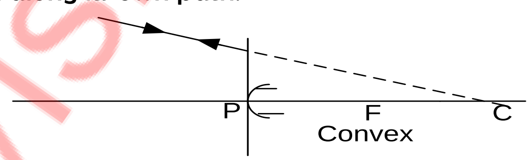
3. A ray through the principal focus is reflected parallel to the principal axis.





4. A ray through the centre of curvature is reflected along its own path.





5. Rays incident to the pole are reflected back, making the same angle with the principal axis.
6. At least two rays are used i.e. (1 and 2) or (1 & 3). Their point of intersection is where the image is, and it is always the top of the image.

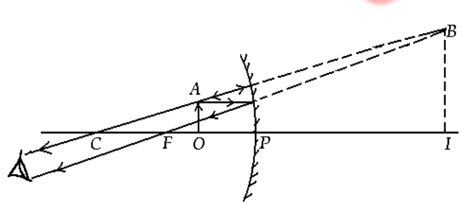
NOTE:

(i) The normal due to reflection at the mirror surface at any point must pass through the centre of curvature.

(ii) The image position can be located by the intersection of two reflected rays initially coming from the object.

IMAGES FORMED BY A CONCAVE MIRROR

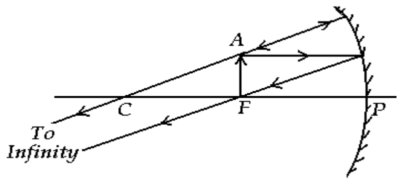
The nature of the image formed by a concave mirror is either real or virtual depending on the object distance from the mirror as shown below;



Object between F and P the image is

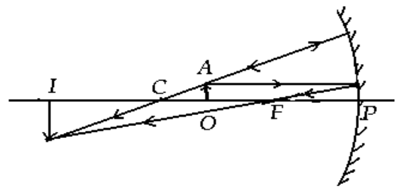
1) Behind the mirror
2) Virtual
3) Erect
4) Magnified

The property of a concave mirror to form erect, virtual and a magnified image when the object is nearer to the mirror than its focus makes it useful as a shaving mirror and also used by dentists for teeth examination.



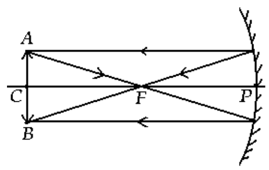
Object at F the image is

1) at infinity, virtual and upright



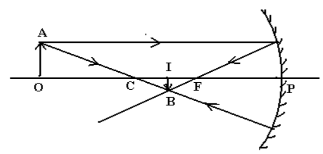
Object between F and C the image is

1) Beyond C
2) Real
3) Inverted
4) Magnified



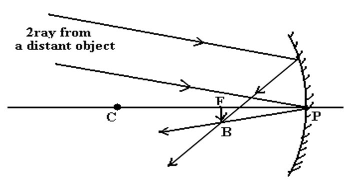
Object at C the image is

1) At C
2) Real
3) Inverted
4) Same size as the object



Object beyond C the image is

1) Between C and F
2) Real
3) Inverted
4) Diminished



Object at infinity the image is

1) At F
2) Real
3) Inverted
4) Diminished

USES OF CONCAVE MIRRORS

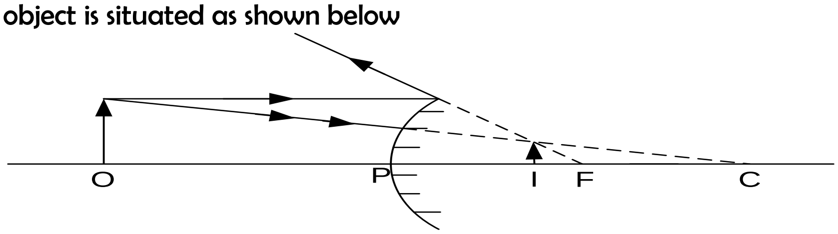
(i) They are used as shaving mirrors.
(ii) They are used by dentists for teeth examination.
(iii) They are used as solar concentrators in solar panels.
(iv) They are used in reflecting telescopes, a device for viewing distant objects
(v) They are used in projectors, a device for showing slides on a screen.

Advantage

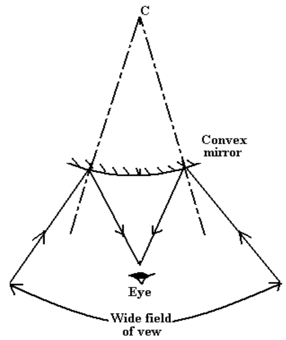
It forms magnified and erect images

IMAGES FORMED BY A CONVEX MIRROR

The image of an object in a convex mirror is erect, virtual, and diminished in size no matter where the object is situated as shown below



In addition to providing an erect image, convex mirrors have got a wide field of view as illustrated below.



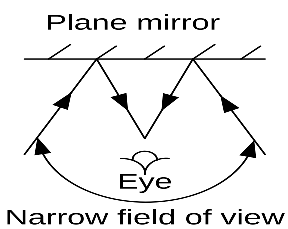
Note. A convex mirror can form real images if it receives converging rays targeting any point between Focal point &and its optical centre like in the above case(If the screen was placed at the position of the eye above a real image will be formed on it)

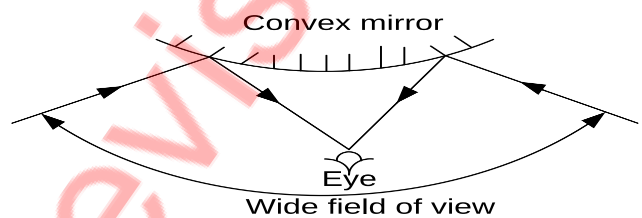
Convex mirrors

(i) They are used as driving mirrors. This is because they always form erect images and have a wide field of view.
(ii) Used in super markets to observe the activities of customers
(iii) Used in security check points to inspect under vehicles

Advantages of convex mirrors over plane mirrors

i) They have a wide field of view
ii) They form erect images



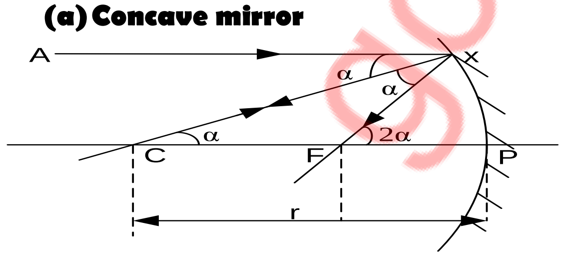


Disadvantages of convex mirrors

It diminished images ,giving a wrong impression to drivers that the object behind is very far

Relation between focal length and radius of curvature

(a) Concave mirror



A ray AX close and parallel to the principal axis is reflected through the principal focus F

$$FP = focal\ lenght\ (f)$$

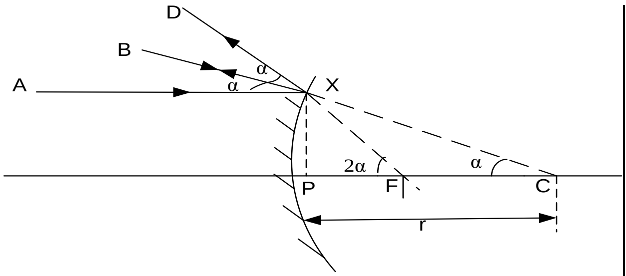
If C is the centre of curvature, then CP is the radius of the mirror cx

$$< AXC =< CXF = \alpha (law\ of\ reflcetion)$$
$$< XCP =< AXC = \alpha (alternate\ angles\ )$$
$$FC = FX$$

For AX close to CP

$$FX \approx FP$$
$$\therefore CF = FP$$
$$2\ FP = CP = r$$
$$r = 2f$$

(b) Convex mirror



A ray AX close and parallel to the principal axis is reflected through the principal focus F

$$FP = focal\ lenght\ (f)$$

If C is the centre of curvature, then CP is the radius of the mirror cx

$$< AXB =< BXD = \alpha(law\ of\ reflcetion)$$
$$< AXB =< XCP = \alpha(alternate\ angles\ )$$
$$FC = FX$$

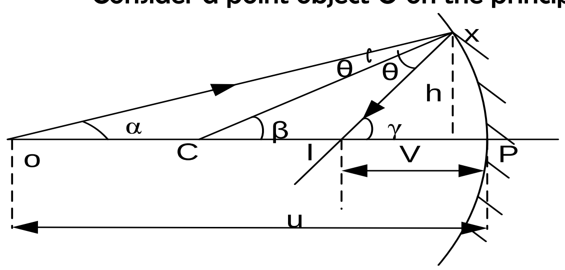
For AX close to CP

$$FX \approx FP$$
$$\therefore CF = FP$$
$$2\ FP = CP = r$$
$$r = 2f$$

MIRROR FORMULAR

(a) Concave mirror

Consider a point object O on the principal axis of a concave mirror



From triangle OXC; $\theta = \beta - \alpha$..................(1)

From triangle CXI; $\theta = \gamma - \beta$..................(2)

From eqn1 and eqn2

$$\beta - \alpha = \gamma - \beta$$
$$2\beta = \gamma + \alpha \text{................(3)}$$

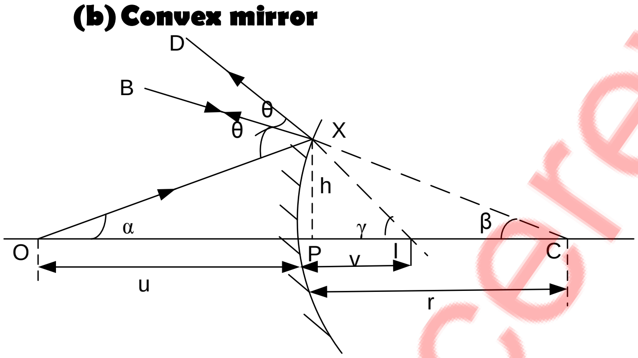
$for\ small\ angles\ in\ radians\ tan\alpha \approx \alpha,$ $tan\beta \approx \beta, tan\gamma \approx \gamma$

$$\frac{2\ h}{CP} = \frac{h}{IP} + \frac{h}{OP}$$
$$\frac{2}{r} = \frac{1}{v} + \frac{1}{u}$$

But

$$r = 2f$$
$$\frac{2}{2f} = \frac{1}{v} + \frac{1}{u}$$
$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

(b) Convex mirror



From triangle OXC; $\theta = \beta + \alpha$..................(1)

From triangle CXI; $2\theta = \gamma + \ \alpha$..................(2)

From eqn1 and eqn2

$$\beta - \alpha = \gamma - \beta$$
$$2(\beta + \alpha) = \gamma + \alpha$$
$$2\beta = \gamma - \alpha \text{................(3)}$$

$for\ small\ angles\ in\ radians\ tan\alpha \approx \alpha,$ $tan\beta \approx \beta, tan\gamma \approx \gamma$

$$\frac{2\ h}{CP} = \frac{h}{IP} - \frac{h}{OP}$$
$$\frac{2}{-r} = \frac{1}{-v} - \frac{1}{u}$$

But $r = 2f$

$$\frac{2}{2f} = \frac{1}{v} + \frac{1}{u}$$
$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

Where

u =object distance

v = image distance

f = focal length

r = radius of curvature

Sign convention

- Distances of real objects and real images are positive ie u and v for real objects and real images are positive.
- Distances of virtual objects and virtual images are negative ie u and v for virtual objects and virtual images are negative.
- Focal length f, for a concave mirror is positive and negative for a convex mirror.

LINEAR MAGNIFICATION

It is defined as the ratio of the image height to object height.

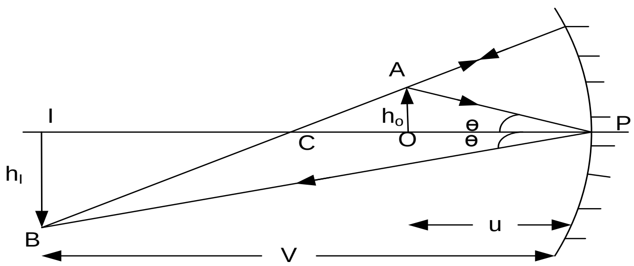
$$m = \frac{height\ image}{height\ object}$$

Magnification can also be obtained by determining the ratio of distance of the image from the mirror (v) to the distance of the object from the mirror (u)

$$m = \frac{image\ distance\ (v)}{object\ distance\ (u)}$$

PROOF

Consider the incidence of ray **AP** on to the pole of a concave mirror from an object of height **h** placed a distance, **u,** from the mirror and then reflected back making the same angle with the principal axis to form an image of height **h₁**, located at distance, **v**, from the mirror as shown



Ray **AP** makes an angle θ with the normal **OP**, then,

From Δ OAP, $\tan\theta = \frac{h_o}{u}$ --------------(i)

From Δ IPB, $\tan\theta = \frac{h_1}{v}$--------------------------.(ii)

Equating equation (i) and (ii) gives. $\frac{h_o}{u} = \frac{h_1}{v}$

Thus magnification, $m, = \frac{v}{u} = \frac{h_I}{h_o}$

NOTE:

(i) No signs need be inserted in the magnification formula.

(ii) Using the mirror formula, a connection relating magnification to the focal length of the mirror with either the object distance or the image distance can be established.

Example

1. An object 1cm tall is placed 30cm in front of a concave mirror of focal length 20cm. find **;**

(i) The position of the image

(ii) The size of the image formed

(iii) The magnification of the image

Solution

ho= 1cm, hi=?, u=30cm, v=?, f=20cm (concave mirror)

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{30}$

$\frac{1}{v} = \frac{1}{60}$

V =60cm

Positive sign means the image is real (60cm in front of the mirror)

M = $\frac{h_i}{h_o} = \frac{v}{u}$

$\frac{h_i}{1} = \frac{60}{30}$

h_i =2cm

m = $\frac{v}{u}$ = $\frac{60}{30}$

m =2

Or

M = $\frac{h_i}{h_o}$ = $\frac{2}{1}$

M = 2

2. An object 10cm tall is placed 30cm in front of a convex mirror of focal length 20cm. Find;

(i) The position

(ii) The size of the image formed.

Solution

ho= 10cm, hi=?, u=30cm, v=?, f= - 20cm (convex mirror)

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{-20} - \frac{1}{30}$

$\frac{1}{v} = \frac{-5}{60}$

V = - 12cm

Negative sign means the image is virtual (12cm behind the mirror)

M = $\frac{h_i}{h_o} = \frac{v}{u}$

$\frac{h_i}{10} = \frac{12}{30}$

h_i =4cm

3. A small object is placed on the principal axis of a convex mirror of curvature of 20cm. Determine the position of the image when the object is 15cm from the mirror

Solution

u=15cm, v=?, r=20cm, f=$\frac{r}{2}$=$\frac{20}{2}$

f= - 10cm (convex mirror)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{-10} - \frac{1}{15}$$

$$\frac{1}{v} = \frac{-5}{30}$$

V = - 6cm

Negative sign means the image is virtual (6cm behind the mirror)

4. A concave mirror with radius of curvature 40cm forms an image of real object which is placed 25cm from the mirror

(a) What is the focal length of the mirror

(b) Calculate the distance of the image from the mirror and give its nature

Solution

$$f = \frac{r}{2}$$

$$f = \frac{40}{2}$$

$$f = 20cm$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{25}$$

$$\frac{1}{v} = \frac{1}{100}$$

$$v = 100cm$$

5. An object is placed 10cm in front of a concave mirror of focal length 15cm. Find the image position and magnification.

Solution:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{10}$$

$$v = -30cm$$

The negative sign implies that the image formed is **virtual** and it is formed 30cm from the mirror

Magnification, M= $\frac{v}{u}$

$$m = \frac{30}{10} = 3$$

6. The image of an object in a convex mirror is 6cm from the mirror. If the radius of curvature of the mirror is 20cm, find the object position and the magnification.

Solution:

For a convex mirror, $f = \frac{r}{2}$

$$f = \frac{-20}{2}$$

$$f = -`10cm$$

v = – 6cm (The image in a convex mirror is always virtual)

Using the mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{u} = \frac{1}{-10} - \frac{1}{-6}$$

$$u = 15cm$$

∴ **Magnification,** M= $\frac{v}{u} = \frac{6}{15} = 0.4$

7. Show that an object and its image coincide in position at the centre of curvature of a concave mirror. Hence find the magnification produced in this case.

Solution

At the centre of curvature of a concave mirror, object distance $\boldsymbol{u} = \boldsymbol{r}$ and $r = 2f$ where r is the radius of curvature of the mirror.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{2}{r} = \frac{1}{r} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{2r} - \frac{1}{r}$$

$$v = r$$

Thus the image is also formed at the centre of curvature and therefore it coincides in position with its object.

Hence, M= $\frac{v}{u} = \frac{r}{r} = 1$

∴ The object and its image are of the same size in this case.

RELATIONSHIP CONNECTING m, v and f

From $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

multiplying all through by v

$$\frac{v}{f} = \frac{v}{v} + \frac{v}{u}$$

$$\boxed{\boldsymbol{m} = \frac{v}{f} - 1}$$

RELATIONSHIP CONNECTING m, u and f

From $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

multiplying all through by u

$$\frac{u}{f} = \frac{u}{v} + \frac{u}{u}$$

$$\boxed{\frac{\mathbf{1}}{\boldsymbol{m}} = \frac{u}{f} - 1}$$

Examples

1. A small objects placed infront of a spherical mirror gives a real image and 4 times the size of the object. When the object is moved 10cm towards the mirror a similarly magnified virtual image is formed. Find the focal length of the mirror

Solution

$\frac{\mathbf{1}}{\boldsymbol{m}} = \frac{u}{f} - 1$

$u_1 = \left(\frac{1}{m_1} + 1\right) f$.............(1)

$u_2 = \left(\frac{1}{m_2} + 1\right) f$.............(2)

For object moved towards the mirror, $u_2 < u_1$ then

$u_1 - u_2 = 10$................(3)

$f = \frac{u_1 - u_2}{\frac{1}{m_1} - \frac{1}{m_2}}$

$f = \frac{10}{\frac{1}{4} - \frac{1}{-4}} = 20cm$

2. A concave mirror forms a real image which is 3 times the linear size of the real object. When the object is displaced a distance d, the real image formed is now 4 times its linear size of the object. If the distance between the two images position is 20cm. find
   (i) focal length of the mirror
   (ii) Distance d

Solution

$\boldsymbol{m} = \frac{v}{f} - 1$

$v_1 = (m_1 + 1)f$.............(1)

$v_2 = (m_2 + 1)f$.............(2)

For $m_2 > m_1, v_2 > v_1$ then

$v_2 - v_1 = 20$................(3)

$f = \frac{v_2 - v_1}{m_2 - m_1}$

$f = \frac{20}{4 - 3} = 20cm$

For $m_2 > m_1, u_1 > u_2$ then

$\frac{\mathbf{1}}{\boldsymbol{m}} = \frac{u}{f} - 1$

$u_1 = \frac{f}{m_1} + 1$.............(1)

$u_2 = \frac{f}{m_2} + 1$.............(1)

$d = u_1 - u_2$

$d = \frac{f}{m_1} - \frac{f}{m_2}$

$d = \frac{20}{3} - \frac{20}{4}$

$\boldsymbol{d = 1.67cm}$

3. A concave mirror forms on a screen a real image of three times the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is **30cm,** determine the
   (i) focal length of the mirror
   (ii) shift of the object

Solution.

(i) $\boldsymbol{m} = \frac{v}{f} - 1$

$v_1 = (m_1 + 1)f$.............(1)

$v_2 = (m_2 + 1)f$.............(2)

For $m_2 > m_1, v_2 > v_1$ then

$v_2 - v_1 = 30$................(3)

$f = \frac{v_2 - v_1}{m_2 - m_1}$

$f = \frac{30}{5-3} = 15cm$

(ii) For $m_2 > m_1, u_1 > u_2$ then

$\frac{\mathbf{1}}{\boldsymbol{m}} = \frac{u}{f} - 1$

$u_1 = \frac{f}{m_1} + 1$.............(1)

$u_2 = \frac{f}{m_2} + 1$.............(1)

$d = u_1 - u_2$

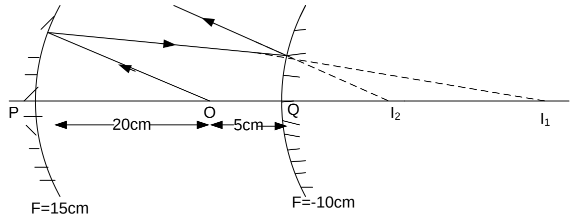
$d = \frac{f}{m_1} - \frac{f}{m_2}$

$d = \frac{15}{3} - \frac{15}{5}$

$\boldsymbol{d = 2cm}$

3. A concave mirror **P** of focal length 15cm faces a convex mirror **Q** of focal length 10cm placed 25cm from it. An object is placed between **P** and **Q** at a point 20cm from **P**.
   (i) Determine the distance from **Q** of the image formed by reflection, first in **P** and then in **Q**
   (ii) Find the magnification of the image formed in **(i)** above

Solution



Consider the action of a concave mirror

$u = 20cm,$ and $f = 15cm$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{20}$$

$$v = 60cm$$

∴ The image distance from a concave mirror $= \mathbf{60cm}$

Thus, the image distance behind a convex mirror = $60 - (5 + 20)cm = 35cm.$

Consider the action of a convex mirror

The image formed by a concave mirror acts as a virtual object for the convex mirror. Thus

$u = -35cm$ and $f = -10cm$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{-10} - \frac{1}{-35}$$

$$v = -14cm$$

∴ A final virtual image is **14cm** behind the convex mirror

(ii) magnification $m = m_1 m_2$

$$m = \frac{v_1}{u_1} x \frac{v_2}{u_2}$$

$$m = \frac{60}{20} x \frac{14}{35} = 1.2$$

4. A small convex mirror is placed **60cm** from the pole and on the axis of a large concave mirror of radius of curvature **200cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

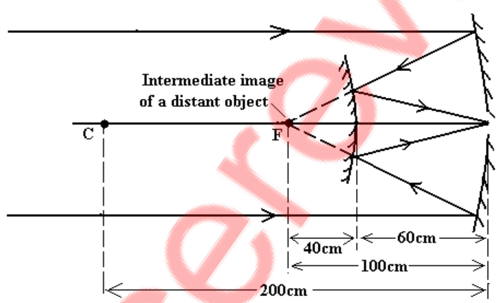
(ii) Suggest a practical application for the arrangement of the mirrors in **a (i)** above**.**

(b) Calculate the

(i) radius of curvature of the convex mirror.

(ii) height of the real image if the distant object subtends an angle of **0·5**° at the pole of the convex mirror.

Solution(b) i)



(ii) The mirror arrangement finds application in a reflecting telescope, a device for viewing distant objects

(b) (i) Consider the action of a concave mirror

The image of a distant object is formed at the principal focus of the concave mirror. This image acts as a virtual object for a convex mirror.

Consider the action of a convex mirror

$u = -40cm$ and $v = 60cm$

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ where $r = 2f$

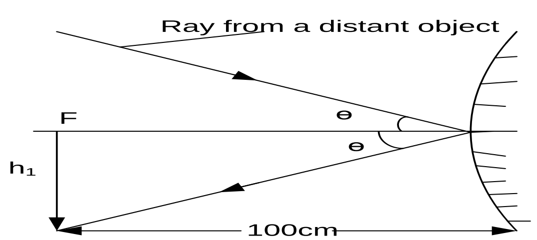
$$\frac{2}{r} = \frac{1}{v} + \frac{1}{u}$$

$$r = \frac{2uv}{u+v} = \frac{2x - 40x60}{-40 + 60} = -240cm$$

The required radius of curvature r = 240cm

(ii) Consider the magnification produced by a convex mirror

Let h_1 = height of the intermediate image formed by a concave mirror as shown.



$$tan\ 0.5° = \frac{h_1}{100}$$

$$h_1 = 0.8727cm$$

h_1 =height of image formed by a convex mirror

$$m = \frac{h_2}{h_1} = \frac{v}{u}$$

$$h_2 = \frac{60}{40}x0.727 = 1.3cm$$

Required image height = 1·3cm

EXERCISE:2

1. Define the terms centre of curvature, radius of curvature, principal focus and focal length of a converging mirror.
2. Distinguish between real and virtual images.
3. Explain with the aid of a concave mirror the term **a caustic surface.**
4. Explain why a parabolic mirror is used in searchlights instead of a concave mirror
5. An object is placed a distance **u** from a concave mirror. The mirror forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:
   (i) real
   (ii) virtual
6. Give two instances in each case where concave mirrors and convex mirrors are useful.
7. **(i)** Explain the suitability of a concave mirror as a shaving mirror.
   (ii) Explain with the aid of a ray diagram why a convex mirror is used as a car driving mirror.
8. Show with the aid of a ray diagram, that the radius of curvature of a concave mirror is twice the focal length of the mirror
9. Use a geometrical ray diagram for a concave to derive the relation $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
10. Derive the relation connecting the radius of curvature **r** object distance **u** and image distance **v** of a diverging mirror.
11. An object is placed perpendicular to the principal axis of a concave mirror of focal length f at a distance $(f + x)$ and a real image of the object is formed at a distance $(f + y)$. Show that the radius of curvature **r** of the mirror is given by $r = 2\sqrt{xy}$
12. (i) Define the term **linear magnification.**
   (ii) Show that in a concave mirror, **linear magnification**= $\frac{image\ distance}{object\ distance}$.
   (iii) A concave mirror of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.
   (iv) A concave mirror of focal length 10cm forms an image five times the height of its object. Find the possible object and corresponding image positions.

[**Ans: (iii)** $u = 10cm, v = -30cm$ **, (iv)** $u = 12cm, v = 60cm$ **OR** $u = 8cm, v = -40cm$]

13. A concave mirror forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 30cm, determine the
   (i) focal length of the mirror
   (ii) shift of the object

[**Answers: (i)** $f = 10cm$ **(ii)** $3cm$]

14. A concave mirror of radius of curvature **20cm** faces a convex mirror of radius of curvature **10cm** and is **28cm** from it. If an object is placed midway between the mirrors, find the nature and position of the image formed by reflection first at the concave mirror and then at the convex mirror.

[**Answer:** A final virtual image is **17·5cm** behind the convex mirror]

15. A small convex mirror is placed **100cm** from the pole and on the axis of a large concave mirror of radius of curvature **320cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

(ii) Suggest a practical application for the arrangement of mirrors in **a (i)** above.

(iii) Calculate the radius of curvature of the convex mirror

(b) If the distant object subtends an angle of 3×10^{-3} **radians** at the pole of the concave mirror, calculate the

(i) size of the real image that would have been formed at the focus of the concave mirror.

(ii) size of the image formed by the convex mirror

[**Ans: (a) (iii) 150cm (b) (i) 0·48cm (ii) 0·8cm**]

16. A converging mirror produces an image whose length is 2.5 times that of the object. If the mirror is moved through a distance of 5cm towards the object, the image formed is 5 times as long as the object. Calculate the focal length of the mirror. **An(f=25cm)**

17. A concave mirror forms an image half the size of the object. The object is then moved towards the mirror until the image size is three quarters that of the object. If the image is moved by a distance of 0.8cm. find the;

(i) Focal length of the mirror

(ii) New position of the object **An(f=3.2cm, 7.47cm)**

18. A real image is formed 40 cm from a spherical mirror, the image being twice the size of the object. What kind of mirror is it and what is the radius of the curvature.

19. An object is 4cm high. It s desired to form a real image 2cm high and 96cm from the object. Determine the type of mirror required and focal length of the mirror.

20. A dentist holds a concave mirror of focal length 4cm at a distance of 1.5 cm from the tooth. Find the position and magnification of the image which will be formed.

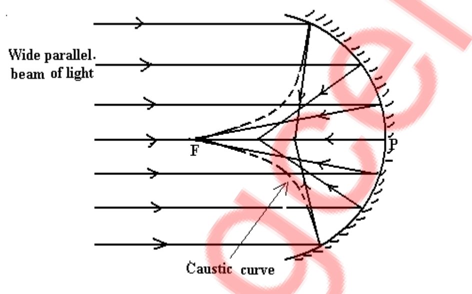
21. A concave mirror of radius of curvature 25cm faces a convex mirror of radius of curvature 20cm with the convex mirror 30cm from a concave mirror. If the object is placed mid way between two mirrors, find the nature and position of the image formed by reflection

(i) By concave mirror

(ii) By convex mirror

REFLECTION OF A PARALLEL WIDE BEAM OF LIGHT AT CURVED MIRRORS

Consider the reflection of a wide parallel beam of light incident on a concave mirror as shown.



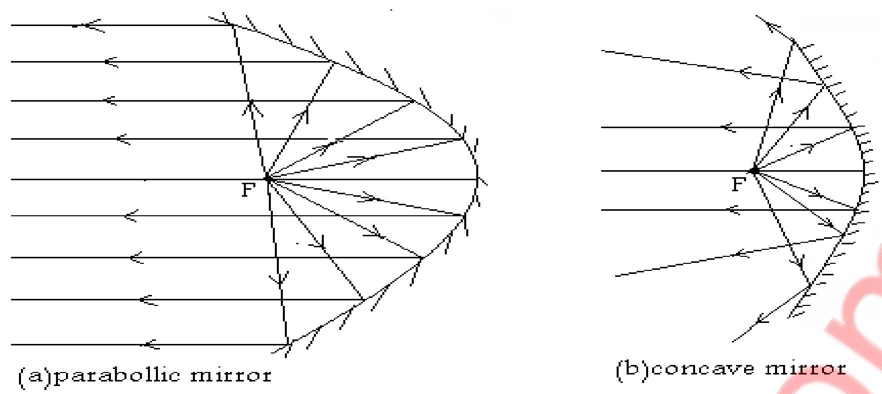
When a wide parallel beam of light is incident on a concave mirror, the different reflected rays are converged to different points. However these reflected rays appear to touch a surface known as a **caustic surface(**A surface on which every reflected ray from the mirror forms a tangent **to)** and has an apex at the principal focus **F**.

NOTE

(i) The marginal rays furthest from the principal axis are converged nearer to the pole of the mirror than the paraxial rays.

(ii) Similarly, if a wide parallel beam of light is incident on a convex mirror, the different reflected rays appear to have diverged from different points.

COMPARISON OF CONCAVE AND PARABOLIC MIRRORS (Reversing light)



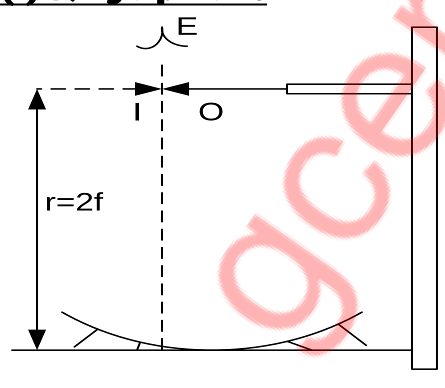
- When a lamp is placed at the principal focus of a concave mirror, only rays from this lamp that strike the mirror at points close to the principle axis will be reflected parallel to the principle axis and those striking at points well away from the principal axis will be reflected in different directions and not as a parallel beam as seen in (b) above. In this case the intensity of the reflected beam practically diminishes as the distance from the mirror increases.
- When a lamp is placed at the principal focus of a parabolic mirror, all rays from this lamp that strike the mirror at points close to and far from the principle axis will be reflected parallel to the principle axis as seen in (a) above. In this case the intensity of the reflected beam remains practically undiminished as the distance from the mirror increases. This accounts for the use of parabolic mirrors as search lights other than concave mirrors.

Note:
A parabolic mirror has the advantage of reflecting the light source placed at the focus parallel to the principal axis with undiminished intensity.

Uses of Parabolic mirrors
They are used as reflectors in search light torches

DETERMINATION OF THE FOCAL LENGTH OF A CONCAVE MIRROR.

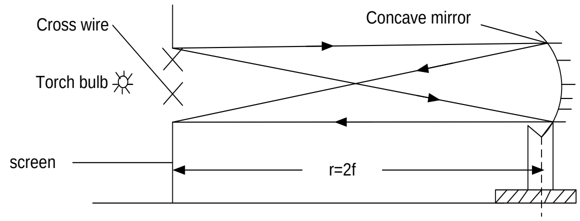
Method (1) Using a pin at C



- Place a concave mirror on horizontal bench with its reflecting surface placed upwards
- A pin is then clamped horizontally on a retort stand such that its pointed end lies along the principal axis of the mirror
- Move the pin vertically until the point is located where the pin coincides with its own image
- Measure the distance r from the mirror to the pin
- The focal length $f = \frac{r}{2}$

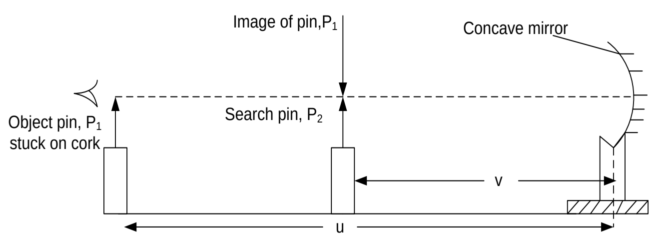
NOTE :
(i) In the position where there is no parallax between the object pin and its image, there is no relative motion between the object and its image when the observer moves the head from side to side.
(ii) When the pin coincides with its image, the rays are incident normal to the mirror and are thus reflected along their own path. Therefore the pin coincides with its image at the centre of curvature of the mirror.

Method (2) Using an illuminated object at C



- An object in this, consists of a hole cut in a white screen made of cross-wire illuminated from behind by a source of light.
- A concave mirror is mounted in a holder, and moved to and from in front of the screen until a sharp image of the cross-wire is formed on the screen adjacent to the cross-wire.
- When this has been done, both the image and the object are at the same distance from the mirror, and hence both must be situated in a plane passing through the centre of curvature and at right angles to the axis.
- The distance between the mirror and the screen is measured and this is the radius of curvature, **r**.
- Half of this distance, **r**, is the focal length, *f* $(f = {}^{r}/_{2})$.

Method (3) Using no parallax method in locating V



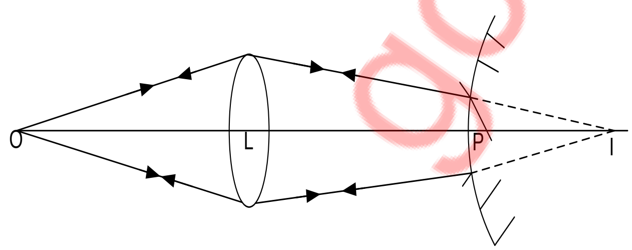
- An object pin **P₁** is placed at a distance **u** in front of a mounted concave mirror so that its tip lies along the principal axis of the mirror and it forms an inverted image.
- The distance of the object pin from the mirror is measured, u.
- A search pin **P₂** placed between the mirror is and pin **p₁** is adjusted until it coincides with the image of pin **p₁** by no-parallax method.
- The distance **v** of pin **p₂** from the mirror is measured.
- The procedure is repeated for several values of **u** and the results are tabulated including values of uv, and $u + v$.
- A graph of uv against $u + v$ is plotted and the slope **s** determined
- The focal length **f** of the mirror if $f = s$.

NOTE:

If a gaph of $\frac{1}{U}$ $agaisnt$ $\frac{1}{V}$ is plotted, then each intercept C of such a graph is equal to $\frac{1}{f}$. Hence $f = \frac{1}{C}$

AN EXPERIMENT TO DETERMINE THE FOCAL LENGTH OF THE CONVEX MIRROR

Method (1) Using a convex lens.



- Place an object O in front of convex lens and locate the position of real image
- Measure the distance LI and record
- The convex mirror is placed between the lens and image I with its reflecting surface facing the lens. Move the mirror along OI until the image I coincides with object
- Measure the distances PI and LP
- Focal length f is obtained from $f = \frac{LI - LP}{2}$

NOTE ;

When the incident rays from an object are reflected back along the incident path, a real inverted image is formed besides the object in which case the rays strike the mirror normally. Therefore they will if produced pass through the centre of curvature of the mirror thus distance **PI = radius of curvature**

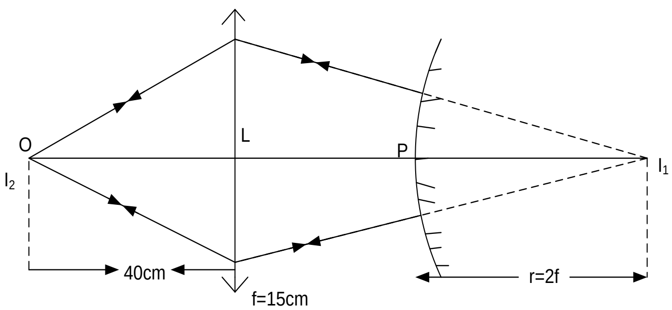
Examples

1. An object **O** is placed **40cm** in front of a convex lens of focal length **15cm** forming an image on the screen. A convex mirror situated **4cm** from the lens in the region between the lens and the screen forms the final image besides object **O**.

   (i) Draw a ray diagram to show how the final image is formed.

   (ii) Determine the focal length of the convex mirror.

Solution



Consider the action of a convex lens

$u = 40cm$, and $f = 15cm$

From $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{40}$$

$$v = 24cm$$

The radius of curvature $\boldsymbol{r} = (\mathbf{24} - \mathbf{4})\boldsymbol{cm}$

$= \mathbf{20}\boldsymbol{cm}$

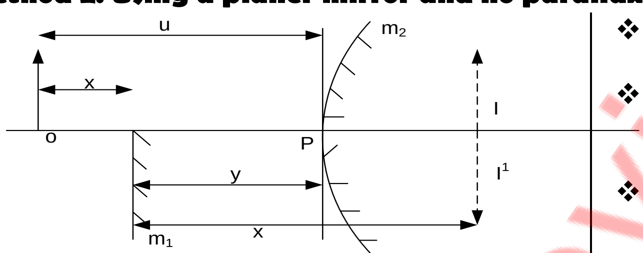
Using the relation $\boldsymbol{r} = \mathbf{2}\boldsymbol{f}$

$\Rightarrow \quad 2f = 20cm$

$\therefore \quad f = 10cm$

Thus $\boldsymbol{f} = -\mathbf{10}\boldsymbol{cm}$ "The centre of curvature of a convex mirror is virtual"

Method 2: Using a planer mirror and no parallax method



- An object pin O is placed in front of convex mirror m_2 such that it forms a virtual diminished image at I
- The distance u of object O from convex mirror is measured
- A plane mirror m_1 is then placed between object O and the convex mirror such that it covers half aperture of convex mirror
- The plane Mirror **M** is adjusted until its own image of **o** coincides with **I** by no parallax method.
- Measure the distance x and y
- F can be calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where $u = (x + y)$ and $v = -(x - y)$

Note:

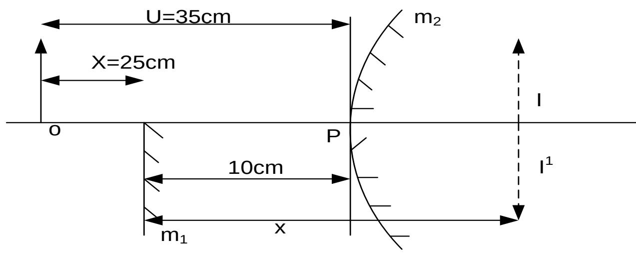
(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

(ii) Substituting for $u = (x + y)$and $v = -(x - y)$ in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$

Examples

1. A plane mirror is placed 10cm infront of a convex mirror so that it covers half of the mirror surface. Apin 25cm infront of a plane mirror gives an image which coincide with that of the pin in the convex mirror. Find the focal length of the convex mirror.

Solution



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Where $u = (x + y)$ and $v = -(x - y)$

$$\frac{1}{f} = \frac{1}{35} + \frac{1}{-(25-10)}$$

$$\frac{1}{f} = \frac{1}{35} - \frac{1}{15}$$

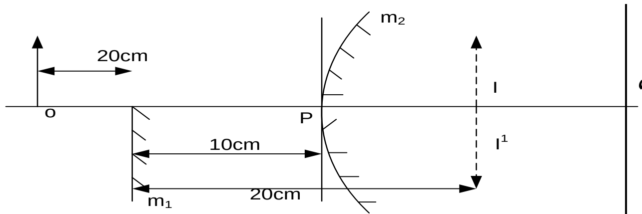
$$\frac{1}{f} = \frac{-20}{525}$$

$$f = \frac{525}{-20}$$

$$f = -26.25cm$$

2. A plane mirror is placed **10cm** in front of a convex mirror so that it covers about half of the mirror surface. A pin **20cm** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.

Solution



Consider the action of a convex mirror

$u = 30cm$ and $v = -(20 - 10) = -10cm$

"The image formed is virtual "

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{-10}$$

$$f = -15cm$$

EXERCISE:3

1. Describe an experiment to determine the focal length of a concave mirror.
2. You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a concave mirror, and a meter ruler. Describe an experiment to determine the focal length of a concave mirror using the above apparatus.
3. Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using a no parallax method.
4. Describe an experiment to measure the focal length of a convex mirror
5. Describe how the focal length of a diverging mirror can be determined using a convex lens.
6. Describe how the focal length of a convex mirror can be obtained using a plane mirror and the no parallax method.
7. A plane mirror is placed at a distance d in front of a convex mirror of focal length f such that it covers about half of the mirror surface. A pin placed at a distance L in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, Show that $2df = d^2 - L^2$

REFRACTION OF LIGHT

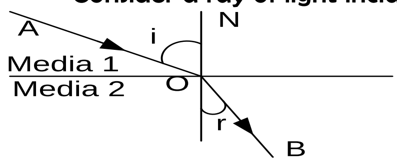
Refraction is the change of direction of light propagation of light as it travels from one medium to another.

Explanation of refraction

The bending of light is as a result of the change in speed as light travels from one medium to another. The change in speed of light usually leads to the change in direction unless if the ray is incident normally. The speed of light in air is higher than the speed of light in glass or water.
Glass and water are therefore said to be denser than air also is denser than water.

LAWS OF REFRACTION

Consider a ray of light incident on an interface between two media as shown.



O = Point of incidence.

OA = Incident ray
OB = Refracted ray.
ON = Normal at O
$\angle i$ = Angle of incidence
$\angle r$ = Angle of refraction

LAW 1: The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

LAW 2: The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media.

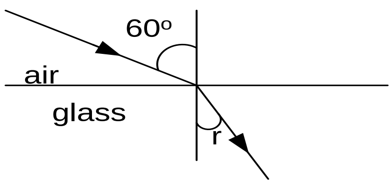
This is called **Snell's law.**
This constant ratio is called Refractive index, (n).

$n = \frac{\sin i}{\sin r}$ where;

i = angle of incidence r = angle of refraction.

Examples

1.



Find the angle of refraction if the refractive index of glass is 1.52

Solution

$n = \frac{\sin i}{\sin r}$

$1.52 = \frac{\sin 60}{\sin r}$

Sin r = $\frac{0.8666}{1.52}$

Sin r = 0.569
r = $sin^{-1}(0.569)$
r = 34.7°

2. The angle of incidence of water of refractive index 1.33 is $45°$. Find the angle of refraction.

Solution

$n = \frac{\sin i}{\sin r}$

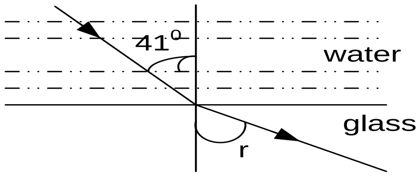
$1.52 = \frac{\sin 45}{\sin r}$

Sin r = $\frac{0.707}{1.33}$

Sin r = 0.531
r = $sin^{-1}(0.531)$
r = 34.7°

Exercise:4

1. The angle of incidence is 30° and angle of refraction is 19°. Find the refractive index of the material
2. A ray of light is incident in air at an angle of 30°. Find the value of angle of refraction, r, if the refractive index is 1.5.
3. A ray of light is incident on a water- glass boundary at an angle of 41° as shown below.



Calculate the angle of refraction, if the refractive indices of water and glass are 1.33 and 1.50 respectively

Refractive index n

Refractive index of a material is the ratio of the sine of angle of incidence to the sine of angle of refraction for a ray of light traveling from a vacuum to a given medium.

OR

Is the ratio of the speed of light in a vacuum to speed of light in a medium.

Thus **Refractive index**, $n = \frac{speed\ of\ light\ in\ a\ vacuum\ (c)}{speed\ of\ light\ in\ a\ medium\ (v)}$

Where speed of light in a vacuum $c = 3.0\ x\ 10^8\ ms^{-1}$.

NOTE:

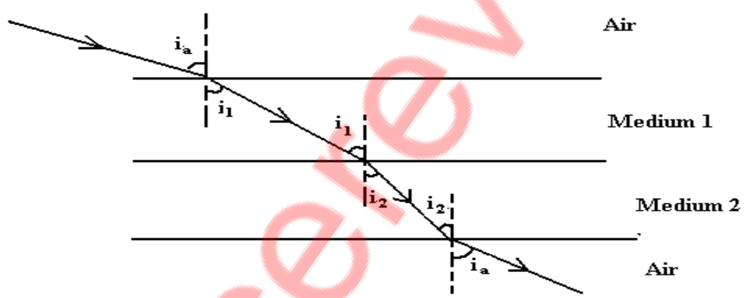
The refractive index, **n** for a vacuum is **1**. However if light travels from air to another medium, the value of **n** is slightly greater than **1**. For example, $n = 1{\cdot}33$ for water and $n = 1{\cdot}5$ for glass.

THE PRINCIPLE OF REVERSIBILITY OF LIGHT.

It states that the paths of light rays are reversible.
This means that a ray of light can travel from medium **1** to **2** and from **2** to **1** along the same path.

GENERAL RELATION BETWEEN n AND sin i

Consider a ray of light moving from air through a series of media **1, 2** and then finally emerge into air as shown.



At **air – medium 1** interface, Snell's gives $\frac{\sin i_a}{\sin i_1} = n_1$

$\Rightarrow \quad \sin i_a = n_1 \sin i_1$ -- (i)

At **air - medium 2** interface, Snell's gives $\frac{\sin i_a}{\sin i_2} = n_2$

$\Rightarrow \quad \sin i_a = n_2 \sin i_2$ ------------------------------------(ii)

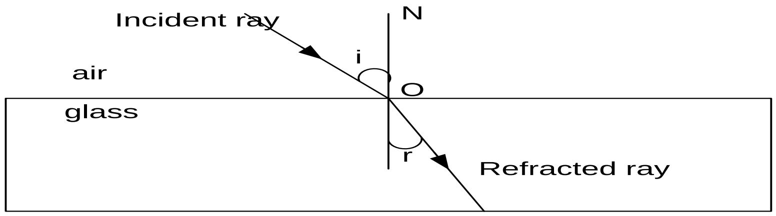
Equating equation (i) and (ii) gives

$n_1 \sin i_1 = n_2 \sin i_2$

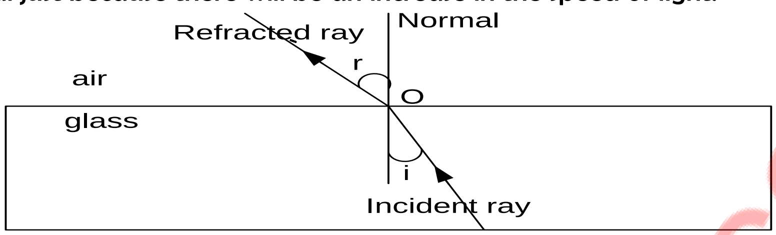
$\therefore$ $\boxed{n\ sin\ i = a\ constant.}$

N.B:

1. For a ray travelling from a less dense medium to a denser medium e.g from air to glass, it is refracted towards the normal since there is a decrease in speed of light.



2. If the ray travels from a denser to a less dense medium e.g. from glass to air, it will be reflected away from the normal just because there will be an increase in the speed of light.

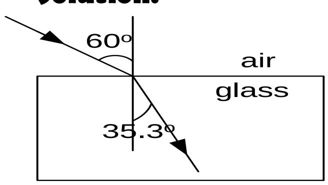


3. When the incident ray meets a refracting surface at 90°, it is not refracted at all
4. The speed of light reduces when it travels into a dense medium

Examples:

1. Calculate the refractive index of the glass if a ray of light is incident on it at an angle of $60°$ and it is refracted at an angle of $35.3°$.

Solution:



$n \sin i = constant$

$n_a sini_a = n_g sini_g$

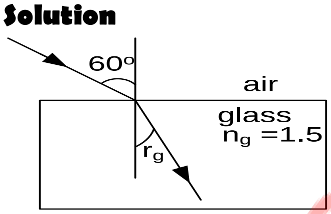
$1 \text{ x } sin60 = n_g \text{x } \sin 35.3$

$\frac{0.866}{0.5778} = \frac{0.5778}{0.5778} ng$

$\therefore$ The refractive index of glass is 1.5

2. A ray of light in air makes an angle of incidence of $60°$ with the normal to glass surface of refractive index 1.5. What is the angle of refraction?

Solution



$n_a sini_a = n_g sini_g$

$1 \text{ x } sin60 = 1.5 sinr_g$

$\frac{0.866}{1.5} = \frac{1.5}{1.5} sinrg$

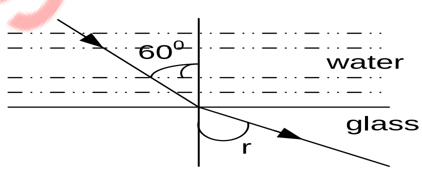
$sinr_g = 0.5773$

$r_g = sin^{-1} 0.5773$

$r_g = 35.3°$

The angle of refraction is 35.3°

3. A monochromatic beam of light is incident at **60°** on a water-glass interface of refractive index $1{\cdot}33$ and $1{\cdot}5$ respectively as shown



Calculate the angle of reflection **r.**

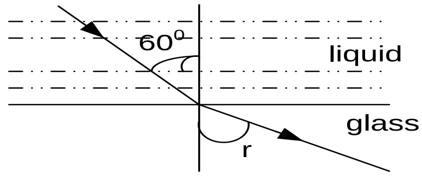
Solution:

Applying Snell's law gives $n_w sini_w = n_g sini_g$

$1.33 sin60 = 1.5 sinr$

Thus $\angle r = 50{\cdot}2°$

4. A monochromatic ray of light is incident from a liquid on to the upper surface of a transparent glass block as shown.



Given that the speed of light in the liquid and glass is $2.4 \times 10^8\ ms^{-1}$ and $1.92 \times 10^8\ ms^{-1}$ respectively, find the angle of refraction, **r.**

Solution:

$$n_l sini_l = n_g sini_g$$

$$\frac{c}{v_l} sin60 = \frac{c}{v_g} sinr$$

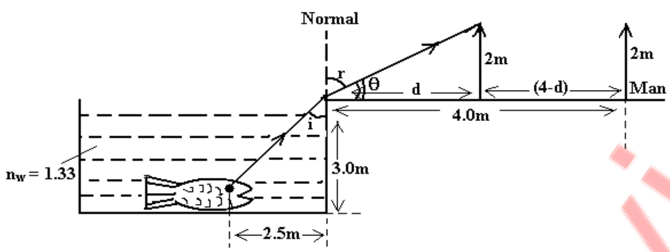
$$sinr = \frac{v_g}{v_l} sin60$$

$$sinr = \frac{1.92 \times 10^8}{2.4 \times 10^8} sin60$$

$$\Rightarrow \quad \angle r = 43.9°.$$

5. A small fish is 3·0m below the surface of the pond and 2·5m from the bank. A man 2·0m tall stands 4·0m from the pond. Assuming that the sides of the pond are vertical, calculate the distance the man should move towards the edge of the pond before movement becomes visible to the fish. **(Refractive index of water = 1·33).**

Solution



From the diagram, $tan\ i = \frac{2.5}{3}$

$\Rightarrow \quad \angle i = 39.81°$

Applying Snell's law at the edge of the pond gives

$$n_w sini_w = n_g sini_g$$

$$1{\cdot}33\ sin\ 39{\cdot}81° = 1\ sin\ r$$

$$\Rightarrow \angle r = 58.4°$$

Thus $\angle\theta = 90° - 58.4° = 31.6°$

From the diagram $tan\ \theta = \frac{2}{d}$

$$\Rightarrow \quad d = \frac{2}{\tan 31.6}$$

$$\therefore \quad d = 3{\cdot}2m.$$

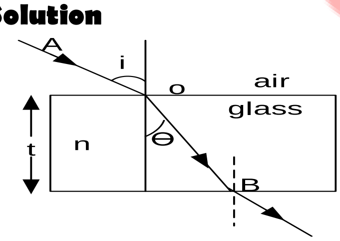
Thus required distance traveled $= 4 - d$

$= 4 - 3{\cdot}2$

$= 0{\cdot}8m$

6. A monochromatic light incident on a block of material placed in a vacuum is refracted through an angle of θ. If the block has a refractive index **n** and is of thickness **t**, show that light takes time of $\frac{n\,t\,sec\theta}{c}$ to emerge

Solution



Distance travelled in the block is OB

$$cos\theta = \frac{t}{OB}$$

$$OB = \frac{t}{cos\theta} = t\ sec\theta$$

$$speed = \frac{distance}{time}$$

$$time = \frac{OB}{V} = \frac{t\ sec\theta}{V}$$

$$n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{C}{V}$$

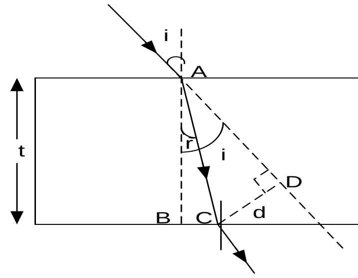
$$V = \frac{C}{n}$$

$$time = \frac{t\ sec\theta}{V} = \frac{t\ sec\theta}{\left(\frac{C}{n}\right)} = \frac{\boldsymbol{n\,t\,sec\theta}}{\boldsymbol{c}}$$

SIDE WISE DISPLACEMENT OF LIGHT RAYS.

When light travels from one medium to another, its direction is displaced side ways. This is called lateral displacement.

Consider a ray of light incident at an angle **i** on the upper surface of a glass block of thickness **t**, and then suddenly refracted through an angle **r** causing it to suffer a sidewise displacement **d**.



From Δ ABC, $AC = \frac{t}{\cos r}$ -----------------.(i)

From the diagram, $\angle CAD = (i - r)$

From Δ ACD, $AC = \frac{d}{\sin(i-r)}$ ----------(ii)

Equating equation (i) and (ii) gives

$$\frac{t}{\cos r} = \frac{d}{\sin(i-r)}$$

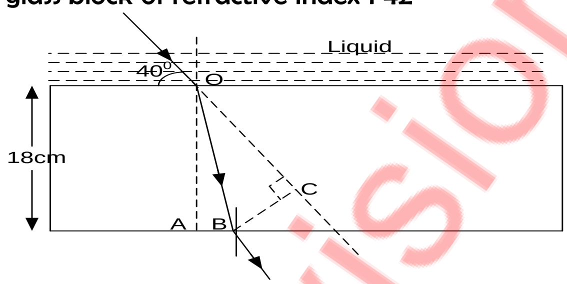
$$\Rightarrow \quad \boldsymbol{d} = \frac{\boldsymbol{t}\sin(i-r)}{\boldsymbol{\cos r}}$$

NOTE :

The horizontal displacement of the incident ray , $\boldsymbol{BC} = \boldsymbol{t.\tan r}$

Example:

1. The figure below shows a monochromatic ray of light incident from a liquid of refractive index 1·33 onto the upper surface of a glass block of refractive index 1·42

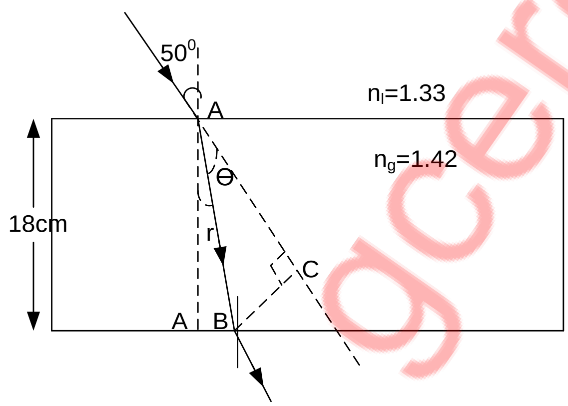


Calculate the;

(i) horizontal displacement AB.

(ii) lateral displacement BC of the emergent light.

Solution



(i) Applying Snell's law at the liquid- glass interface gives,

$$n_l sin i_l = n_g sin i_g$$

$$1{\cdot}33 \sin 50^{\circ} = 1{\cdot}42 \sin r$$

$$\Rightarrow \quad \angle r = 45.8^{\circ}$$

Horizontal displacement $AB = t \tan r$

$= 18 \tan 45{\cdot}8^{\circ}$

$= \mathbf{18.51} \boldsymbol{cm}$

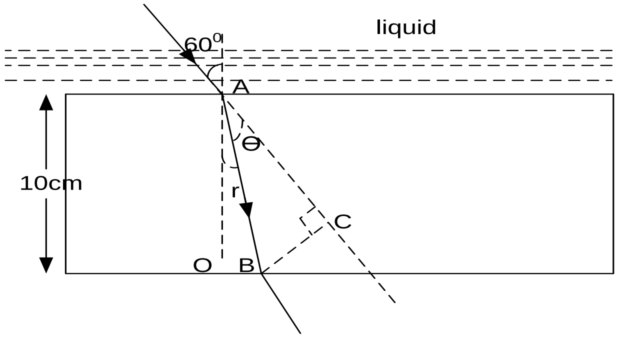
(ii) Lateral displacement $d = \frac{\boldsymbol{t}\sin(i-r)}{\boldsymbol{\cos r}}$

$$\boldsymbol{d} = \frac{\mathbf{18}\sin(50-45.8)}{\boldsymbol{\cos}\,\mathbf{45.8}}$$

$$\boldsymbol{d} = \mathbf{1.89}\boldsymbol{cm}.$$

2. Monochromatic light is incident from the liquid on the upper surface of transparent glass block where the sides of block are plane and parallel. If the speed of light in the liquid $2.42x10^{8}ms^{-1}$ and speed of light in glass sis $1.92x10^{8}ms^{-1}$. Calculate the lateral displacement of the emergent beam

Solution



$$n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a medium}} = \frac{C}{V}$$

$$n_L = \frac{3.0x10^8}{2.42x10^8} = 1.239$$

$$n_G = \frac{3.0x10^8}{1.92x10^8} = 1.563$$

$n \sin i = constant$

$1.239x\sin 60 = 1.563\sin r$

$r_R = 34.1°$

$1 \text{ x } \sin 60 = 1.596\sin r_B$

$r = 43.4°$

$\theta = 60 - 43.4 = 16.6°$

$Cos 43.4 = \frac{10}{AB}$

$AB = 13.76cm$

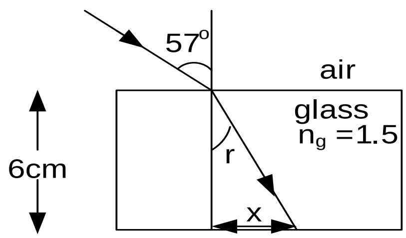
$Sin 16.6 = \frac{BC}{13.76}$

$BC = 3.93cm$

Further examples

1. Calculate the horizontal displacement of ray of light incident at an angle of 57° on a glass block 6cm thick whose refractive index is 1.5

Solution



$n_a \sin i_a = n_g \sin i_g$

$1 \text{ x } \sin 57 = 1.5\sin r_g$

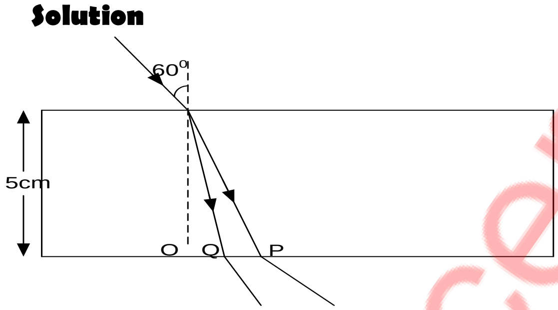
$r_g = 35.3°$

$\tan 34 = \frac{x}{6}$

$x = 4.04cm$

2. Light consisting of blue and red light is incident in air glass interface. The two colours emerge from glass block at two points P and Q respectively. If the speed of blue and red light in glass are $1.88x10^8 ms^{-1}$ and $1.94x10^8 ms^{-1}$ respectively. Calculate the distance PQ

Solution



$$n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a medium}} = \frac{C}{V}$$

$$n_B = \frac{3.0x10^8}{1.88x10^8} = 1.596$$

$$n_R = \frac{3.0x10^8}{1.94x10^8} = 1.546$$

$n \sin i = constant$

$1 \text{ x } \sin 60 = 1.546\sin r_R$

$r_R = 34.1°$

$1 \text{ x } \sin 60 = 1.596\sin r_B$

$r_B = 32.9°$

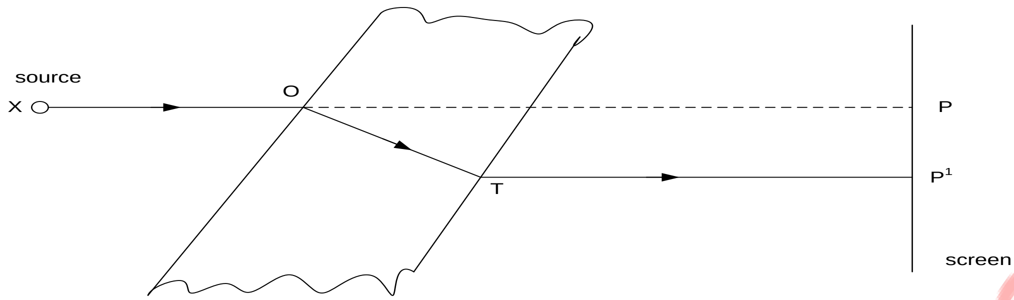
$\tan r_R = \frac{5}{OQ}$

$OQ = 5 \tan 34.1° = 3.38cm$

$OQ = 5 \tan 32.9° = 3.23cm$

$PQ = 3.28 - 3.23 = 0.15cm$

3. A monochromatic source in air sends narrow beam of light perpendicular to a screen $2.0m$ away. The beam strikes the screen at P. A glass block of refractive index 1.5 and thickness $10cm$ is inserted as shown below so that the beam strikes it at an angle of $30°$



Find;

(i) The angle of refraction at the first surface

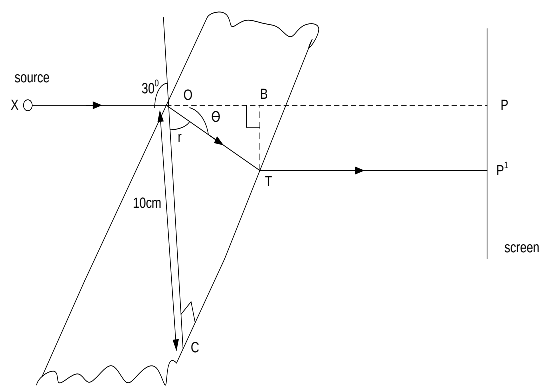
(ii) The distance OT

(iii) The speed of the beam through glass block

(iv) Time taken to cover distance OT

(v) Lateral displacement of the beam

Solution



(i) $n \sin i = constant$

$1x\sin 30 = 1.5\sin r$

$r = 19.47°$

(ii) $\Delta OCT; \cos 19.47 = \frac{0.1}{OT}$

$OT = 0.11m$

(iii) $n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{C}{V}$

$V_g = \frac{3.0x10^8}{1.5} = 2.0x10^8 ms^{-1}$

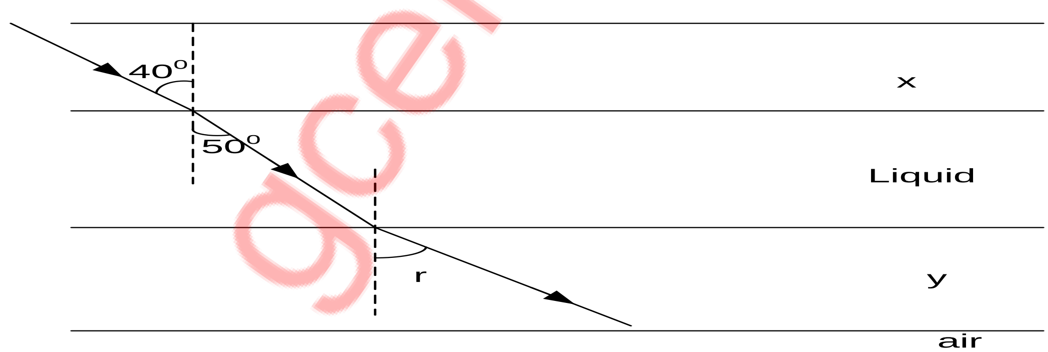
(iv) $time = \frac{distance}{speed} = \frac{0.11}{2.0x10^8} = 5.5x10^{-10}s$

(v) $\theta = 30 - 19.47 = 10.53°$

$\sin 10.53 = \frac{BT}{0.11}$

$BT = 0.02m$

4. The figure below shows a layer of liquid confined between two transparent plates X and Y of refractive index of 1.54 and 1.44 respectively. A ray of monochromatic light making an angle of $40°$ with normal to the interface between medium X and liquid is refracted through an angle of $50°$ by the liquid. Find



(i) Refractive index of the liquid

(ii) The angle of refraction in medium y

(iii) Minimum angle of incidence x for which the ray light will not emerge from the medium y

Solution

(i) $n \sin i = constant$

glass x- liquid interface

$1.54\sin 40 = n_l \sin 50$

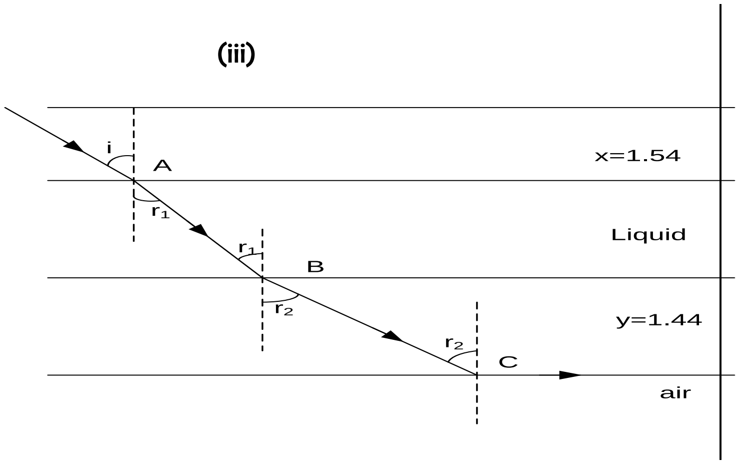
$n_l = 1.29$

(ii) liquid – glass y interface

$1.29\sin 50 = 1.44\sin r$

$r = 43.3°$

(iii)



At C: $1xsin90 = 1.44x\sin r_2$

$r_2 = 44.0°$

At B: $1.44xsin44 = 1.29x\sin r_1$

$r_1 = 50.8°$

At A: $1.29xsin50.8 = 1.54x\sin i$

$i = 40.5°$

Exercise:5

(1) What is meant by **refraction of light?**

(2) **(i)** State the laws of refraction of light.

(ii) State what brings about refraction of light as it travels from one medium to another.

3. A beam of light is incident on a surface of water at an angle of 30° with the normal to the surface. The angle of refraction in water is 22°. Find the speed of light in water if it is 3×10^8 ms^{-1}.

4. A ray of light in air makes an angle of 30° with the surface of a rectangular glass block of refractive index 1.5. What is the angle of refraction?. **An($19.5°$)**

5. A ray of light travelling from a liquid to air has an angle of incidence of $40°$ and an angle of refraction of $60°$. Find the refractive index of the liquid. **An(1.35)**

6. **(i)** What is meant by the **refractive index** of a material**?**

(ii) Light of two colours blue and red is incident at an angle γ from air to a glass block of thickness **t**. When blue and red lights are refracted through angles of θ_b and θ_r respectively, their corresponding speeds in the glass block are V_b and V_r**.** Show that the separation of the two colours at the bottom of the glass block $d = \frac{t}{C}\left(\frac{V_r}{cos\theta_r} - \frac{V_b}{cos\theta_b}\right) sin\gamma$. Where $\theta_r > \theta_b$ and **C** is the speed of light in air.

(iii) Light consisting of blue and red is incident at an angle of **60°** from air to a glass block of thickness **18cm.** If the speeds of blue and red light in the glass block are $1.86 \times 10^8\ ms^{-1}$ and $1.92 \times 10^8\ ms^{-1}$ respectively, find the separation of the two colours at the bottom of the glass block. [**Answer: 0·54cm**]

7. Show that when the ray of light passes through different media separated by plane boundaries, $nsin\phi = constant$ where n is the absolute refractive index of a medium and ϕ is the angle made by the ray with the normal in the medium.

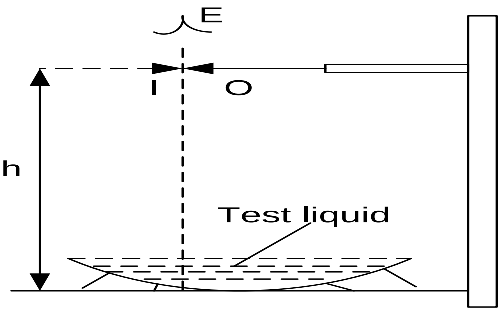
8. Show that when the ray of light passes through different media **1** and **2** separated by plane boundaries, $_1n_2 \times {_2n_1} = 1$ where **n** is the refractive index of a medium.

9. Show that when the ray of light passes through different media **1,2** and **3** separated by plane boundaries, $_1n_3 = {_1n_2} \times {_2n_3}$ where **n** is the refractive index of a medium.

10. Show that a ray of light passing through a glass block with parallel sides of thickness **t** suffers a sidewise displacement $d = \frac{t\sin(\phi-\lambda)}{\cos\lambda}$, where ϕ is the angle of incidence and λ is the angle of refraction.

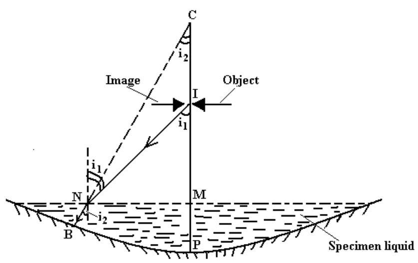
DETERMINATION OF REFRACTIVE INDEX

(i) Concave mirror method (refractive index of small quantity of liquid)



- The concave mirror is placed on a table with its reflecting surface upwards.
- An optical pin is clamped horizontally on a retort stand above the mirror with one end on the principal axis of the mirror.
- The pin is adjusted vertically while viewing from above until when a point is located whee the pin coincides with its image without parallax
- The distance of the pin from the pole of the mirror, R cm is measured and recorded
- The test liquid is poured into the mirror to a depth, dcm
- The pin is adjusted to locate the point of coincidence of the pin and the image .
- The distance of the pin above the liquid hcm is measured and recorded.
- Refractive index of the liquid n is obtained from $n = \frac{R-d}{h}$ or $n = \frac{R}{h}$

Theory



PROOF

For refraction at **N,** $n_a sin i_1 = n_l \sin i_2$------- (i)

From the diagram,

$\sin i_2 = \frac{NM}{NC}$ and $\sin i_1 = \frac{NM}{NI}$

Equation (i) now becomes

$$n_a\left(\frac{NM}{NI}\right) = n_l\left(\frac{NM}{NC}\right)$$

On simplifying, $n_l = \frac{NC}{NI}$

But **N** is very close to **M** hence NC ≈ MC and NI ≈ MN

$\Rightarrow \quad n_l = \frac{MC}{MI}$

Also for a small quantity of the liquid, **M** is close to **P** $\Rightarrow$ MC ≈ CP = **r**, and MI ≈ IP.

Thus $n_l = \frac{r}{IP}$

NOTE :

If the specimen liquid is of reasonable quantity, then its depth **d** can not be ignored. In this case,

$$\boxed{n_l = \frac{MC}{MI} = \frac{r-d}{IP-d}}$$

Examples

1. A liquid is placed in a concave mirror to a depth of **2cm**. An object held above the liquid coincides with its own image when it is **45.5cm** from the pole of the mirror. If the radius of curvature of mirror is **60cm**, calculate the refractive index of the liquid

$$n = \frac{r-2}{x-2} = \frac{60-2}{45.5-2} = 1.33$$

2. A liquid is poured in to a concave mirror to a depth of **2·0cm**. An object held above the liquid coincides with its own image when its **27·0cm** above the liquid surface. If the radius of curvature of the mirror is **40·0cm**, calculate the refractive index of the liquid.

$$n = \frac{r-2}{x} = \frac{40-2}{27} = 1.4$$

3. A small concave mirror of focal length **8cm** lies on a bench and a pin is moved vertically above it .At what point will the pin coincide with its image if the mirror is filled with water of refractive index $^4/_3$.

Solution

For a small concave mirror, the quantity of water is small that its depth **d** can be ignored

Using the relation $n = \frac{r}{x}$

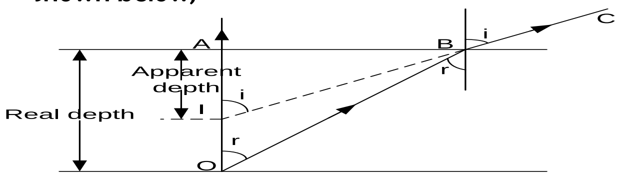
Where $r = 2f = 2 \times 8 = 16cm$

$$x = \frac{16}{^4/_3} = 12cm$$

Therefore the pin coincided with its image at a height of **12cm** above the mirror

REAL AND APPARENT DEPTH

When a glass block e.g. is placed on top of the object, the object when viewed appears displaced as shown below;



- A ray OB is refracted into the air along BC. To an observer at C, ray BC appears to come from IB
- O is the apparent position and O is the real position of the object

  Applying sneils law at B

$$n\, sin\, i = constant$$

$$n_g sinr = n_a \text{sin}i \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Consider ΔIBA; sin$i = \frac{AB}{IB}$

Consider ΔOBA; sin$r = \frac{AB}{OB}$

Putting into equation 1; $n_g \frac{AB}{OB} = \frac{AB}{IB}$

$$n_g = \frac{OB}{IB}$$

For small angles $OB \approx OA$ and $IB \approx AI$

$$n_g = \frac{OA}{IA} = \frac{Real\ depth}{Apparent\ depth} = \frac{t}{t-d}$$

$$\boxed{d = t\left(1 - \frac{1}{n}\right)}$$

Where d is the apparent displacement and t is the real depth.

NOTE

(i) The apparent displacement **d** of an object **O** is independent of the position of **O** below the glass block. Thus the same expression above gives the displacement of an object which is some distance in air below a parallel-sided glass block.

(ii) If there are different layers of different transparent materials resting on top of each other, the apparent position of the object at the bottom can be found by adding the separate displacements due to each layer.

Examples

1. A microscope is focused on a mark on a table. When the mark is covered by a plate of glass 2cm thick, the microscope has to be raised 0.67cm for the mark to be once more in focus.

Solution

$$n = \frac{t}{t-d}$$

$$n = \frac{2}{2-0.67}$$

n = 1.5

2. An object 6cm below the tank of water of refractive index of 1.33. determine the displacement of object to the observer directly above the tank

Solution

$$d = t\left(1 - \frac{1}{n}\right)$$

$$d = 6\left(1 - \frac{1}{1.33}\right)$$

$$d = 1.48cm$$

3. A tank contains slab of glass 8cm thick and of refractive index 1.52, above this is a liquid 10cm thick of refractive index 1.45 and floating on it is 3cm of water of refractive index 1.33. find the apparent position of the mark below the tank

Solution

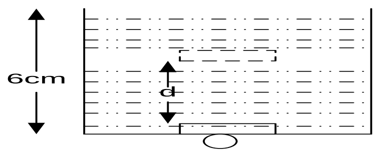
$$d = t\left(1 - \frac{1}{n}\right)$$

$$d = 8\left(1 - \frac{1}{1.52}\right) + 10\left(1 - \frac{1}{1.45}\right) + 3\left(1 - \frac{1}{1.33}\right)$$

$$d = 6.583cm$$

4. An object at a depth of $6{\cdot}0cm$ below the surface of water of refractive index $^4/_3$ is observed directly from above the water surface. Calculate the apparent displacement of the object

Solution



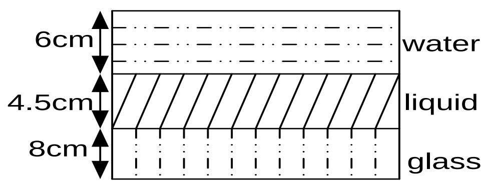
Using the relation $d = t\left(1 - \frac{1}{n}\right)$

$$d = 6\left(1 - \frac{3}{4}\right)$$

$$d = 1.5cm$$

5. A tank contains a slab of glass **8cm** and refractive index **1·6**. Above this is a depth of **4·5cm** of a liquid of refractive index **1·5** and upon this floats **6cm** of water of refractive index $^4/_3$ calculate the apparent displacement of an object at the bottom of the tank to an observer looking down wards directly from above.

Solution



Using the relation $d = t\left(1 - \frac{1}{n}\right)$

Apparent displacement $d = d_w + d_l + d_g$

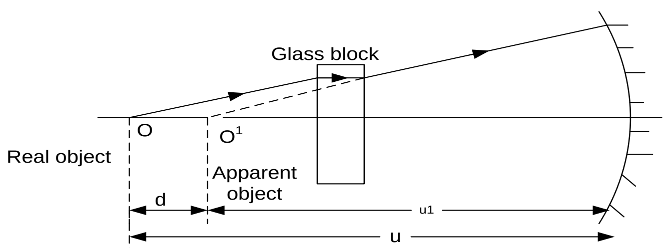
$$6\left(1 - \frac{3}{4}\right) + 4.5\left(1 - \frac{1}{1.5}\right) + 8\left(1 - \frac{31}{1.6}\right)$$

$$\boldsymbol{d = 1.5 + 1.5 + 3}$$

$$\boldsymbol{d = 6cm.}$$

6. A small object is placed **20cm** in front of a concave mirror of focal length **15cm.** A parallel-sided glass block of thickness **6cm** and refractive index **1·5** is then placed between the mirror and the object. Find the shift in the position and size of the image

Solution



Consider the action of a concave mirror in the absence of a glass block

$u = 20cm$ and $f = 15cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{15} = \frac{1}{20} + \frac{1}{v}$$

$$v = 60cm$$

Thus in the absence of a glass block, **image distance** $= 60cm$

In this case, **magnification** $m = \frac{v}{u} = \frac{60}{20} = 3$

Consider the action of a glass block

$$d = t\left(1 - \frac{1}{n}\right)$$

$$d = 6\left(1 - \frac{1}{1.5}\right) = 2cm$$

Thus in the presence of a glass block, object distance $u^1 = (20 - 2)\ cm = 18cm$

"The object is displaced and it appears to be 18cm in front of the mirror"

Consider the action of a concave mirror in the presence of a glass block

$u^1 = 18cm$ and $f = 15cm$

$$\frac{1}{f} = \frac{1}{u^1} + \frac{1}{v^1}$$

$$\frac{1}{15} = \frac{1}{18} + \frac{1}{v^1}$$

$$v^1 = 90cm$$

$\therefore$ **The required shift in the image position**

$$= v^1 - v = (90 - 60)\ cm = 30cm$$

The magnification now becomes $m^1 = \frac{v^1}{u^1} = \frac{90}{18} = 5$

Exercise: 6

1. A block of glass is 5.8cm thick. A point particle at its lower surface is viewed from above. The particle appears to be 3.9cm near. Calculate the refractive index of this glass. **An(1.49)**
2. A mark is made at the bottom of the beaker. Water of refractive index 1.33 is poured into the beaker to level of 5cm, to the water in the beaker is added a liquid which does not mix with water up to level of 8cm above the bottom of the beaker. When viewed normally form above, the mark appears to be $6.2cm$ below the upper level of the liquid, calculate the refractive index of the liquid added to the water. **An(1.23)**
3. A cube of glass $15cm$ thick is placed in water of refractive index 1.33 in an open container so that the upper surface of the cube is parallel to water surface of depth 10cm. a scratch at the bottom of the

cube appear to be $17.5cm$ below the water surface when viewed from vertically above. Calculate the refractive index of glass **An(1.5)**

4. A microscope is focused on a scratch on the bottom of the beaker. Turpentine is poured into the beaker to depth of 4cm and it is found to raise the microscope through a vertical distance of 1.28cm to bring the scratch back into focus. Find the refractive index of turpentine. **An(1.47)**
5. A microscope is first focused on a scratch on the inside of the bottom of an empty glass dish. water is then poured in and it is found that the microscope has to be raised by 1.2cm for refocusing. Chalk dust is sprinkled on the surface of water and this dust comes into focus when the microscope is raised an additional 3.5cm. Find the refractive index of water **An(1.34)**

Determination of refractive index of a glass block using real and apparent depth (travelling microscope)

- A cross is made on a sheet of white paper and the paper is placed under a travelling microscope
- The microscope is adjusted until the cross is focused clearly. The reading on the microscope scale is taken, $a\ cm$
- The test glass block is placed on the paper and the microscope adjusted again until the cross is clearly seen. The scale reading is recorded $b\ cm$
- Lycopodium powder is now sprinkled at the bottom of the glass block. The microscope is again adjusted until the particle are seen clearly, the scale reading $c\ cm$ is recorded
- Refractive index n is calculated from $n = \frac{c-a}{c-b}$

Determination of refractive index of a liquid using real and apparent depth (travelling microscope)

- A scratch is made at the bottom of a beaker and the beaker is placed under a travelling microscope
- The microscope is adjusted until the scratch is focused clearly. The reading on the microscope scale is taken, $a\ cm$
- The test liquid is poured in the beaker and the microscope adjusted again until the scratch is clearly seen. The scale reading is recorded $b\ cm$
- Some particles that can float on surface of the liquid are sprinkled on the liquid surface. The microscope is again adjusted until the particle are seen clearly, the scale reading $c\ cm$ is recorded
- Refractive index n is calculated from $n = \frac{c-a}{c-b}$

EXAMPLE:

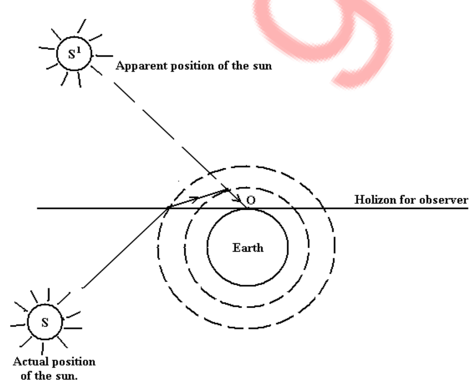
A microscope is focused on a mark at the bottom of the beaker. Water is poured in to the beaker to a depth of **8cm** and it is found necessary to raise the microscope through a vertical distance of **2cm** to bring the mark again in to focus. Find the refractive index of water.

Solution

Using the relation $n = \frac{c}{c-b} = \frac{8}{8-2} = 1.33$

Explanation of Some effects of refraction

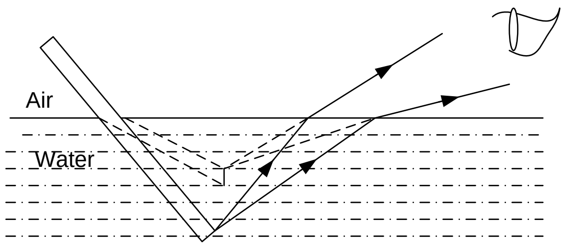
1. **Appearance of the sun when setting in the West**



- As the sun sets, layers of air near the earth get cooler and therefore more denser than layers of air higher up. Light from the sun is therefore continuously refracted towards the normal
- Rays of light which would propagate away from the earth are therefore refracted on to the earth
- When received by an observer on earth, they give an impression of presence of the sun above the horizon

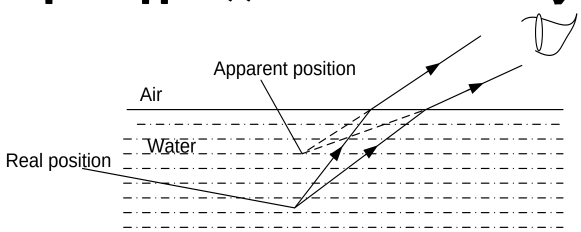
  A similar effect is seen when the sun is rising in the morning.

2. **A stick partially immersed in water**



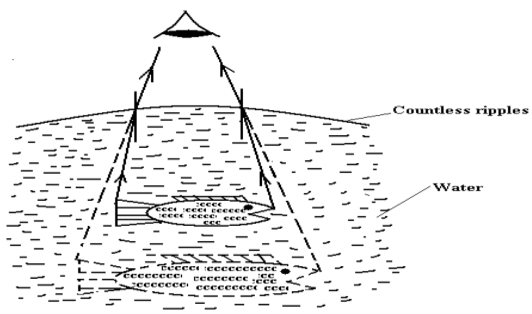
Any stick held in a slanting position in water appears to be bent.
When light rays pass from an object under water to air i.e. from a denser medium to a less dense medium, they are refracted away from the normal. As a result part of the stick under H_2O appears to be raised up. The part outside is seen in its normal position and the end result is that the stick appears

3. **A pond appears shallower than it really is**



Light rays proceeding from an object at the bottom of the pond travel from water to air. They bend away from the normal and reach the observer's eye. To an observer, the object (at the bottom of the pond) appears to be in the same straight line along which it entered his eyes, so it appears to be raised and the pond appears shallower than it really is.

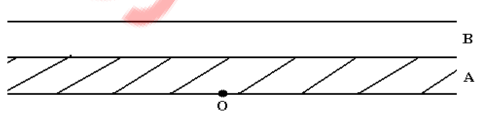
THE APARENT SIZE OF A FISH SITUATED IN WATER.



A large surface of water is not completely flat but consists of count less ripples whose convex surface on air acts as a convex lens of long focal length. In consequence the fish is with in the focal length of the lens hence it appears magnified to an observer viewing it from above.

EXERCISE: 7

1. Show that for an object viewed normally from above through a parallel sided glass block, the refractive index of the glass material is given by
$$n_g = \frac{real\ depth}{apparent\ depth}$$
2. Derive an expression for the apparent displacement of an object when viewed normally through a parallel sided glass block.
3. A vessel of depth **2d cm** is half filled with a liquid of refractive index μ_1, and the upper half is occupied by a liquid of refractive index μ_2. Show that the apparent depth of the vessel, viewed perpendicularly is $\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) d$
4. Two parallel sided blocks **A** and **B** of thickness **4·0cm** and **5·0cm** respectively are arranged such that **A** lies on an object **O** as shown in the figure below



Calculate the apparent displacement of **O** when observed directly from above, if the refractive indices of **A** and **B** are **1·52** and **1·66**.
5. A tank contains liquid **A** of refractive index **1·4** to a depth of **7·0cm**. Upon this floats **9·0cm** of liquid **B**. If an object at the bottom of the tank appears to be **11·0cm** below the top of liquid **B** when viewed directly above from, calculate the refractive index of liquid **B**.
6. Describe how the refractive index of a small quantity of a liquid can be determined using a concave mirror.

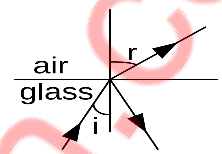
7. Describe how the refractive index of a glass block can be determined using the apparent depth method.
8. A small liquid quantity is poured into a concave mirror such that an object held above the liquid coincides with its image when it is at a height **h** from the pole of the mirror. If the radius of curvature of the mirror is **r**, show with the aid of a suitable illustration, that the refractive index of the liquid $n = \frac{r}{h}$
9. Explain how light from the sun reaches the observer in the morning before the sun appears above the horizon
10. Explain the apparent shape of the bottom of a pool of water to an observer at the bank of the pool.
11. Explain why a fish appears bigger in water than its actual size when out of water.

Total Internal Reflection and Critical angle

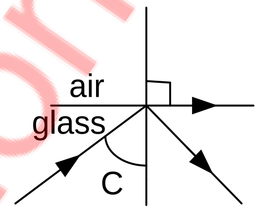
Total internal reflection if light is a reflection within the dense medium when the angle of incidence exceeds the critical angle of the medium.

Explanation of Total Internal Reflection and Critical angöle

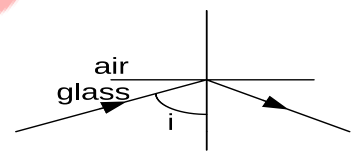
- When light travels from a more optically dense medium to a less optically dense medium, some light is reflected and some is refracted with the refracted beam being bright



- When the angle of incidence, i, is gradually increased, the angle of refraction, r also increases. At a certain value of angle of incidence the angle of refraction becomes 90° and the ray grazes the air – glass boundary. The angle of incidence for which the angle of refraction is 90° is called **critical angle(C)**



- If the angle of incidence is further increased beyond the critical angle, the light ray becomes totally reflected back to the more optically dense medium



Critical angle, C

Critical angle ,C of the medium is the angle of incidence for which the angle of refraction is 90° for a ray of light travelling from a more optically dense medium to a less optically dense medium.

Condition for total internal reflection to occur

(i) The ray of light must travel from a more dense medium to less dense eg from glass to water.

(ii) The angle of incidence must be greater than the critical angle of the medium.

Relationship between critical angle, C , and the refractive index, n

$n_g sini = n_a \sin r$

$nsinc = 1x\sin 90$

$$Sin\ C = \frac{1}{n}$$

Example

1. The refractive index of glass is 1.5, find the critical angle

Solution

$n_g sin\ c = n_a sin 90$

$1.5 sin\ c = 1$

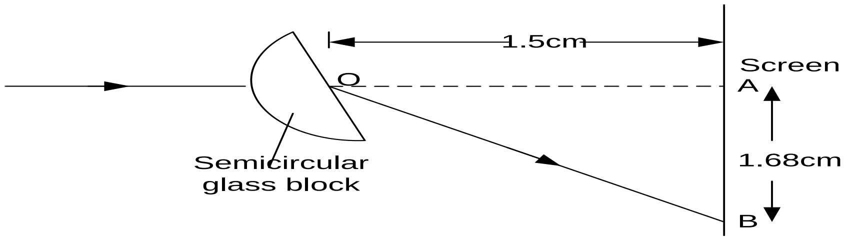
C = $\sin^{-1}\left(\frac{1}{1.5}\right)$ = 41.8°

2. Find the critical angle for the ray of light moving from water of refractive index 1.33 to air

Solution

$\sin C = \frac{1}{n}$ | $C = \sin^{-1}\left(\frac{1}{1.33}\right)$ | C = 40.75°

3. The figure below shows monochromatic light x incident towards a vertical screen



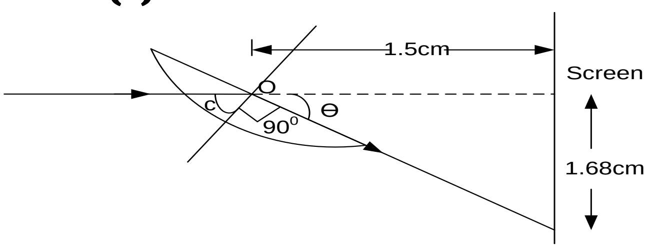
When the semi circular glass block is placed across the path of light with its flat face parallel to screen, bright spot is formed at A. when the glass block is rotated about horizontal axis through O, the bright spot moves downwards from A towards B then just disappears at B a distance 1.68cm from A.

(i) Explain why the bright spot disappears
(ii) Find the refractive index of the material of glass block
(iii) Explain whether AB will be longer if the block of glass of higher refractive index was used

Solution

(i) The spot disappears because the angle of incidence has just exceeded the critical angle

(ii)



$\tan\theta = \frac{1.68}{1.5}$

$\theta = \tan^{-1}\left(\frac{1.68}{1.5}\right) = 48.2°$

$c + 90 + 48.2 = 180$

$c = 41.8°$

$n_g \sin c \ = \ n_a \sin 90$

$n_g \sin 41.8 = 1$

$n_g = \frac{1}{\sin 41.8} = 1.5$

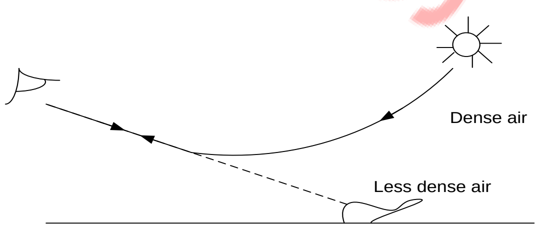
(iii) The distance will be longer because a ray that moves in a more dense medium takes a longer time in the medium

APPLICATION OF TOTAL INTERNAL REFLECTION.

(i) It is responsible for the formation of a mirage.
(ii) It is responsible for the formation of a rainbow.
(iii) It is responsible for the transmission of light in optical fibres.
(iv). It is responsible for the transmission of sky radio waves
(v). It is responsible for the transmission of light in prism binoculars.

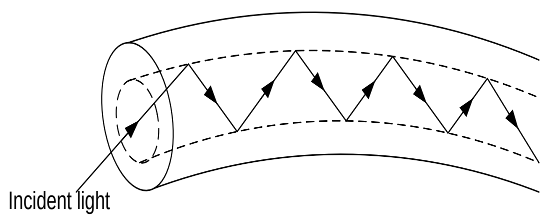
FORMATION OF A MIRAGE

Mirage: is an optical illusion of a pool of water appearing on a hot road surface



- On a hot day, The air layers near the earth's surface are hot and are less denser than the air layers above the earth's surface.
- Therefore as light from the sky pass through the various layers of air, light rays are continually refracted away from the normal till some point where light is totally internally reflected.
- An observer on earth receiving the totally internally reflected light gets an impression of a pool of water on the ground and this is the virtual image of the sky.

AN OPTICAL FIBRE



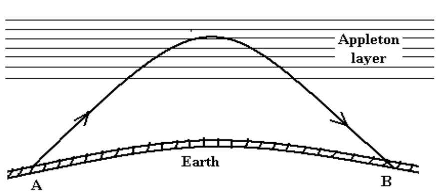
- An Optical cable is made of a transparent material coated with anther of less otptical density.
- Light entering the pipe strikes the boundary of he media at angle of incidence greater than the critical angle.
- Total internal reflection takes. This takes place repeatedly until in the pipe until the light becomes emergent form the pipe.

NOTE

(i) An optical fibre finds a practical application in an endoscope, a device used by doctors to inside the human body.

(ii) Optical fibres are used in telecommunication systems (i.e. Telephone or TV signals are carried along optical fibers by laser light).

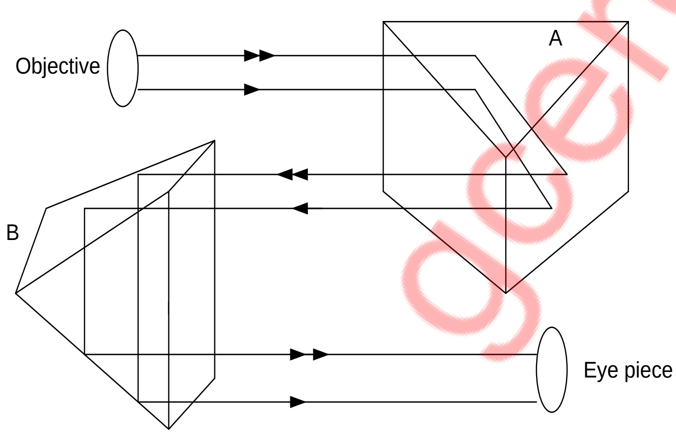
SKY RADIO WAVES



❖ A radio wave sent skyward from a station transmitter **A** is continually refracted away from the normal on entering the ionosphere(Appleton layer) that exists above the earth's surface.

❖ Within the ionosphere(Appleton layer), the wave is totally internally reflected causing it to emerge from the ionosphere(Appleton layer) and finally returns to the earth's surface where it's presence can be detected by a radio receiver at B

Prisms binoculars

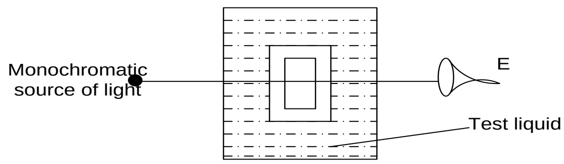


❖ A and B are isosclese right angled totally reflecting prisms
❖ A causes lateral inversion of the image formed by the objective
❖ B inverts the image vertically so that the final image is the same way up and same way round as object
❖ A and B reflect the light each through 180° making the effective length of the telescope three times the distance between the object and the eye piece

Advantages of prisms over plane mirrors

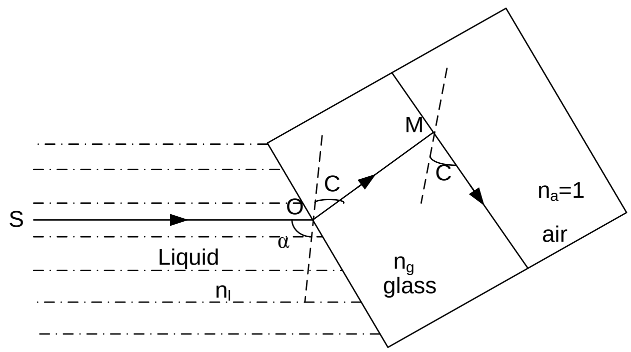
❖ Silvering on the plane mirror wears off with time while no silvering is required in a prism
❖ There is no loss of energy in prisms while thre is loss of energy at plane mirrors
❖ Plane mirrors form multiple images while prisms do not

Determination of refractive of a liquid using air cell method



- The liquid is poured into a parallel sided transparent vessel
- Light from monochromatic source is made incident normally on the vessel and viewed from the opposite side. The air cell is placed in the vessel so that it is illuminated normally on one side
- The position of the air cell, T_1 is noted. The air cell is now rotated in one direction keeping the light in view until light is suddenly cut off from the observer
- The angle, θ_1 of rotation of the air cell is noted
- The air cell is restored to position, T_1 . It is again rotated in opposite direction until light is suddenly cut off.
- The angle, θ_2 of rotation of the air cell is noted
- The average angle of rotation $\theta = \frac{\theta_1 + \theta_2}{2}$
- Refractive index of liquid is got from $n_l = \frac{1}{sin\theta}$

Theory of the air cell method



When the light is first cut off from the observer, it first grazes the glass air boundary as above.
From sneils law

$$n_l sin\alpha = n_g sinC = n_a sin90 = 1$$

$$n_l sin\alpha = 1$$

$$n_l = \frac{1}{sin\alpha}$$

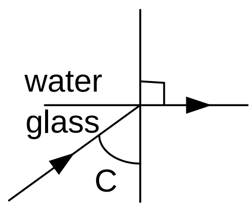
The angle between the two positions, $\theta = 2\alpha$

$$n_l = \frac{1}{sin\,{}^{\theta}/_{2}}$$

Examples:

1. The critical angle for water-air interface is $48°42^{11}$ and that of glass-air interface is $38°47^{1}$. Calculate the critical angle for glass-water interface.

Solution



$n_g sin\, c \;= n_w\, sin\, 90º$..............................(i).

Given that for water-air interface $C_w \;= 48°42^1$.

$$\therefore n_w\, sin\, C_w = 1$$

$$n_w\, sin\, \left(48 + \frac{42}{60}\right) = 1$$

$\Rightarrow \quad n_w = \; 1{\cdot}33$-----------------------(ii)

Also for glass-air interface $C_g \;= 38°47^1$

$$n_g\, sin\, C_g = 1$$

$$n_g\, sin\, \left(38 + \frac{47}{60}\right) = 1$$

$\Rightarrow \quad n_g \;=\; 1{\cdot}67$-----------------------(iii)

Substituting equation (ii) and (iii) into equation (i) gives

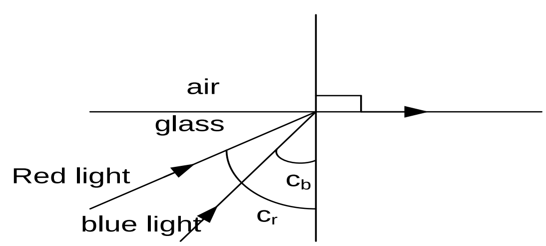
$$1{\cdot}67 sin\, c \;=\; 1{\cdot}33\, sin\, 90\,°$$

$$\Rightarrow \quad c \;=\; 52{\cdot}8\,°$$

2. The refractive index for red light is 1·634 of crown glass and the difference between the critical angles of red and blue light at the glass-air interface is $0°56^1$.What is the refractive index of crown glass for blue light

Solution

Analysis the critical angle between two media for red light is greater than that for any other light colour. This gives rise to the ray diagram below



Since $C_r > C_b$, then $C_r \;-\; C_b \;=\; 0°56^1$-------(i)

Applying Snell's law to red light gives

$n_r \sin C_r = 1$

$1.63 \sin C_r = 1$

$\Rightarrow \quad C_r = 37{\cdot}73^{\circ}$

Equation (i) now becomes

$C_r - C_b = 0°56^1$

$37{\cdot}73^{\circ} - C_b = \left(0 + \frac{56}{60}\right)$

$C_b = 36{\cdot}8^{\circ}$

Applying Snell's law to blue light gives

$n_b \sin C_b = 1$

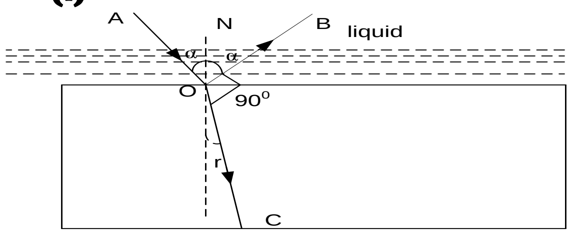
$n_b \sin 36.8 = 1$

$\Rightarrow \quad n_b = 1{\cdot}67$

4. A glass block of refractive index n_g is immersed in a liquid of refractive index n_l. A ray of light is partially reflected and partially refracted at interface such that the angle between the reflected ray and refracted ay is $90°$.
   (i) Show that $n_g = n_l$ tanα where α is the angle of incidence at the liquid glass- interface
   (ii) When the above procedure is repeated with the liquid removed, the angle of incidence increases by $8°$. find α and n_g given that $n_l = 1.33$

Solution

(i)



From 2nd law of reflection $< AON =< NOB = \alpha$

$n_l sin\alpha = n_g sinr$

$n_g = \frac{n_l sin\alpha}{sinr}$

But $r + 90° + \alpha = 180°$

$r = 90° - \alpha$

$n_g = \frac{n_l sin\alpha}{sin(90° - \alpha)} = \frac{n_l sin\alpha}{cos(\alpha)} = n_l\ tan\alpha$

(ii) From $n_g = n_l$ tanα ---------------------(i)

When liquid is removed $n_l = n_a = 1$

$\Rightarrow \quad n_g = 1xtan(\alpha + 8)$

$\therefore n_g = \frac{\tan\alpha + \tan 8}{1 - \tan 8 \tan\alpha}$

$n_g\,(1 - \tan 8 \tan\alpha) = \tan\alpha + \tan 8$

$n_g - n_g tan\ \alpha \tan 8° = \tan \alpha + \tan 8°$------(ii)

from equation (i) **tan**$\alpha = \frac{n_g}{n_l}$

Substituting for tan α in equation (ii) gives

$n_g - \frac{n_g{}^2 \tan 8}{n_l} = \frac{n_g}{n_l} + \tan 8°$

but $n_l = 1{\cdot}33$.

$\therefore n_g{}^2 - 2.340\, n_g + 1{\cdot}326 = 0$ ------------(iii)

Equation (iii) is quadratic in n_g and solving it gives

$n_g = 1{\cdot}39$ **or** n_g not physically possible.

Using equation (i)

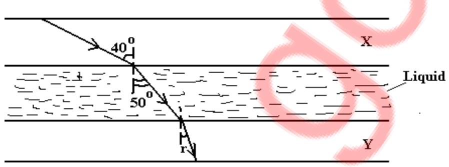
tan$\alpha = \frac{n_g}{n_l} = \frac{1.39}{1.33}$

$\alpha = \tan^{-1}(1.045) = 46{\cdot}3°$

The required angle of incidence = α + 8°

= **54·3°**

5. The figure below shows a liquid layer confined between two transparent plates **X** and **Y** of refractive index 1.54 and 1.44 respectively.



A ray of monochromatic light making an angle of **40**° with the normal to the interface between media **X** and the liquid is refracted through an angle of **50**° by the liquid. Find the
   (i) refractive index of the liquid.
   (ii) angle of refraction ,**r** in the medium **Y.**
   (iii) minimum angle of incidence in the medium **X** for which the light will not emerge from medium **Y.**

Solution

(i) Applying Snell's law at the plate **X** – liquid interface gives

$n_x sini = n_l \sin r$

$1.54 sin 40 = n_l \sin 50$

$\therefore \quad n_l = \mathbf{1.29}$

(ii) Applying Snell's law at the liquid – plate **Y** interface gives

$$n_l sini = n_y \sin r$$

$$1.29 sin50 = n_l \sin r$$

$$\Rightarrow \quad \angle r \;=\; 43.3°$$

(iii) For light not to emerge from plate Y, it grazes the liquid – plate Y interface.

$$\Rightarrow \quad \angle r = 90°$$

Applying Snell's law at the liquid – plate Y interface gives

$$n_l sini_l = n_y \sin r$$

$$1.29 sini_l = 1.44 \sin 90$$

$$sini_l = \frac{1.44}{1.29} \text{-----------------(i)}$$

More over, applying Snell's law at the plate X – liquid interface gives

$$n_x sini_x = n_l \sin i_l$$

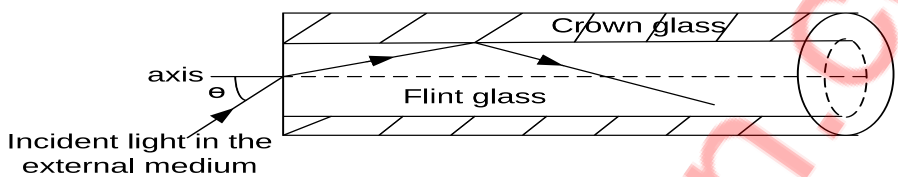
$$1.54 sini_x = 1.29 \sin i_l \text{--------------(ii)}$$

Substituting equation (i) in (ii) gives

$$1.54 sini_x = 1.29 x \frac{1.44}{1.29}$$

$$\Rightarrow \quad \angle i_x = 40.5°$$

6. The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



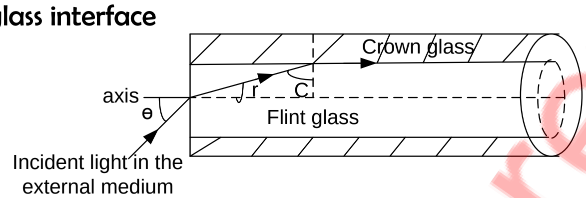
The refractive indices for flint glass, crown glass and the external medium are n_1 , n_2 and n_3 respectively. Show that a ray that enters the pipe is totally reflected at the flint-crown glass interface provided

$$\sin\theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$$

Where θ is the maximum angle of incidence in the external medium.

Solution

Analysis for light to be totally reflected, it must be incident at a critical angle on the flint-crown glass interface



Applying Snell's law at the external medium-flint glass interface gives

$$n_3 \sin\theta \;=\; n_1 \sin r$$

but $r + c \;=\; 90°$

$$\Rightarrow \quad n_3 \sin\theta \;=\; n_1 \sin(90° - c)$$

$$\therefore \quad n_3 \sin\theta \;=\; n_1 \cos c$$

$$\Rightarrow \quad \cos c = \frac{n_3 \sin\theta}{n_1} \text{-----------------(i)}$$

Applying Snell's law at the flint-crown glass interface gives

$$n_1 \sin C \;=\; n_2 \sin 90$$

$$\Rightarrow \quad \sin C \;=\; \frac{n_2}{n_1} \text{------------------------------------(ii)}$$

Using the trigonometric relation

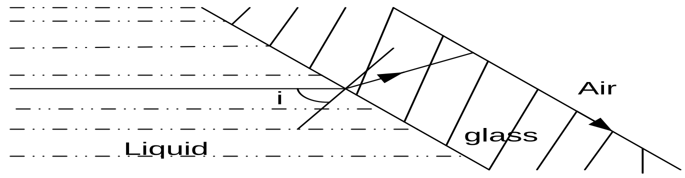
$\sin^2 c + \cos^2 c \;=1$, then

$$\left(\frac{n_2}{n_1}\right)^2 + \left(\frac{n_3 \sin\theta}{n_1}\right)^2 = 1$$

$$\sin\theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$$

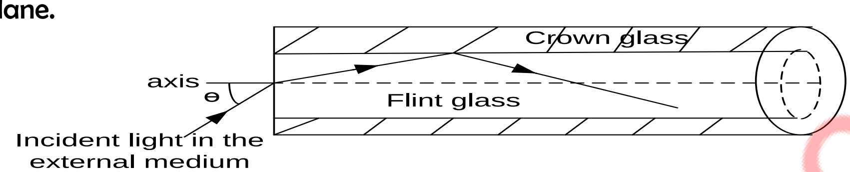
Exercise:8

1. Explain the term total internal reflection and give three instances where it is applied.
2. With the aid of suitable ray diagrams, explain the terms critical angle and total internal reflection.
3. Show that the relation between the refractive index **n** of a medium and critical angle **c** for a ray of light traveling from the medium to air is given by $n = \frac{1}{Sin\ C}$
4. Show that the critical angle, **c** at a boundary between two media when light travels from medium 1 to medium 2 is given by $Sin\ C = \frac{n_2}{n_1}$ where **n_1** and **n_2** are the refractive indices of the media respectively.
5. Explain how a mirage is formed.
6. Explain briefly how sky radio waves travel from a transmitting station to a receiver.
7. Describe how you would determine the refractive index of the liquid using an air cell.
8.



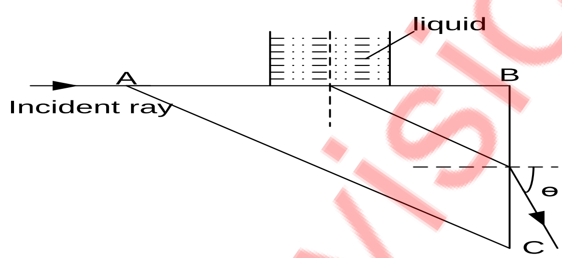
In the figure above, a parallel sided glass slide is in contact with a liquid on one side and air on the other side. A ray of light incident on glass slide from the liquid emerges in air along the glass-air interface. Derive an expression for the absolute refractive index **,n,** of the liquid in terms of the angle of incidence **i** in the liquid-medium.

9. The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



The refractive indices for flint glass and crown glass are $\mathbf{n_1}$ and $\mathbf{n_2}$ respectively. Show that a ray which enters the pipe is totally reflected at the flint-crown glass interface provided $\sin\theta = \sqrt{n_1^2 - n_2^2}$ where θ is maximum angle of incidence at the air-flint glass interface

10. A liquid of refractive index n_l is tapped in contact with the base of a right-angled prism of refractive index n_g by means of a transparent cylindrical pipe as shown.

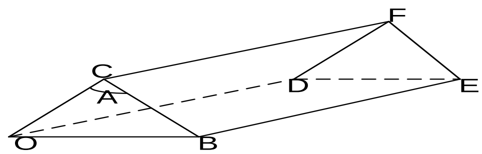


Show that a ray of light which is at a grazing incidence on the liquid-glass interface emerges in to air through face **BC** at an angle θ below the horizontal provided $n_l = \sqrt{n_g{}^2 - sin^2\theta}$ **.** Hence find n_l**,** if $n_g = \mathbf{1.52}$ and $\theta = \mathbf{47.4^o}$

REFRACTION IN A GLASS PRISM

A prism is a geometrical object with at least two plane surfaces. A prism is made up of glass.

Terms used in a prism



- Triangular face; OBC=DEF
- Refracting surface; CBEF= CODF

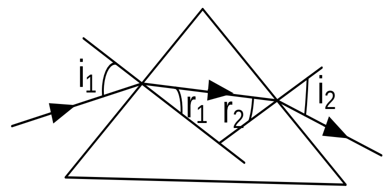
- Angle of a prism or refracting angle;

This is the angle between any two inclined surfaces of a prism and its denoted by A

Eg $< OCB =< DFE = A$

- Base; OBED

Path followed by a ray of light in a glass prism



Explain why a prism deviates light towards the base

This is because a ray that moves form a less optically dense medium bends towards the normal and a ray that moves from a more optically dense medium bends away from the normal

Deviation by a prism

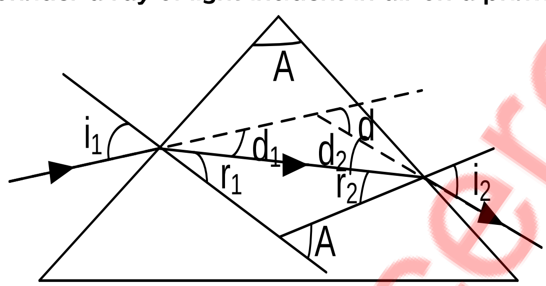
It is the change in direction of a ray of light produced by a prism.

When light passes through a glass prism, the direction of the emergent ray is altered from the initial direction. The angle through which the beam direction is altered is called **deviation, d**

Definition

The angle of deviation caused by the prism is the angle between the incident ray and the emergent ray.

Consider a ray of light incident in air on a prism of refracting angle **A** and finally emerges into air as shown



$$\boldsymbol{d = d_1 + d_2}$$

Since $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$\boldsymbol{d = (i_1 - r_1) + (i_2 - r_2)}$$

$$\boxed{\boldsymbol{d = (i_1 + i_2) - (r_1 + r_2)}}$$

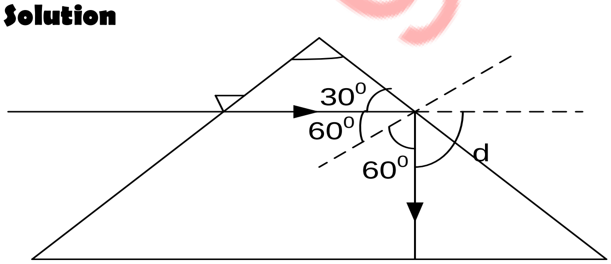
Since $A = r_1 + r_2$

$$\boxed{\boldsymbol{d = (i_1 + i_2) - A}}$$

Examples

1. A beam of monochromatic light is incident normally on glass prism of refractive angle 60°. If the refractive index of glass is 1.62, calculate the deviation caused by the prism

Solution



$$n_g sinc = 1$$

$$c = sin^{-1}\left(\frac{1}{1.62}\right) = 38°$$

The angle of incidence at the second surface is 60° which is greater than the critical angle 38, hence total internal reflection will occur

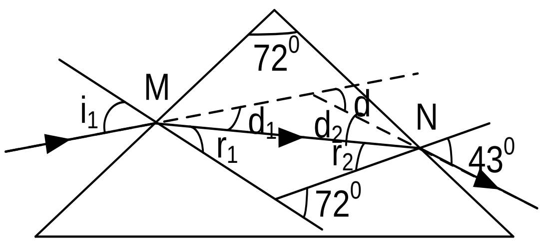
Total deviation $d = 30° + 30° = 60°$

2. A ray of light is incident on a prism of refracting angle **72°** and refractive index of **1·3.** The ray emerges from the prism at **43°.** Find

(i) the angle of incidence.

(ii) the deviation of the ray.

Solution



(i) Applying Sneil's law at N :

$n\ Sin\ i = constant$

$n_g \sin r_2 = n_a \sin 43$

$1.3 \sin r_2 = 1x \sin 43$

$r_2 = sin^{-1}\left(\frac{sin43}{1.3}\right)$

$r_2 = 31.64°$

$A = r_1 + r_2$

$r_1 = 72° - 31.64°$

$r_1 = 40.36°$

Applying Sneil's law at M

$n\ Sin\ i = constant$

$n_g \sin r_1 = n_a \sin i_1$

$1.3x \sin 40.36° = 1x \sin i_1$

$i_1 = sin^{-1}\left(\frac{1.3xsin40.36}{1}\right)$

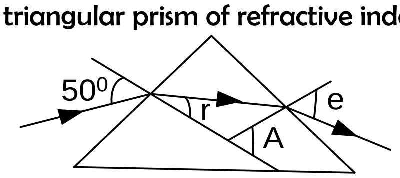
$i_1 = 57.34°$

(ii) $\boldsymbol{d = (i_1 + i_2) - A}$

$d = (57.34° + 43) - (72°)$

$d = 28.34°$

3. The diagram below shows a ray of monochromatic light incident at an angle of 50° on an equilateral triangular prism of refractive index 1.52

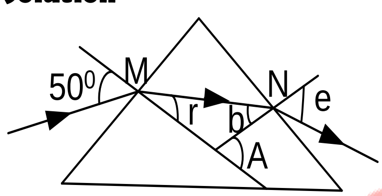


(i) Calculate the angles marked **r** and **e**

(ii) Find the deviation produced

(iii) Explained what could be observed if the ray above were of white light

Solution



i) Applying Sneil's law at M

$n\ Sin\ i = constant$

$n_a \sin 50 = n_g \sin r$

$1x \sin 50 = 1.52x \sin r$

$r = sin^{-1}\left(\frac{sin50}{1.52}\right)$

$r = sin^{-1}\left(\frac{0.766}{1.52}\right)$

$r = sin^{-1}(0.504)$

$r = 30.26°$

Since it equilateral, $A = 60°$

$A = r + b$ hence $b = A - r$

$b = 60° - 30.26°$

$b = 29.74°$

Applying Sneil's law at N

$n\ Sin\ i = constant$

$n_g \sin 50 = n_a \sin r$

$1.52x \sin 29.74 = 1x \sin e$

$e = sin^{-1}\left(\frac{1.52xsin29.74}{1}\right)$

$e = sin^{-1}\left(\frac{1.52\ x\ 0.4961}{1}\right)$

$e = sin^{-1}(0.754)$

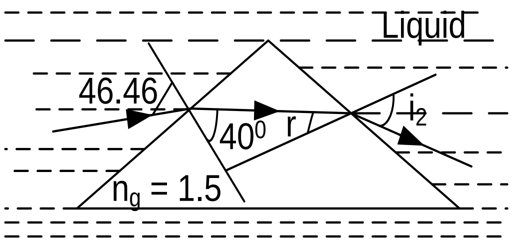
$e = 48.94°$

ii) $\boldsymbol{d = (i_1 + i_2) - A}$

$d = (50° + 48.94°) - (60°)$

$d = 38.94°$

4. The diagram below shows a ray of light incident at an angle of $46.46°$ on one side of an equilateral triangular prism immersed in liquid of refractive index n_l,



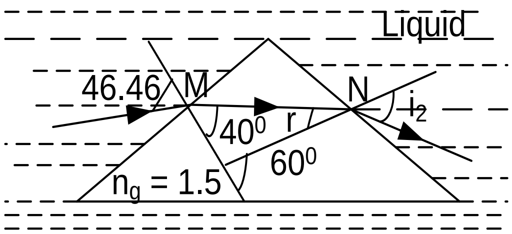
Given that the refractive index of glass is 1.5 and the angle of refraction at the first face is $40°$, calculate

(i) The value of refractive index of the liquid

(ii) The value of i_2 and

(iii) The angle of deviation

Solution



i) Applying Sneil's law at M

$$n\ Sin\ i = constant$$
$$n_l \sin 46.46 = n_g \sin 40$$
$$n_l\ x \sin 46.46 = 1.5x \sin 40$$
$$n_l = \left(\frac{1.5x0.6428}{0.7249}\right)$$
$$n_l = 1.33$$

ii) Since it equilateral, $A = 60°$

$A = r + 40°$ hence $r = A - 40°$

$$r = 60° - 40°$$
$$r = 20°$$

Applying Sneil's law at N

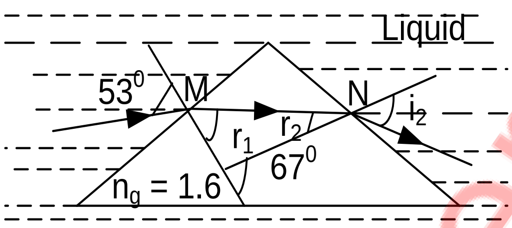
$$n\ Sin\ i = constant$$
$$n_g \sin 20 = n_l sin i_2$$
$$1.5x \sin 20 = 1.33x sin i_2$$
$$i_2 = sin^{-1}\left(\frac{1.5xsin20}{1.33}\right)$$
$$i_2 = sin^{-1}\left(\frac{1.5\ x\ 0.342}{1.33}\right)$$
$$i_2 = sin^{-1}(0.3857)$$
$$i_2 = 22.69°$$

iii) $\boldsymbol{d = (i_1 + i_2) - A}$

$$d = (46.46° + 22.69°) - (60°)$$
$$d = 9.15°$$

4. A prism of refracting angle 67° and refractive index of 1·6 is immersed in a liquid of refractive index 1·2. If a ray of light traveling through the liquid makes an angle of incidence of 53° at the left face of the prism, Determine the total deviation of the ray

Solution



iii) Applying Sneil's law at M

$$n\ Sin\ i = constant$$
$$n_l \sin 53 = n_g \sin r_1$$
$$1.2x \sin 53 = 1.6x \sin r_1$$
$$r_1 = sin^{-1}\left(\frac{1.2xsin53}{1.6}\right)$$
$$r_1 = 36.8°$$

$$A = r_1 + r_2$$
$$r_2 = 67° - 36.8°$$
$$r_2 = 30.2°$$

Applying Sneil's law at N

$$n\ Sin\ i = constant$$
$$n_g \sin 36.8 = n_l sin i_2$$
$$1.6x \sin 36.8 = 1.2x sin i_2$$
$$i_2 = sin^{-1}\left(\frac{1.6xsin36.8}{1.2}\right)$$
$$i_2 = 42.12°$$

iv) $\boldsymbol{d = (i_1 + i_2) - A}$

$$d = (53° + 42.12°) - (67°)$$
$$d = 28.12°$$

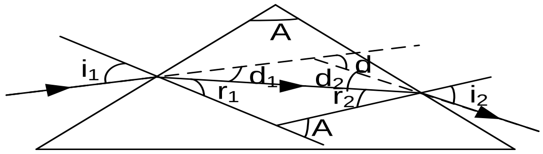
Minimum Deviation by a prism

Minimum deviation occurs when;

- The ray of light passes symmetrically
- The angle of incidence must be equal to the angle of emergence i.e $\boldsymbol{i_1 = i_2 = i}$ and $\boldsymbol{r_1 = r_2 = r}$

Relation of angle of prism A, minimum deviation and refractive index

Consider a ray on one face of the prism at an angle $\mathbf{i_1}$ and leaves it at an angle $\mathbf{i_2}$ to the normal as shown



$$\boldsymbol{d = d_1 + d_2}$$

Since $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$\boldsymbol{d = (i_1 - r_1) + (i_2 - r_2)}$$

$$\boxed{\boldsymbol{d = (i_1 + i_2) - (r_1 + r_2)}}$$

Since $A = r_1 + r_2$

$$\boldsymbol{d = (i_1 + i_2) - A}$$

But for minimum deviation $\boldsymbol{i_1 = i_2 = i}$ and

$$\boldsymbol{r_1 = r_2 = r}$$

$$\boldsymbol{d_{min} = 2i - A} \quad \therefore \quad \boldsymbol{i = \frac{d_{min}+A}{2}}$$

Also $A = r_1 + r_2 = 2r$

$$r = \frac{A}{2}$$

$$n_a \sin i = n_g sinr$$

$$n_g = n_a \frac{\sin(\boldsymbol{i})}{sin(r)}$$

$$n_g = n_a \frac{\sin\left(\frac{\boldsymbol{d_{min} + A}}{\boldsymbol{2}}\right)}{sin\left(\frac{A}{2}\right)}$$

But $n_a = 1$

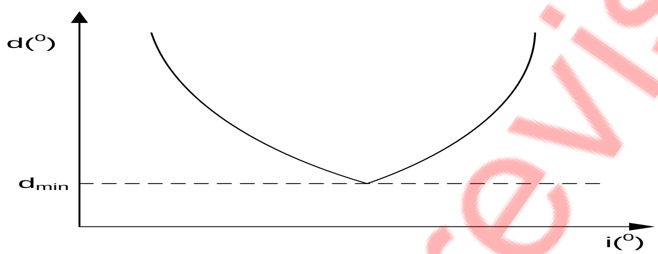
$$\boxed{n_g = \frac{\sin\left(\frac{\boldsymbol{d_{min} + A}}{\boldsymbol{2}}\right)}{sin\left(\frac{A}{2}\right)}}$$

Note: If the prism is surrounded by a medium of refractive index n_l

$$\boxed{n_g = n_l \frac{\sin\left(\frac{\boldsymbol{d_{min} + A}}{\boldsymbol{2}}\right)}{sin\left(\frac{A}{2}\right)}}$$

A graph of deviation against angle of incidence

Experiments show that as the angle of incidence **i** is increased from zero, the deviation **D** reduces continuously up to a minimum value of deviation $\mathbf{D_{min}}$ and then increases to a maximum value as the angle of incidence is increased as shown below:



Examples

1. Calculate the angle of incidence at minimum deviation for light passing through a Prism of refracting angle **70°** and refractive index of **1·65**.

Solution

$$n_g = n_a \frac{\sin\left(\frac{\boldsymbol{d_{min} + A}}{\boldsymbol{2}}\right)}{sin\left(\frac{A}{2}\right)}$$

$$1.65 = 1x \frac{\sin\left(\frac{\boldsymbol{d_{min} + 70}}{\boldsymbol{2}}\right)}{sin\left(\frac{70}{2}\right)}$$

$$\boldsymbol{d_{min} = 72.33°}$$

$$\boldsymbol{i = \frac{d_{min} + A}{2}}$$

$$i = \frac{72.33 + 70}{2} = 71.17°$$

2. An equilateral glass prism of refractive index **1·5** is completely immersed in a liquid of refractive index **1·3**. if a ray of light passes symmetrically through the prism, calculate the:
   (i) angle of deviation of the ray.
   (ii) angle of incidence

Solution

(a) For an equilateral prism, its refracting angle A =60°

(b) If the ray passes through the prism symmetrically, then the angle of deviation is minimum

$$n_g = n_l \frac{\sin\left(\frac{\boldsymbol{d_{min} + A}}{\boldsymbol{2}}\right)}{sin\left(\frac{A}{2}\right)}$$

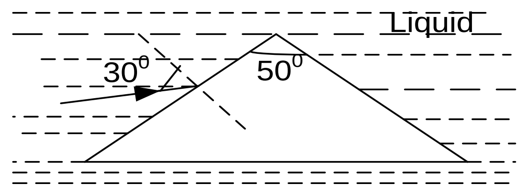
$$1.5 = 1.3x\frac{\sin\left(\frac{d_{min}+60}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$d_{min} = 10.47°$$

$$i = \frac{d_{min}+A}{2}$$

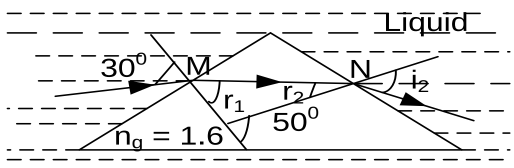
$$i = \frac{10.47+60}{2} = 35.24°$$

3. A ray of light propagating in a ;liquid is incident on prims of refractive angle 50° and refractive index 1.6 at an angle of 30° as shown below



If the ray passes symmetrically in the prism, find refractive index of liquid

Solution



For symmetrical ray $A = r_1 + r_2 = 2r$

$$r = \frac{A}{2} = \frac{50}{2} = 25°$$

$$i_1 = i_2 = i$$

$$n_l \sin i = n_g \sin r$$

$$n_l \sin 30 = 1.6 \sin 25$$

$$n_l = 1.35$$

Alternatively

$$n_g = n_l\frac{\sin\left(\frac{d_{min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$1.6 = n_l x\frac{\sin(30)}{\sin\left(\frac{50}{2}\right)}$$

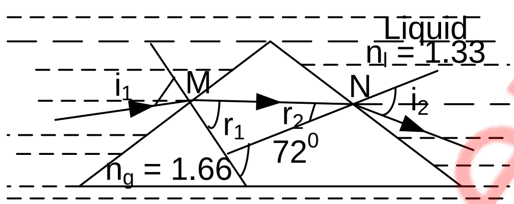
$$n_l = 1.35$$

3. A glass of refractive angle 72° and refractive index 1.66 and it is immersed in the liquid of refractive index 1.33. Calculate;

(i) Angle of incidence of ray of light, if its passes through symmetrical

(ii) The minimum deviation

Solution



For symmetrical ray $A = r_1 + r_2 = 2r$

$$r = \frac{A}{2} = \frac{72}{2} = 36°$$

$$i_1 = i_2 = i$$

$$n_l \sin i = n_g \sin r$$

$$1.33 \sin i = 1.66 \sin 36$$

$$i = 47.2°$$

$$n_g = n_l\frac{\sin\left(\frac{d_{min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$1.33\sin\left(\frac{d_{min}+72}{2}\right) = 1.66\sin\left(\frac{72}{2}\right)$$

$$d_{min} = 22.38°$$

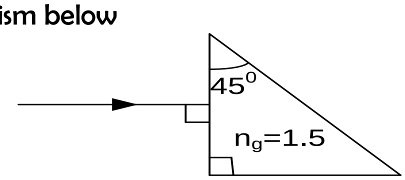
Or $d_{min} = (2i) - A$

$$d_{min} = 2x47.2 - 72$$

$$d_{min} = 22.4$$

Exercise;9

1. Calculate the total deviation in the prism below



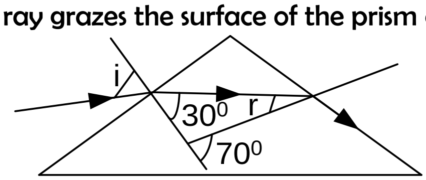
An(90°)

2. Light propagation in air is incident at 12° on a glass prism of refractive index 1.54 and refracting angle 60 ° as show. The emergent beam glass faces of the prism in contact with the liquid. Find

(i) refractive index of liquid

(ii) deviation produced by the prism. **An(1.2, 42°)**

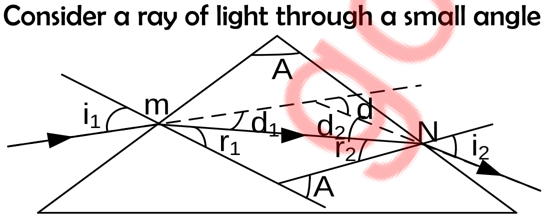
3. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive indices 1.54 and 1.48 for the two wave lengths. Find the angular separation of the two wave's length after refraction by the prism. **An(0.3°)**
4. A ray of light just undergoes total internal reflection at the second plane of the prism of refracting angle 60° and refractive index 1.5. what is its angle of incidence on the face
5. A ray of monochromatic light is incident at an angle of 30° on a prism of which the refractive index 1.52.What is the maximum refracting angle of the prism if light is just to emerge from the opposite face. **An(60.34°)**
6. Calculate the critical angle for a glass air surface if a ray of light which is incident in air is deviated through 15.5° when its angle of incidence is 40°.
7. Calculate the angular separation of the red and violet rays which image from a 60° glass prism when a ray of white light is incident on the prism at an angle of 45°. Glass has a refractive index of 1.64 for red light and 1.66 for violet light.
8. Monochromatic light is incident at an angle of 45° on a glass prism of refracting angle 70° in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass. **An(1.5)**
9. Monochromatic light propagating in aliquid is incident at an angle of 40° on a glass prism of refracting angle 60° and refractive index 1.50. if the ray passes symmetrically through the prism, find the refractive index of the liquid. **An(1.2)**
10. A glass of refractive angle 60° and refractive index 1.5 and it is immersed in the liquid of refractive index 1.3. Calculate;
    (i) Angle of incidence of ray of light, if its passes through symmetrical
    (ii) The minimum deviation **An(35.2°, 10.4°)**
11. A monochromatic beam of light is incident at an angle i, on a glass prism of refracting angle $70°$, the emergent ray grazes the surface of the prism as shown below



Find ;
    (i) The refractive index of the prism
    (ii) Angle i
12. A ray of red light is incident on a prism of refractive index 1.48 and refracting angle 60°. The ray emerges from a prism at an angle of 43°. Find;
    (i) The angle of incidence
    (ii) The angle of deviation

Deviation produced by small angle prism (A$\le 10°$)

Consider a ray of light through a small angle prism



At m: $\sin i_1 = n \sin r_1$

For small angles i_1 and r_1 measured in radians, $\sin i_1 \approx i_1$ and $\sin r_1 \approx r_1$

$$i_1 = nr_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

But $r_2 = A - r_1$, since A and r_1 are both small then r_2 is also small

At N: $\sin i_2 = n \sin r_2$

Since r_2 *is* small angles and $\sin i_2 = n \sin r_2$ then i_2 is also small, $\sin i_2 \approx i_2$ and $\sin r_2 \approx r_2$

$$i_2 = nr_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Deviation produced by prism ,

$$\boldsymbol{d} = (\boldsymbol{i_1} - \boldsymbol{r_1}) + (\boldsymbol{i_2} - \boldsymbol{r_2})$$

$$\boldsymbol{d}\ \boldsymbol{d} = nr_1 - r_1 + nr_2 - r_2$$

$$\boldsymbol{d} = (\boldsymbol{n} - \mathbf{1})\,(\boldsymbol{r_1} + \boldsymbol{r_2})$$

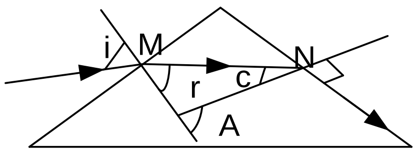
$$\boxed{\boldsymbol{d} = (\boldsymbol{n} - \mathbf{1})\boldsymbol{A}}$$

GRAZING PROPERTY OF LIGHT RAYS AS APPLIED TO PRISMS.

If a ray of light is either such that the incident angle or the emergent angle is equal to 90° to the normal of the prism, then the ray is said to graze the refracting surface of the prism.

Consider a ray of light incident at an angle i on a glass prism of refracting angle A situated in air with the emergent light grazing the other refracting surface of the prism as shown.

Maximum deviation diagram



From the diagram, $r + c = A$

$\therefore\ r = A - C$-----------------------------------(a)

At M Snell's law becomes

$n_a \sin i = n_g sinr$ -------------(b)

Substituting equation (a) in (b) gives

$\sin i = n_g sin(A - C)$

$\Rightarrow sin\ i = n_g\ (sin\ A\ cos\ C - sin\ C\ cos\ A\ )$ ------(c)

At N, Snell's law becomes

$n_g\ sin\ c = n_a\ sin\ 90°$.

$\therefore sin\ c = \frac{1}{n_g}$

But $\cos C = \sqrt{1 - sin^2\ c} = \sqrt{1 - \left(\frac{1}{n_g}\right)^2} = \frac{\sqrt{(n_g{}^2 - 1)}}{n_g}$

Substituting $sin\ c$ and $cos\ c$ in equation c gives

$$sin\ i = n_g\ (sin\ A \left[\frac{\sqrt{(n_g{}^2 - 1)}}{n_g}\right] - \frac{1}{n_g} cos\ A\ )$$

On simplifying we have $\sqrt{(n_g{}^2 - 1)} = \frac{\sin i + cosA}{\sin A}$

Squaring both sides and simplifying for n_g gives

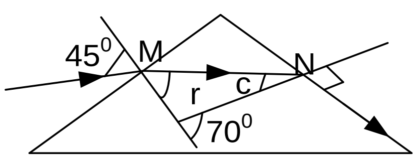
$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

Knowing the angles i and A, the refractive index n_g of a material of a prism can be determined.

EXAMPLES

13. Monochromatic light is incident at an angle of **45**° on a glass prism of refracting angle **70**° in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass material.

Solution



At M, Snell's law becomes

$n_a \sin 45 = n_g sinr$ ----------------------(a)

From the diagram, $r + c = 70°$

$\Rightarrow\ r = 70° - c$ ----------------(b)

Substituting equation (b) in (a) gives

$Sin\ 45° = n_g\ sin\ (\ 70° - c\ )$ ----------(c)

At N, Snell's law becomes

$n_g sin\ c = n_a\ sin\ 90°$

$n_g = \frac{1}{\sin C}$-----------------------------------(d)

Substituting equation (d) in (c) gives

$Sin\ 45° = \frac{sin\ (\ 70° - c\ )}{\sin C}$

$sin\ 45° sin\ c = sin\ 70° cos\ c - sin\ c\ cos\ 70°$

$(\ sin\ 45° + cos\ 70°)\ sin\ c = sin\ 70° cos\ c$

Dividing $cos\ c$ throughout gives

$$\tan C = \frac{\sin 70}{(\ sin\ 45° + cos\ 70°)}$$

$c = 41.9°$

$n_g = \frac{1}{\sin C}$

$n_g = \frac{1}{\sin 41.9}$

$n_g = 1.497$

Alternatively

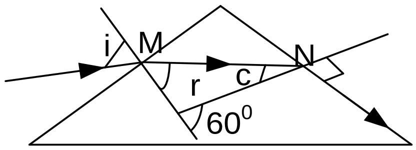
$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

$$n_g = \sqrt{1 + \left(\frac{\sin 45 + \cos 70}{\sin 70}\right)^2}$$

$n_g = 1.497$

14. A ray of light is incident on one refracting face of a prism of refractive index 1·5 and refracting angle 60°. Calculate the minimum angle of incidence for the ray to emerge through the second refracting face.

Solution



for minimum angle of incidence, the emergent ray grazes the second refracting face.
At **N**, Snell's law becomes

$$1{\cdot}5 \sin c = n_a \sin 90^\circ$$
$$\Rightarrow c = 41{\cdot}8^\circ$$
But $r + c = 60^\circ$
$$\Rightarrow r = 60^\circ - c$$
$$= 60^\circ - 41{\cdot}8^\circ$$
$$\therefore r = 18{\cdot}2^\circ$$

At **M**, Snell's law becomes

$$1{\cdot}5 \sin 18{\cdot}2^\circ = n_a \sin i$$
$$\therefore i = 27{\cdot}9^\circ$$

Alternatively

$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

$$1.5 = \sqrt{1 + \left(\frac{\sin i + \cos 60}{\sin 60}\right)^2}$$

$i = 27{\cdot}9^\circ$

EXERCISE:10

1. (i) Obtain an expression relating the deviation of a ray of light by the prism to the refracting angle and the angles of incidence and emergence.

(ii) The deviation of a ray of light incident on the first face of a **60°** glass prism at an angle of **45°** is **40°.** Calculate the angle of emergence of a ray on the second face of the prism. [**Ans** i_2 = 65°]

(iii) A prism of refractive index **1·64** is immersed in a liquid of refractive index **1·4**. A ray of light is incident on one face of the prism at an angle of **40°.** If the ray emerges at an angle of **29°,** determine the angle of the prism. [**Answer: 57·7**°]

2. **(**i) For a ray of light passing through the prism, what is the condition for minimum deviation to occur?

(ii) Derive an expression for the refractive index of a prism in terms of the refracting angle, **A**, and the angle of minimum deviation **D**.

(iii) A glass prism of refractive index n and refracting angle ,**A**, is completely immersed in a liquid of refractive index n_l. If a ray of light that passes symmetrically through the prism is deviated through an angleφ**,** Show that

$$\frac{n_l}{n} = \frac{\sin\left(\frac{A}{2}\right)}{\sin\left(\frac{\varphi + A}{2}\right)}$$

3. (a) A glass prism with refracting angle **60°** is made of glass whose refractive indices for red and violet light are respectively $1{\cdot}514$ and $1{\cdot}530$**.** A ray of white light is set incident on the prism to give a minimum deviation for red light.

Determine the:

(i) angle of incidence of the light on the prism.
(ii) angle of emergence of the violet light.
(iii) angular width of the spectrum.

(b) A certain prism is found to produce a minimum deviation of **51°.** While it produces a deviation of **62·8**° for a ray of light incident on its first face at an angle of **40·1**° and emerges through its second face at an angle of **82·7°.**

Determine the:

(i) refracting angle of the prism.
(ii) angle of incidence at minimum deviation.
(iii) refractive index of the material of the prism.

[**Ans (i) 60**° **(ii) 55·5**° **(iii) 1·648**]

4. **(i)** A ray of monochromatic light is incident at a small angle of incidence on a small angle prism in air. Obtain the expression $\boldsymbol{D = (n-1)A}$ for the deviation of light by the prism.

   (ii) A glass prism of small angle ,**A**, and refractive index n_g and is completely immersed in a liquid of refractive index n_l. Show that a ray of light passing through the prism at a small angle of incidence suffers a deviation given by $D = \left(\frac{n_g}{n_l} - 1\right)A$

5. Explain why white light is dispersed by a transparent medium.
6. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. find the angular separation of the two wave lengths after refraction by the prism. [**Ans ϕ = 0·2°**]
7. A point source of white light is placed at the bottom of a water tank in a dark room. The light from the source is observed obliquely at the water surface. Explain what is observed.
8. Monochromatic light is incident at an angle ϕ on a glass prism of refracting angle ,A, situated in air. If the emergent light grazes the other refracting surface of the prism, Show that the refractive index, n_g, of the prism material is given by

$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

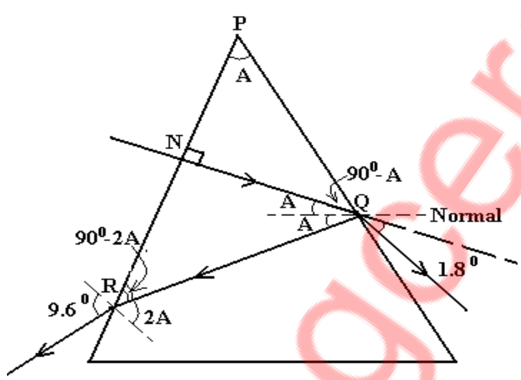
9. A ray of light is incident at angle of 30° on a prism of refractive index 1·5 .calculate the limiting angle of the prism such that the ray does not emerge when it meets the second face. [**Ans A = 61·3°**]

EXAMPLES:

A ray of light that falls normally upon the first face of a glass prism of a small refracting angle under goes a partial refraction and reflection at the second face of the prism. The refracted ray is deviated through an angle $1.8°$ and the reflected ray makes an angle of $9.6°$ with the incident ray after emerging from the prism through its first face. Calculate the refracting angle of the prism and its refractive index of the glass material.

Solution

Let **A** be the required refracting angle of the prism as shown



Consider the deviation suffered by the incident light

$\boldsymbol{D = (n-1)A}$

$\Rightarrow 1.8° = (n-1)A$ --------------------(i)

From ΔPQN, $\angle PQN = 90° - A$

$\Rightarrow$ At Q, the angle of incidence $= A$

From ΔNQR, $\angle QRN = 90° - 2A$

$\Rightarrow$ At R, the angle of incidence $= 2A$

∴At R, Snell's becomes $n_a \sin 9{\cdot}6° = n \sin 2A$

For small angles, $\boldsymbol{\sin 9.6° \approx 9.6°}$ **and** $\boldsymbol{\sin 2A \approx 2A}$

$\Rightarrow 9.6° = 2nA$ ----------------------------(ii)

Equation (i) ÷ Equation (ii) gives

$$\frac{1.8°}{9.6°} = \frac{(n-1)A}{2nA}$$

$\Rightarrow 3.6° n = 9.6°(n-1)$

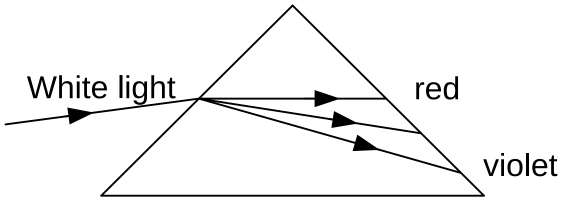
Thus $\boldsymbol{n = 1.6}$

Equation (i) now becomes $1.8° = (1.6-1)A$

$\therefore \boldsymbol{A = 3°}$

DISPERSION OF WHITE LIGHT BY A TRANSPARENT MEDIUM

Dispersion of whit light is the separation of white light in to its component colours by a transparent medium due to their speed differences in the medium



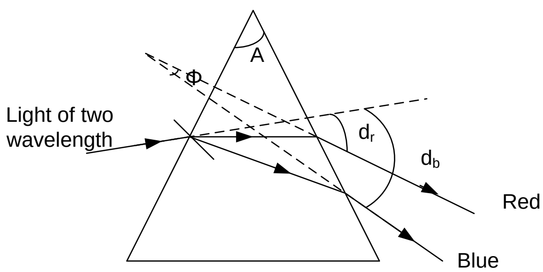
When white light falls on a transparent medium, its different component colours travel with different speeds through the medium. They are there fore deviated by different amounts on refraction at the surface of the medium and hence dispersion.

NOTE :

(i) White light is a mixture of various colours. This is called the spectrum of white light.

(ii) The spectrum of white light consists of red, orange, yellow, green ,blue, indigo and violet light bands. On refraction, violet is the most refracted colour away from the normal (violet is the most deviated colour) while red is least deviated

(iii) When light of two wavelengths say red and blue light is incident at a small angle on a small angle prism of refracting angle A having refractive indices of n_r and n_b for the two wave lengths respectively, then the two wave lengths are deviated as shown below.



The deviation of red and blue light is given by $d_r = (n_r - 1)A$ and $d_b = (n_b - 1)A$

The quantity $\phi = d_b - d_r$ is called the **Angular separation (Angular dispersion)** produced by the prism.

$\Rightarrow \phi = (n_r - 1)A - (n_b - 1)A$

on simplifying $\phi = (n_r - n_b)A$

EXAMPLES:

1. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

Solution

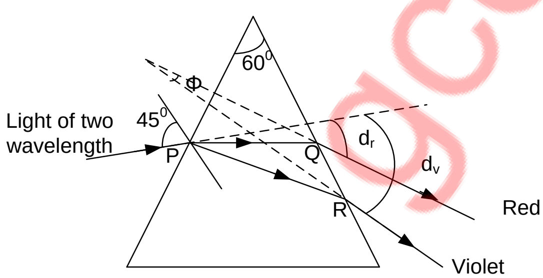
For a small prism,

Angular separation $\phi = (n_1 - n_2)A$

$\phi = (1{\cdot}52 - 1.48) \times 5^\circ$

$\Rightarrow \phi = \mathbf{0{\cdot}2^\circ}$

2. A glass prism with refracting angle 60° has a refractive index of 1·64 for red light and 1·66 for violet light. Calculate the angular separation of the red and violet rays which emerge from the prism when a ray of white light is incident on the prism at an angle of 45°



Case I: Consider the deviation suffered by red light

At **P,** Snell's law becomes .

$n_a \sin 45^\circ = 1{\cdot}64 \sin r_1$

$\therefore r_1 = 25{\cdot}54^\circ$

But $r_1 + r_2 = 60^\circ$

$\Rightarrow r_2 = 60^\circ - 25{\cdot}54^\circ$

$\therefore r_2 = 34{\cdot}46^\circ$

At **Q**, Snell's law becomes

$n_a \sin i_2 = 1{\cdot}64 \sin 34{\cdot}46^\circ$

$\therefore i_2 = \mathbf{68{\cdot}13^\circ}$

Total Deviation $\boldsymbol{D_r = d_2 + d_1}$

where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\boldsymbol{D_r} = (45^\circ - 25{\cdot}54^\circ) + (68{\cdot}13^\circ - 34{\cdot}46^\circ)$

$\therefore \boldsymbol{D_r} = 53{\cdot}13^\circ$

Case II: Consider the deviation suffered by violet light

At **P,** Snell's law becomes .

$n_a \sin 45^\circ = 1{\cdot}66 \sin r_1$

$\therefore r_1 = 25{\cdot}21^\circ$

But $r_1 + r_2 = 60^\circ$

$r_2 = 60^\circ - 25{\cdot}21^\circ$

$\therefore r_2 = 34{\cdot}79^\circ$

At **R**, Snell's law becomes

$n_a \sin i_2 = 1{\cdot}66 \sin\ 34{\cdot}79^{\circ}$

$\therefore i_2 = \mathbf{71.28^{\circ}}$.

Total Deviation $D_v = d_2 + d_1$

where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\boldsymbol{D_v} = (45^{\circ} - 25{\cdot}21^{\circ}) + (71{\cdot}28^{\circ} - 34{\cdot}79^{\circ})$

$\therefore \boldsymbol{D_v} = 56{\cdot}28^{\circ}$.

Thus required angular separation $\boldsymbol{\phi} = \boldsymbol{D_v} - \boldsymbol{D_r}$

$\boldsymbol{\phi = 56{\cdot}28^{\circ} - 53{\cdot}13^{\circ}}$

$\Rightarrow \quad \phi = \mathbf{3{\cdot}15^{\circ}}$

APPEARANCE OF WHITE LIGHT PLACED IN WATER

OBSERVATION:

A coloured spectrum is seen inside the water surface with violet on top and red down.

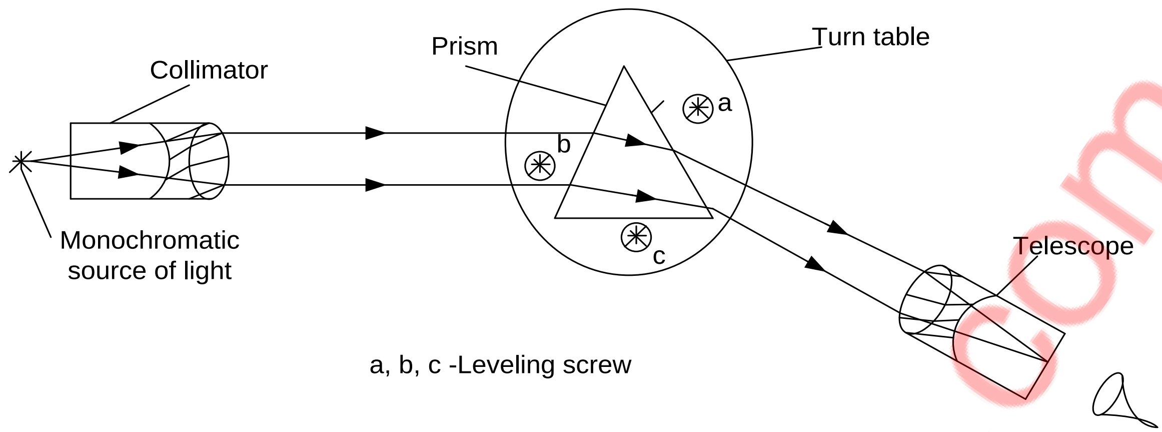
EXPLANATION:

The different component colours of white light travel with different speeds through water. They are there fore deviated by different amounts on refraction at the water surface. Hence different coloured images are formed at different points inside the water surface with a violet coloured image on top.

SPECTROMETER

It is an instrument used to measure accurate determination of deviation of a parallel beam of light which has passed through a prism. This provides a mean of studying optical spectra as well as measuring the angle of prism, minimum deviation and refractive index of glass prism

It consists of a collimator, a telescope, and a turn table on which the prism is placed as shown.



Initial adjustments:

Before the spectrometer is put in to use, 3 adjustments must be made onto it and these include,

(i) The collimator is adjusted to produce parallel rays of light.
(ii) The telescope is adjusted to receive light from the collimator on its cross wire.
(iii) The turn table is leveled.

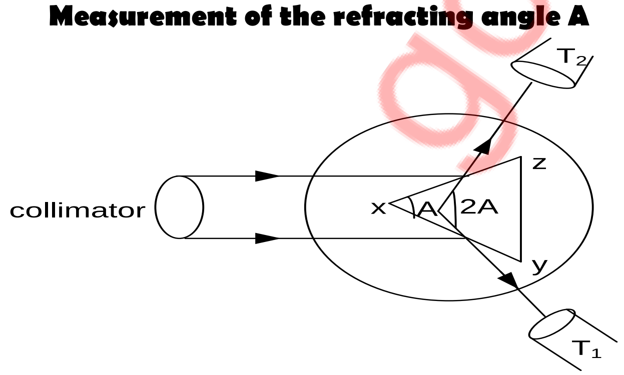
Explanation of the adjustments

i) The eye piece of the telescope is adjusted so that the cross wires are in sharp focus. The telescope is turned to face a distant object and the length of the telescope is adjusted until the image of the image of the distant object is clearly seen on the cross wires. This means that the telescope receives parallel light.

ii) The prism is removed and the collimator slit is now illuminated using a strong source of monochromatic light. The telescope is now turned to face the collimator and collimator length adjusted until the image of its slit is seen clearly on the cross wires. This means that the collimator is set to produce parallel light.

iii) The prism is now placed on the table. If the image of the slit seen is off the field of view, the screws are adjusted in or out to bring the image to the centre of the field of view. This way the spectrometer is adjusted and ready for use

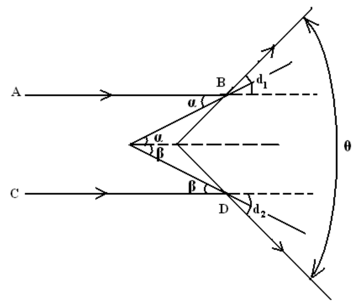
Measurement of the refracting angle A



- The collimator is adjusted to produce parallel rays of light.
- The telescope is adjusted to receive light from the collimator on its cross wire.
- The turn table is leveled
- The prism is placed on the turn table with its refracting angle facing the collimator as shown.
- With the table fixed, the telescope is moved to position T_1 to receive the light from the collimator on its cross wire. This position T_1 is noted
- The telescope is now turned to a new position T_2 to receive light on its cross wire. The angle θ between T_1 and T_2 is measured.
- The prism angle A is given by $A = \frac{\theta}{2}$

PROOF OF THE RELATION

Consider a parallel beam of light incident on to a prism of refracting angle A making glancing angles α and β as shown.



From the geometry, $\alpha + \beta = A$---------.(i).

Deviation d_1 of ray AB $= 2\alpha$

Deviation d_2 of ray CD $= 2\beta$.

Total deviation $\theta = d_1 + d_2$

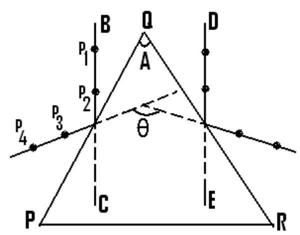
$\theta = 2\alpha + 2\beta$

$\theta = 2(\alpha + \beta)$------------------(ii)

Combining equation (i) and (ii) gives

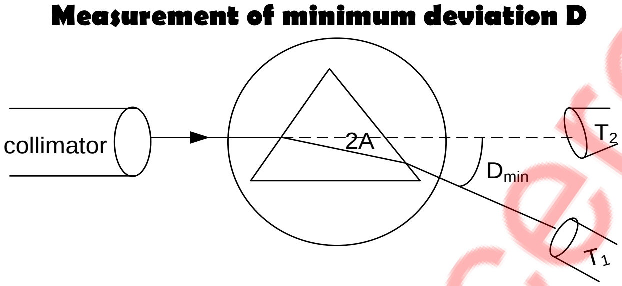
$$\theta = 2A.$$

METHOD 2: USING OPTICAL PINS



- A white paper is stuck to the soft board using top-headed pins. Two parallel line BCand DE are drawn on the paper and the prism is placed with its apex as shown.
- Two optical pins **P_1**and **P_2** are placed along BC and pins **P_3** and **P_4** are placed such that they appear to be in line with the images of **P_1** and **P_2** as seen by reflection from face PQ .
- The procedure is repeated for face QR.
- The prism is removed and angle θ is measured.
- The required refracting angle $A = \frac{\theta}{2}$

Measurement of minimum deviation D



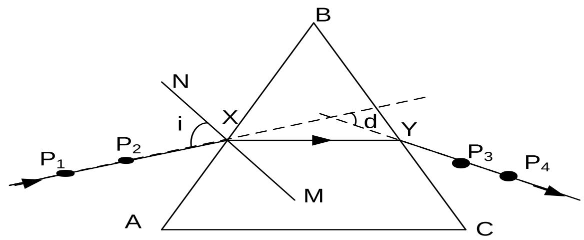
- The collimator is adjusted to produce parallel rays of light.
- The telescope is adjusted to receive light from the collimator on its cross wire
- The turn table is leveled.
- The prism is placed with the refracting angle pointing a way from the collimator as shown above.
- The telescope is turned to receive refracted light form the opposite face of the prism.
- The table is now turned while keeping the refracted light in view until a point when the ray begins to move backwards. Position T_1 of the telescope is noted.
- The prism is removed and the telescope is turned to receive light directly from the collimator. The new position T_2 is marked.
- The angle between T_1 and T_2 is determined and this is the angle of minimum deviation d_{min}.

Note :

The refractive index of the material of the prism is calculated from

$$n = \frac{\sin\left(\frac{d_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

METHOD 2: USING OPTICAL PINS



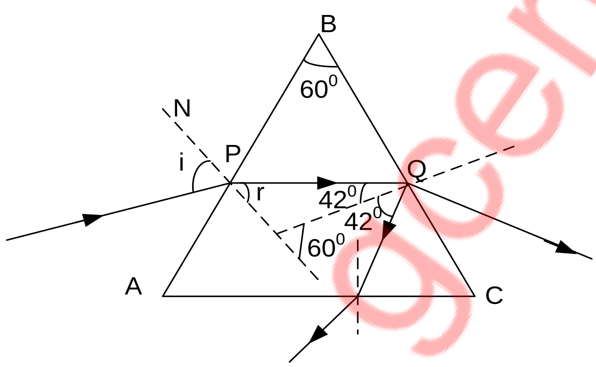
- The prism is palced on a plane sheet of paaper on a soft board and its outline ABC is traced out as shown above.
- A normal NM is drawn through point X on side AB of the prism and a line PX is drawn making an angle i
- Two optical pins **P_1** and **P_2** are placed along the lines that make different angles of incidence **i.**
- Pins **P_3** and **P_4** are placed such that they appear to be in line with the images of **P_1** and **P_2** as seen through the prism.
- The angles of deviation **d** are measured for different angles of incidence.
- A graph of **d** against **i** is plotted to give a curve whose angle of deviation at its turning point is the angle of minimum deviation d_{min} of the prism.

USES OF A GLASS PRISM.

- They enable the refractive index of a glass material to be measured accurately.
- They are used in the dispersion of light emitted by glowing objects.
- They are used as reflecting surfaces with minimal energy loss.
- They are used in prism binoculars.

More worked out examples

1. A ray of monochromatic light is incident on one face of a glass prism of refracting angle 60° and is totally internally reflected at the next face.
   (i) Draw a diagram to show the path of light through the prism.
   (ii) Calculate the angle of incidence at the first face of the prism if its refractive index is 1·53 and the angle of incidence at the second face is 42°.
   Solution



From the diagram, $r + 42° = 60°$

$\therefore r = 18°$

At P, Snell's becomes

$$n_a \sin i = 1{\cdot}53 \sin 18°$$

$$\therefore i = 28{\cdot}2°$$

NOTE

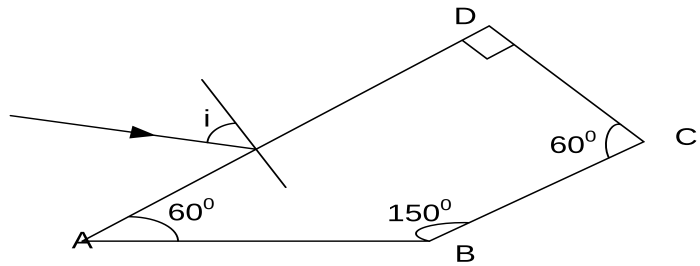
For $n_g = 1{\cdot}53$, then the critical angle c for the above glass material is given by the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.53}$$

$$c = 40.8°$$

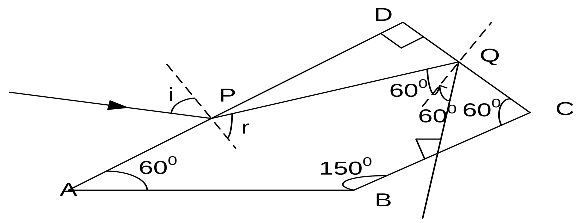
Thus total internal reflection occurs at **Q** since the angle of incidence is greater than the critical angle **c**

2. A ray of light is incident on the face AD of a glass block of refractive index 1·52 as shown.



If the ray emerges normally through face BC after total internal reflection, calculate the angle of incidence, i.

Solution



- After a total internal reflection at Q, t the ray emerges through face BC
- At R, there is no refraction. Therefore Snell's law does not hold at this point.

From ΔRCQ, $\angle RQC + 60^\circ + 90^\circ = 180^\circ$

$\therefore \angle RQC = 30^\circ$

$\Rightarrow$ At Q, the angle of reflection $= 60^\circ$

Hence at Q, the angle of incidence $= 60^\circ$

Solving ΔQDP gives $\angle QPD = 60^\circ$

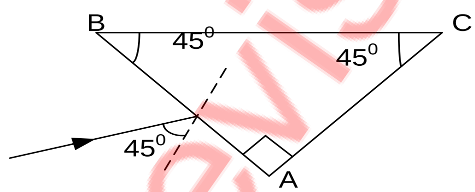
Hence at P, the angle of refraction $\boldsymbol{r} = 30^\circ$

$\Rightarrow$ At P, Snell's law becomes

$n_a \sin i = 1{\cdot}52 \sin 30^\circ$

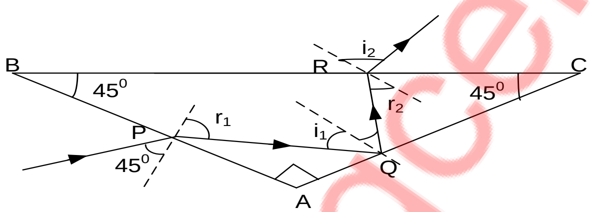
$\therefore \; i = 49{\cdot}5^\circ$

3. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



At P, Snell's becomes $n_a \sin 45^\circ = 1{\cdot}5 \sin r$

$\therefore \; r_1 = 28{\cdot}1^\circ$

At P, $\angle APQ + r_1 = 90^\circ$ where $r_1 = 28{\cdot}1^\circ$

$\Rightarrow \boldsymbol{\angle APQ = 61{\cdot}9^\circ}$

From ΔAPQ, $\angle PQA + 61{\cdot}9^\circ + 90^\circ = 180^\circ$

$\therefore \; \angle PQA = 28{\cdot}1^\circ$

$\Rightarrow$ At Q, the angle of incidence $\boldsymbol{i_1 = 61{\cdot}9^\circ}$

Testing for total internal reflection at Q using the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.5}$$

$$c = 41.8°$$

Thus light is totally reflected at **Q** since $\boldsymbol{i_1 > c}$.

$\Rightarrow \boldsymbol{\angle PQA = \angle RQC = 28{\cdot}1^\circ}$

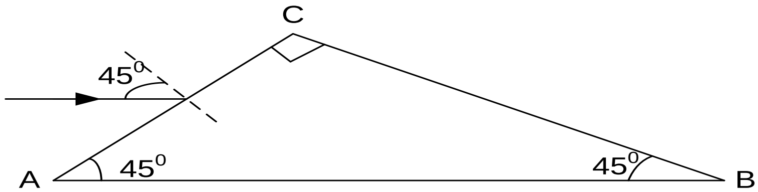
From ΔRQC, $28{\cdot}1^\circ + 45^\circ + 90^\circ + r_2 = 180^\circ$

$r_2 = 16{\cdot}9^\circ$

At R, Snell's becomes $1{\cdot}5 \sin 16{\cdot}9^\circ = n_a \sin i2$

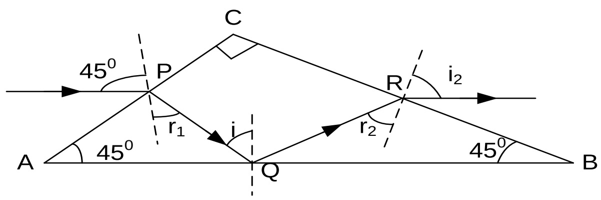
Thus $\boldsymbol{i_2 = 25{\cdot}15^\circ}$

4. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



At P, Snell's becomes $n_a \sin 45° = 1{\cdot}5 \sin r$

$\therefore r_1 = 28{\cdot}1°$

From ΔAPQ, $\angle PQA + 45° + 90° + r_1 = 180°$

where $r_1 = 28{\cdot}1°$

$\therefore \angle PQA = 16{\cdot}9°$

$\Rightarrow$ At Q, the angle of incidence $\boldsymbol{i = 73{\cdot}1°}$

Testing for total internal reflection at Q using the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.5}$$
$$c = 41.8°$$

Thus light is totally reflected at **Q** since $\boldsymbol{i > c}$.

$\Rightarrow \boldsymbol{\angle PQA = \angle RQC = 16{\cdot}9°}$

From ΔRQC, $16{\cdot}9° + 45° + 90° + r_2 = 180°$

$r_2 = 28{\cdot}1°$

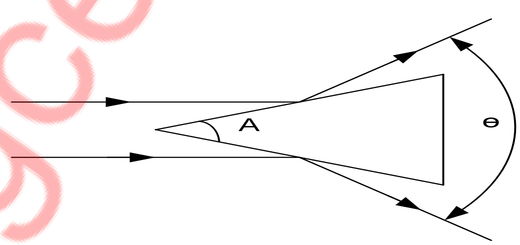
At R, Snell's becomes

$1{\cdot}5 \sin 28{\cdot}1° = n_a \sin i_2$

Thus $\boldsymbol{i_2 = 45°}$

EXERCISE:11

1. Draw a labeled diagram of a spectrometer and State the necessary adjustments that must be made on to it before put in to use.
2. Describe how the refracting angle of the prism can be measured using a spectrometer.
3. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the refracting angle **A** of the prism
4. A parallel beam of light is incident on to a prism of refracting angle, **A**, as shown



Show that θ = **2A**

5. Describe how the minimum deviation, **D**, of a ray of light passing through a glass prism can be measured using a spectrometer.
6. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the angle of minimum deviation, **D,** of a ray of light passing through a glass prism.
7. Describe how the refractive index of a material of a glass prism of known refracting angle can be determined using a spectrometer.

8. Describe briefly two uses of glass prisms
9. Light of two wavelengths is incident at a small angle on a thin prism of refracting angle 5° and refractive indices 1.52 and 1.50 for the two wavelengths. Find the angular separation of the two wavelengths after refraction by the prism **An(0.10°)**

REFRACTION THROUGH LENSES

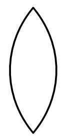
A lens is a piece glass bounced by one or two spherical surfaces.

Types of lenses

There are two types of lenses as shown below

a. Converging (convex) lens

It is a lens which is thicker in the middle than at the edges.

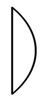


- Its curved outwards
- Thicker in the middle
- Thinner at the edge

Convex lenses are also divided into two namely;

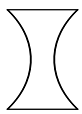
(i) Converging meniscus

(ii) Plano convex



b. Diverging (concave) lens

It is a lens which is thinner in the middle than at the edges.

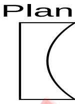


- Its curved inwards
- Thinner in the middle
- Thicker at the edge

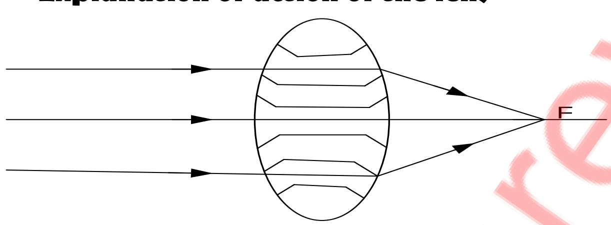
Concave lenses are also divided into two namely;

(i) Diverging meniscus

(ii) Plano concave



Explanation of action of the lens



A thin lens is regarded as made up of a large number of small angle prism whole angles increase from zero at the middle of the lens to a small value at the edge.

The deviation by a small angle prism is $d = (n-1)A$ where n is the refractive index therefore,

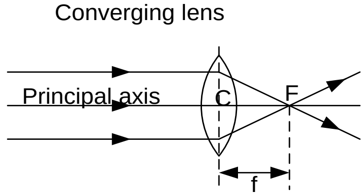
- For light incident on a path of a prism, it bends towards the base and that is why for convex lens, rays converge and for diverging lens, they diverge

Refraction of light in lenses

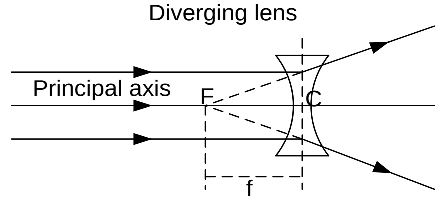
(i) A parallel beam of light, parallel and close to the principal axis of **a converging lens** is converged or brought to focus at the principal focus **F**

(ii) A parallel beam of light, parallel and close to the principal axis of **a diverging lens** is diverged such that the rays appear to come from the principal focus **F**

Converging lens



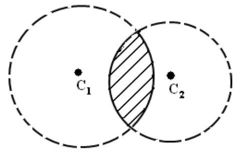
Diverging lens



Terms used in Lenses

Definitions:

1. **Centres of curvature of a lens:** These are centres of the spheres of which the lens surfaces form parts.



Points C_1 and C_2 are the centers of curvature of the lens surfaces.

2. **Radii of curvature of a lens:** These are distances from the centers to the surfaces of the spheres of which the lens surfaces form part.

3. **Principal axis of a lens:** This is the line joining the centers of curvature of the two surfaces of the lens.

4. **Optical centre of the lens:** This is the mid-point of the lens surface through which rays incident on the lens pass un deviated.

5. **Paraxial rays:** These are rays close to the principle axis and make small angles with the lens axis.

6. (i) **Principal focus "F" of a convex lens:** it is a point on the principal axis where where rays originally parallel and closeto the principal axis converge after refraction by the lens.
A convex lens has a real (in front) principal focus.

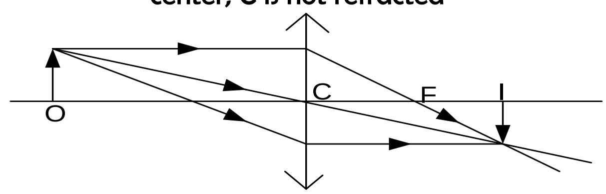
(ii) **Principal focus "F" of a concave lens:** it is a point on the principal axis where where rays originally parallel and closeto the principal axis appear to diverge from after refraction by the lens.
A concave lens has a virtual (behind) principal focus.

7. (i) **Focal length "f" of a convex lens:** it is the distance from the optical centre of the lens to the point where paraxial rays incident and parallel to the principal axis converge after refraction by the lens.
(ii) **Focal length "f" of a concave lens:** it is the distance from the optical centre of the lens to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after refraction by the lens.

Ray diagrams for a converging lens

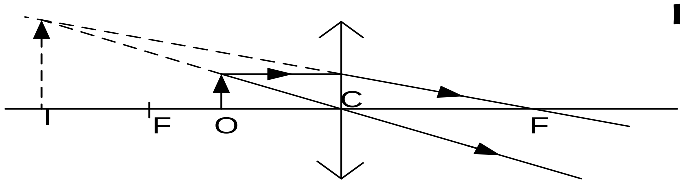
Principal rays for a converging lens

- A ray parallel to the principal axis is refracted to pass through the principal focus F
- A ray passing through the principal focus F is refracted to parallel to the principal axis
- A ray passing through the optical center, C is not refracted



1. **Images formed by a converging lens**

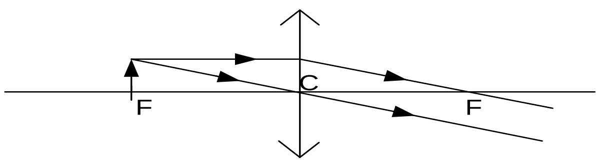
(i) **Object between F and C (Magnifying glass)**



Nature of image

- Virtual
- Erect
- Maginifed

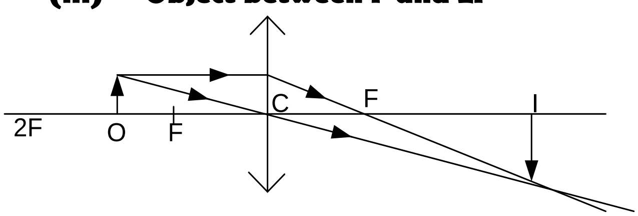
(ii) **Object at F**



Nature of image

- Image at infinity

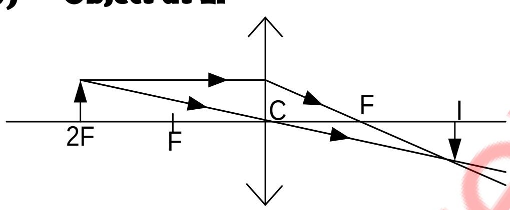
(iii) Object between F and 2F



Nature of image

- Real
- Inverted
- Magnified
- Beyond 2F

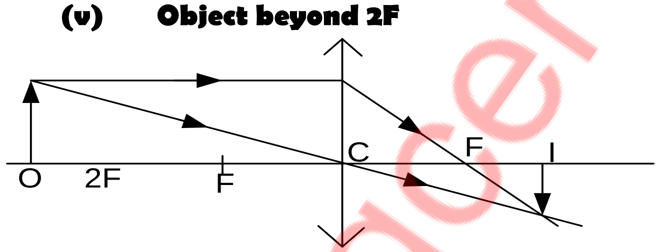
(iv) Object at 2F



Nature of image

- Real
- Inverted
- Same size as object
- Between F and 2F

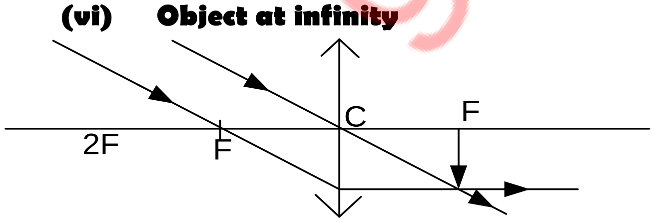
(v) Object beyond 2F



Nature of image

- Real
- Inverted
- Diminished
- Between F and 2F

(vi) Object at infinity



Nature of image

- Real
- Inverted
- Diminished
- At F

USES OF CONVEX LENSES

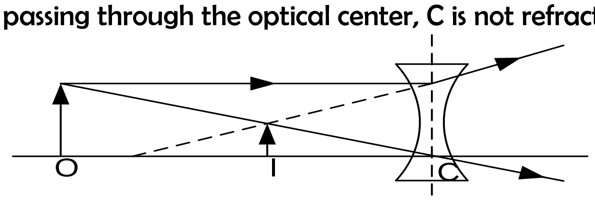
(i) They are used in spectacles for long-sighted people

(ii) They are used in cameras.
(iii) They are used in projectors.
(iv)They are used in microscopes.
(v) They are used in astronomical telescopes.

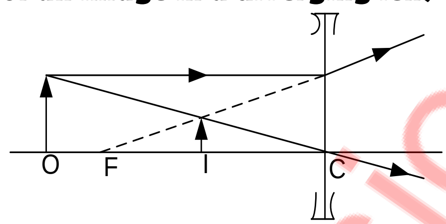
Ray diagrams for a diverging lens

a) Principal rays for a diverging lens

- A ray parallel to the principal axis is refracted to appear to come from the principal focus F
- A ray passing through the optical center, C is not refracted



Formation of an image in a diverging lens



Nature of the image

- Virtual
- Erect
- Diminished
- Between F and C

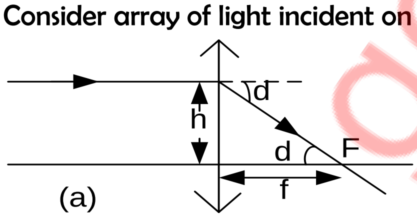
USES OF CONCAVE LENSES

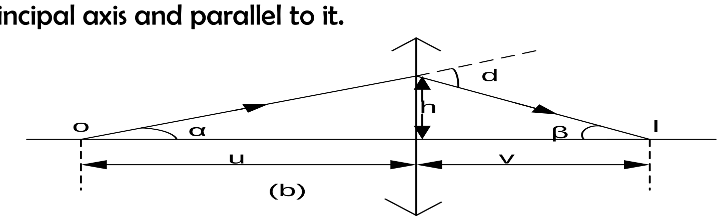
(i) They are used in spectacles for short-sighted people
(ii) They are used in Galilean telescopes.

Thin lens formula

a. Convex lens

Consider array of light incident on a lens close to its principal axis and parallel to it.





From figure (a) above deviation d is small and if it is measured in radians

$$d \approx \tan d = \frac{h}{f} \dots\dots\dots\dots (1)$$

From figure (b): $\alpha + \beta = d \dots\dots\dots\dots (2)$

For small angles α, β measured in radians

$$\alpha \approx \tan\alpha = \frac{h}{u} \dots\dots\dots\dots (3)$$

$$\beta \approx \tan\beta = \frac{h}{v} \dots\dots\dots\dots (4)$$

$$\frac{h}{u} + \frac{h}{v} = \frac{h}{f}$$

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

From which;

$$\boxed{f = \frac{u\,v}{u+v}}$$

Where
u =object distance
v = image distance
f = focal length

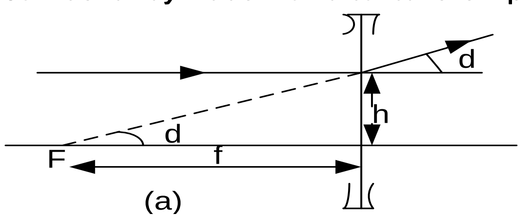
Note:
All distances are measured from the optical center of the lens.

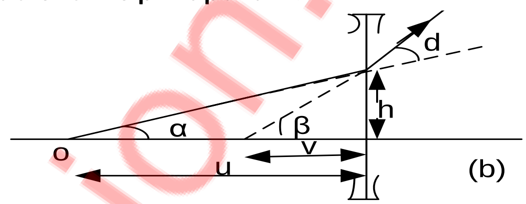
Sign convention

- Distances of real objects and real images are positive ie u and v for real objects and real images are positive.
- Distances of virtual objects and virtual images are negative ie u and v for virtual objects and virtual images are negative.
- Focal length f, for a convex lens is positive and negative for a concave lens.

Concave lens

Consider array incident on a concave lens parallel and close to the principal axis.





From figure (a) above deviation d is small and if it is measured in radians

$$d \approx tand = \frac{h}{f} \dots\dots\dots\dots (1)$$

From figure (b): $\alpha + d = \beta$

$$-d = (\alpha - \beta) \dots\dots\dots\dots (2)$$

For small angles α, β measured in radians

$$\alpha \approx tan\alpha = \frac{h}{u} \dots\dots\dots\dots (3)$$

$$\beta \approx tan\beta = \frac{h}{v} \dots\dots\dots\dots (4)$$

$$-\frac{h}{f} = \frac{h}{u} - \frac{h}{v}$$

$$-\frac{1}{f} = \frac{1}{u} - \frac{1}{v}$$

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

The power of a lenses

The power of a lens,P is the reciprocal of the focal length in metres.
The **S.I unit** of power of a lens is **Dioptres(D)**.
A diopter is the power of a lens of focal length one metre

$$power = \frac{1}{focal\ lenght(metres)}$$

$$\boxed{\boldsymbol{P = \frac{1}{f(m)}}}$$

Examples

(a) Power of a single lens

1. A converging lens has a focal lens 15cm. Calculate the power of the lens.

Solution

$\boldsymbol{f} = 15cm$=0.15m, P =?

$$\boldsymbol{P} = \frac{1}{f(m)}$$

$$\mathrm{P} = \frac{1}{0.15}$$

$$\mathrm{P} = 6.67\mathrm{D}$$

2. Find the power of a diverging lens of focal length 10cm

Solution

$f = -10cm = -0.1m$, $P = ?$

$P = \frac{1}{f(m)}$

$P = \frac{1}{-0.1}$

$P = -`10D$

(b) Power of combination of two lenses in contact

When two lenses are in contact, the power of the combination is obtained by adding the power of the two lenses

$$P = P_1 + P_2$$

1. Two converging lenses of focal length 10cm and 20cm are placed in contact. Find the power of the combination

Solution

For first lens

$f = 10cm = 0.1m$, $P_1 = ?$

$P_1 = \frac{1}{f(m)}$

$P_1 = \frac{1}{0.1}$

$P_1 = 10D$

For second lens

$f = 20cm = 0.2m$, $P_2 = ?$

$P_2 = \frac{1}{f(m)}$

$P_2 = \frac{1}{0.2}$

$P_2 = 5D$

Power of the combination

$P = P_1 + P_2$

$P = 10 + 5$

$P = 15D$

2. A converging lens of focal length 10cm is placed in contact with a diverging lens of focal length 25cm. Find the power of the combination

Solution

For first lens

$f = 10cm = 0.1m$, $P_1 = ?$

$P_1 = \frac{1}{f(m)}$

$P_1 = \frac{1}{0.1}$

$P_1 = 10D$

For second lens

$f = -25cm = -0.25m$, $P_2 = ?$

$P_2 = \frac{1}{f(m)}$

$P_2 = \frac{1}{-0.25}$

$P_2 = -4D$

Power of the combination

$P = P_1 + P_2$

$P = 10 - 4$

$P = 6D$

LINEAR OR LATERAL MAGNIFICATION

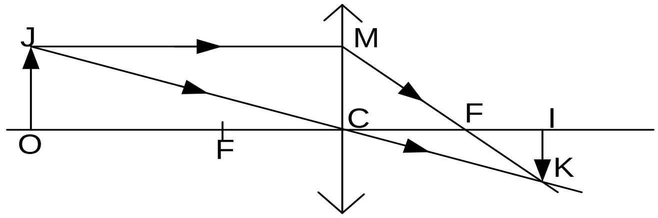
It is defined as the ratio of the image height to object height.

$$m = \frac{height\ image}{height\ object}$$

Magnification is also defined as the ratio of distance of the image from the lens to the distance of the object from the lens.

$$m = \frac{image\ distance\ (v)}{object\ distance\ (u)}$$

Consider formation of real image by a convex lens.



ΔOJC is similar to ΔIKP

$$\frac{IK}{OJ} = \frac{CI}{OC} = \frac{v}{u} = \frac{image\ distance\ (v)}{object\ distance\ (u)}$$

Examples

1. An object is placed 20cm from a converging lens of focal length 15cm. Find the nature, position and the magnification of the image formed

Solution

u = +20cm, f =+15cm, v =?

$i)\ \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{15} = \frac{1}{20} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{15} - \frac{1}{20}$

$\frac{1}{v} = \frac{1}{60}$

v = 60cm

iii) $M = \frac{v}{u}$

$M = \frac{60}{20}$

$m = 3$

- Real image(since v = positive)
- Magnified image (since m > 1)

2. A four times magnified virtual image is formed of an object placed 12cm from a converging lens. Calculate;
   (i) The position of the image and
   (ii) The focal length of the lens

Solution

(i) M = 4, u = 12cm, v =?

$M = \frac{v}{u}$

$4 = \frac{v}{12}$

$v = 48cm$

(ii) u =12cm, v = -48cm(virtual image), f=?

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{12} + \frac{1}{-48}$

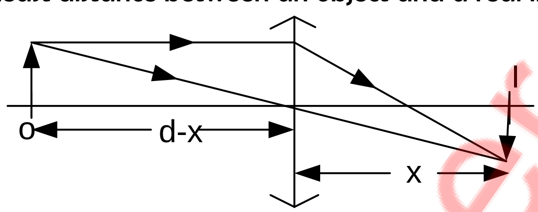
$\frac{1}{f} = \frac{1}{12} - \frac{1}{48}$

$\frac{1}{f} = \frac{3}{48}$

f = 16cm

Least distance between image distance and object distance in a convex lens

Least distance between an object and a real image formed by a convex lens.



Suppose the object and image are a distance d apart with the image a distance x beyond the lens

$u = d - x,\ \ v = x$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{d-x} + \frac{1}{x}$

$x^2 - dx + df = 0$

$x = \frac{-(-d) \pm \sqrt{(-d)^2 - 4df}}{2}$

For real image $d^2 - 4fd \geq 0$

$d \geq 4f$

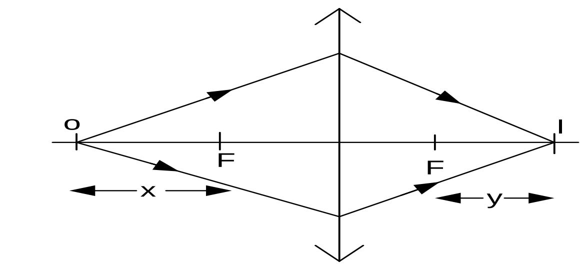
Hence $d = 4f$

The least distance between an object and the real image of the object formed by the convex lens is 4f.

Conjugate points

These are points on the principle axis such that when the object is placed at one , the image is formed at the other.

Suppose a convex lens forms an image of an object O at I ,if the object was placed at I, the lens would form the image of the object at O, then O and I are called conjugate points



$u = f + x, \quad v = y + f$

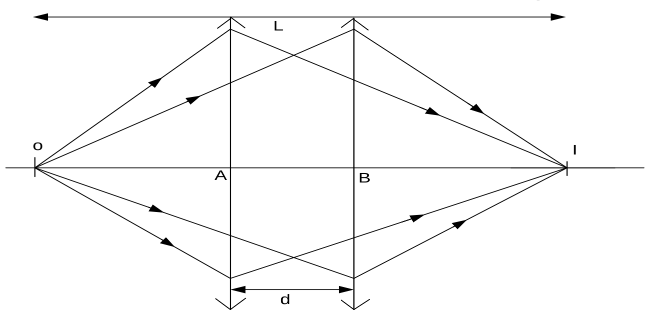
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{f+x} + \frac{1}{y+f}$$

$\boxed{f^2 = xy}$ (Newton's equation)

Displacement of a lens when an object and screen are fixed.

The lens is moved in position A when the image of O is produced at I, it's a gain displaced through a distance d until the image of O is a gain produced at I.



O and I are conjugate points with respect to the lens.

$OB = AI$ and $OA = BI$

When lens is in position A: $OA + AB + BI = L$

$$u + d + u = L$$

$$u = \frac{l-d}{2}$$

Also: $AI = AB + BI$

$$v = d + u$$

$$v = d + \frac{l-d}{2}$$

$$v = \frac{l+d}{2}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

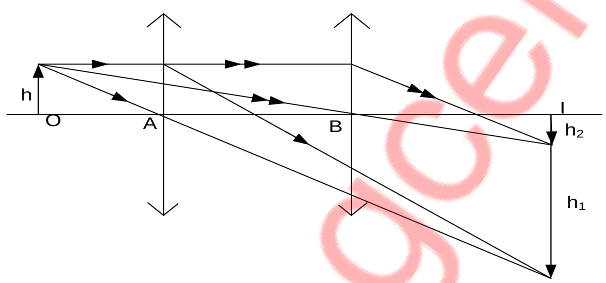
$$\frac{1}{f} = \frac{1}{\left(\frac{l-d}{2}\right)} + \frac{1}{\left(\frac{l+d}{2}\right)}$$

$$\frac{1}{f} = \frac{2}{l-d} + \frac{2}{l+d}$$

$$4lf = l^2 - d^2$$

$$\boxed{f = \frac{l^2 - d^2}{4l}}$$

Displacement of a lens when an erect object and screen are fixed



O and I are conjugate points with respect to the lens.

$OB = AI$ and $OA = BI$

When lens is in position A: $m_A = \frac{AI}{OA} = \frac{h_1}{h}$.........(i)

When lens is in position A: $m_B = \frac{BI}{OB} = \frac{h_2}{h}$.........(ii)

But $AI = 0B$

$$\frac{h_1}{h} OA = \frac{h}{h_2} BI$$

$$h^2 = h_1 h_2$$

$$\boxed{h = \sqrt{h_1 h_2}}$$

This method of measuring **h** is most useful when the object is inaccessible for example

(i) When the width of the slit inside the tube is required.

(ii) When the focal length of a thick lens is required.

CONDITION FOR THE FORMATION OF A REAL IMAGE BY A CONVEX LENS

(i) The object distance must be greater than the focal length of the lens.

(ii) The distance between the object and the screen must be at least four times the focal length of the lens.

EXAMPLES:

1. A real image in a converging lens of focal length 15cm is twice as long as the object. Find the image distance from the lens

Solution

$m = \frac{v}{f} - 1$

$2 = \frac{v}{15} - 1$

$v = 45cm$

2. A convex lens of focal length 15cm forms an image three times the height of its object. Find the possible object and corresponding image positions. State the nature of each image.

Solution

Let the object distance $= u$

$\Rightarrow$ The possible image distances $= +3u$ **or** $-3u$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Case I where the image distance $= +3u$

$$\frac{1}{15} = \frac{1}{u} + \frac{1}{3u}$$

$\therefore$ The object distance $u = 20cm$

The corresponding image distance:

$3u = 3 \times 20cm = 60cm$

The image is real, magnified, inverted and behind the lens

Case II where the image distance $= -3u$

$$\frac{1}{15} = \frac{1}{u} - \frac{1}{3u}$$

$\therefore$ The object distance $u = 10cm$

The corresponding image distance :

$-3u = -3 \times 10cm = -30cm$

The image is virtual, magnified, erect and same side with the object

3. The magnification of an object in a converging lens is **m**. when the lens is moved a distance **d** towards the object, the magnification becomes m^1 show that the focal length **f** of the lens is given by $f = \frac{dmm^1}{m^1 - m}$.

Solution

Let u be the object distance before displacement

$$\frac{1}{m} = \frac{u}{f} - 1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Let $u - d$ be the object distance before displacement

$$\frac{1}{m^1} = \frac{u-d}{f} - 1 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

(1) −(2)

$$\frac{1}{m} - \frac{1}{m^1} = \frac{d}{f}$$

$$f = \frac{dmm^1}{m^1 - m}$$

4. Two thin convex lenses **A** and **B** of focal lengths **5cm** and **15cm** respectively are placed coaxially **20cm** apart. If an object is placed **6cm** from **A** on the side remote from **B**,

(i) Find the position, nature and magnification of the final image.

(ii) Sketch a ray diagram to show the formation of the final image.

Solution

Consider the action of a convex lens A

$u = 6cm$ and $f = 5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{5} - \frac{1}{6}$$

$v = 30cm$

Consider the action of a convex lens B

The image formed by lens **A** acts as a virtual object for lens **B**.

Thus $u = -(30 - 20)cm = -10cm$ and $f = 15cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{-10}$$

$$v = 6cm$$

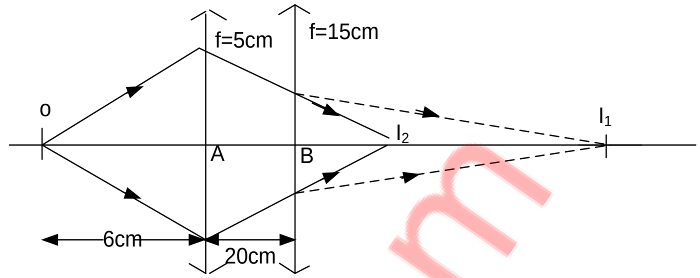
$\Rightarrow$ The image is real and is **6cm** from **B**

$$m = m_1 x m_2$$

$$m = \frac{v_1}{u_1} x \frac{v_2}{u_2}$$

$$m = \frac{30}{6} x \frac{6}{10}$$

$$m = 3$$



5. A thin converging lens **P** of focal length **10cm** and a thin diverging lens **Q** of focal length **15cm** are placed coaxially **50cm** apart. If an object is placed **12cm** from **P** on the side remote from **Q.**
   (i) Find the position, nature and magnification of the final image.
   (ii) Sketch a ray diagram to show the formation of the final image.

Solution

Consider the action of a converging lens P.

$u = 12cm$ and $f = 10cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{12}$$

$$v = 60cm$$

Consider the action of a diverging lens Q.

The image formed by lens **P** acts as a virtual object for lens **Q.**

Thus $u = -(60 - 50) = -10cm$ and $f = -15cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{-15} - \frac{1}{-10}$$

$$v = 30cm$$

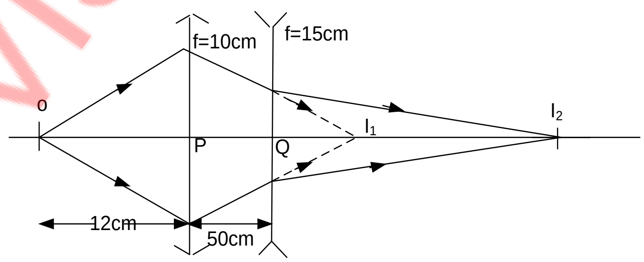
$\Rightarrow$ The image is real and is **30cm** from **Q.**

$$m = m_1 x m_2$$

$$m = \frac{v_1}{u_1} x \frac{v_2}{u_2}$$

$$m = \frac{60}{12} x \frac{30}{10}$$

$$m = 15$$

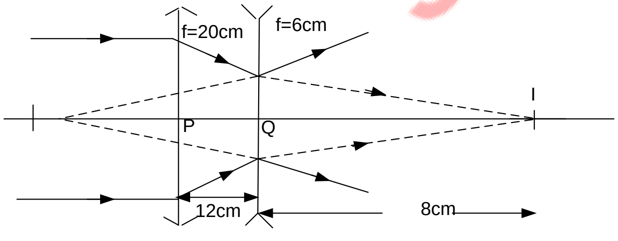


6. Light from a distant object is incident on a converging lens of focal length **20cm** placed **12cm** in front of a diverging lens of focal length **6cm.** Determine the position and nature of the final image.

Solution

Consider the action of a converging lens

The image of a distant object is formed at the principal focus of the converging lens



Consider the action of a diverging lens

The image formed by a converging lens acts as a virtual object for a diverging lens

$\Rightarrow u = -(20 - 12) = -8cm,\ f = -6cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

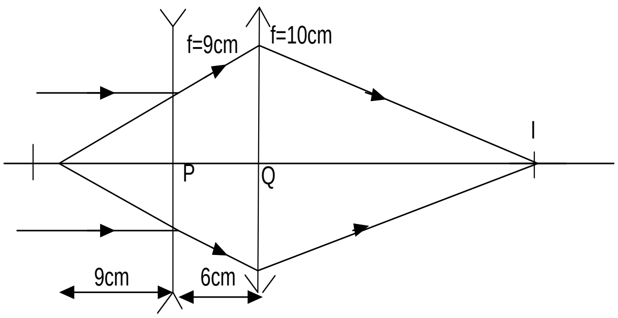
$$\frac{1}{v} = \frac{1}{-6} - \frac{1}{-8}$$

$$v = -24cm$$

$\Rightarrow$ The image is virtual and is 24cm behind the diverging lens.

7. Light from a distant object is incident on a diverging lens of focal length **9cm** placed **6cm** in front of a converging lens of focal length **10cm.** Determine the position and nature of the final image.

Solution



Consider the action of a diverging lens

The image of a distant object is formed at the principal focus of the diverging lens

Consider the action of a converging lens

The image formed by a diverging lens acts as a real object for a converging lens

$\Rightarrow \quad u = (9 + 6)cm = 15cm$ and

$f = 10cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

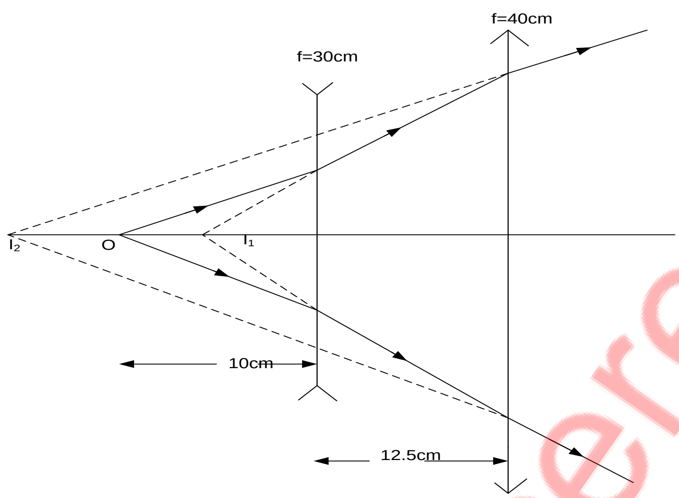
$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15}$$

$$v = 30cm$$

$\Rightarrow$ The image is real and is 30cm from the converging lens.

8. A thin diverging lens of focal length **30cm** and a thin converging lens of focal length **40cm** are placed coaxially **12·5cm** apart. If an object is placed **10cm** from a diverging lens on the side remote from a converging lens**.** Find the position, nature and magnification of the final image.

Solution



Consider the action of a diverging lens

$u = 10cm$, and $f = -30cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{-30} - \frac{1}{10}$$

$$v = 7.5cm$$

Consider the action of a converging lens

The image formed by a diverging lens acts as a real object for a converging lens

$\Rightarrow \quad u = (7.5 + 12.5)cm = 20cm$ and

$f = 40cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{20}$$

$$v = -40cm$$

$\Rightarrow$ The image is virtual and is 40cm from a converging lens.

$$m = m_1 x m_2$$

$$m = \frac{v_1}{u_1} x \frac{v_2}{u_2}$$

$$m = \frac{7.5}{10} x \frac{40}{20}$$

$$m = 1.5$$

9. A luminous object and the screen are placed on a n optical bench and a converging lens is placed between them to show a sharp image of the object on the screen. The linear magnification of the image is found to be 2.5. The lens is now moved 30cm near the screen and a sharp image is again formed on the screen. Calculate the focal length of the lens

Solution

$$m = \frac{v}{u}$$

$$2.5 = \frac{30 + x}{x}$$

$$x = 20cm$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{50}$$

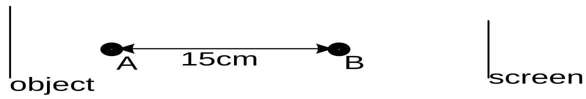
$$f = 14.3cm$$

Or

$$f = \frac{l^2 - d^2}{4l}$$

$$f = \frac{70^2 - 30^2}{4x70} = 14.3cm$$

10.



In the diagram above the image of the object is formed on the screen when a convex lens is placed either at A or B. If A and B are 15cm apart. Find the;

(i) Focal length of the lens

(ii) Magnification of the image when the lens is at B

Solution

$$f = \frac{l^2 - d^2}{4l}$$

$$f = \frac{45^2 - 15^2}{4x45}$$

$$f = 15cm$$

The magnification $\mathbf{m_2}$ produced by the lens in position **B**

$$m_2 = \frac{l - d}{l + d}$$

$$m_2 = \frac{45 - 15}{45 + 15}$$

$$m_2 = 0.5$$

EXERCISE:11

1. (i) Define the terms centres of curvature, radii of curvature, principal focus and focal length of a converging lens.

(ii) What are **Sign conventions?**

2 (a) An object is placed a distance **u** from a convex lens. The lens forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:

(i) **real**

(ii) **virtual**

(b) Draw a ray diagram to show the formation of an image by a diverging lens.

(c) Draw a ray diagram to show the formation of a real image of a virtual point object by a diverging lens

3. Give two instances in each case where concave lenses and convex lenses are useful.

4. Derive an expression for the focal length **f**, of a convex lens in terms of the object distance **u** and the image distance **v.**

5. Define the term **power of a lens.**

6. (i) Define the term **linear magnification.**

(ii) Show that the linear magnification produced by a convex lens is equal to the ratio of the image distance to the object distance.

(iii) A convex lens of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.

(iv) A convex lens of focal length 10cm forms an image five times the height of itsobject. Find the possible object and corresponding image positions.

[**Ans: (iii) u = 10cm, v = −30cm (iv) u = 12cm, v = 60cm OR u = 8cm, v =−40cm**]

7. A convex lens forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 20cm, determine the

(i) focal length of the lens

(ii) shift of the object

[**Answers: (i) f = 10cm (ii) 3cm**]

8. A thin converging lens **P** of focal length 2**0cm** and a thin diverging lens **Q** of focal length **30cm** are placed coaxially **0cm** apart. If an object 3cm tall is placed **70cm** from **Q** on the side remote from **Q.**

(i) Find the position final image.

(ii) The height of the final image. **An(60cm, 4.5cm)**

9. A thin converging lens **A** of focal length **6cm** and a thin diverging lens **B** of focal length **15cm** are placed coaxially **14cm** apart. If an object is placed **8cm** from **A** on the side remote from **B,**

(i) Find the position, nature and magnification of the final image.

(ii) Sketch a ray diagram to show the formation of the final image.

[**Answers: (i) 30cm from lens B, real image and magnification = 9**]

10. An object is placed **24cm** in front of a convex lens **P** of focal length **6cm.** When a concave lens **Q** of focal length **12cm** is placed beyond lens **P,** the screen has to be **10cm** away from lens **P** so as to locate the real image formed**.**

(i) Find the distance between the two lenses **P** and **Q.**

(ii) Sketch a ray diagram to show the formation of the final image**.**

[**Answer: (i) 4cm**]

11. A lens L_1 casts a real image of a distant object on a screen placed at a distance **15cm** away**.** When another lens L_2 is placed **5cm** beyond lens L_1, the screen has to be shifted by **10cm** further away to locate the real image formed**.** Find the focal length and the type of lens L_2**.**

[**Answer: (i) f = −20 cm and therefore the lens is concave**]

12. A thin convex lens is placed between an object and a screen that are kept fixed **64cm** distant apart**.** When the position of the lens is adjusted, a clear focused image is obtained on the screen for two lens positions that are **16cm** distant apart**.**

(i) Draw a ray diagram to show the formation of the images in the two lens positions**.**

(ii) Find the focal length of the lens

(iii) Find the magnification produced in each lens position**.**

[**Answers: (ii) f = 15cm (iii)** $m_1 = 1.67$ **,** $m_2 = 0.6$]

13. (i) State two conditions necessary for a biconvex lens to form a real image of an object.

(ii) Show that the minimum distance between an object and a screen for a real image to be formed on it is **4f,** where **f** is focal length of a convex lens**.** Hence show also that the object and its image are then equidistant from the lens.

14. (i) Explain with the aid of a convex lens the term **conjugate foci.**

(ii) What is meant by **reversibility of light** as applied to formation of a real image by a convex lens?

15. A converging lens of focal length **f** is placed between an object and a screen. The position of the screen is adjusted until a clear magnified image is obtained on the screen. Keeping the screen fixed in this position at a distance **L** from the object, the lens is displaced through a distance **d** to obtain a clear diminished image on the screen.

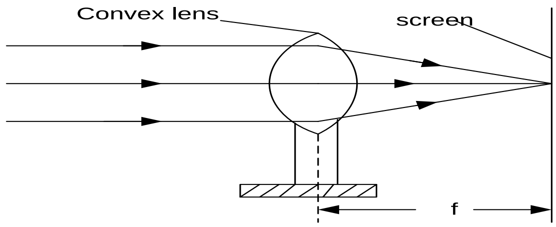
(i) Draw a ray diagram to show the formation of the images in the two cases.

(ii) Show that $l^2 - d^2 = 4lf$

(iii) Find the product of the magnifications produced in the two cases.

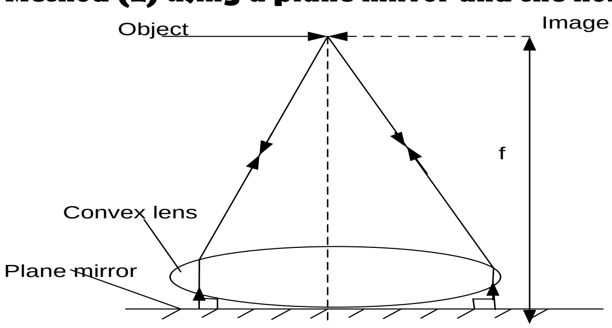
Determining focal length of the converging lens(convex lens)

Method (1) using a distant object



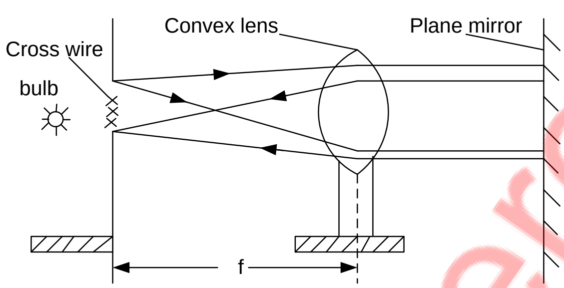
- A distance object such as a window of a tree is focused on to the screen using a convex lens whose focal length is to be determined.
- The distance of the screen from the lens is then the focal length **f** of the lens, which can thus be measured

Method (2) using a plane mirror and the non parallax method.



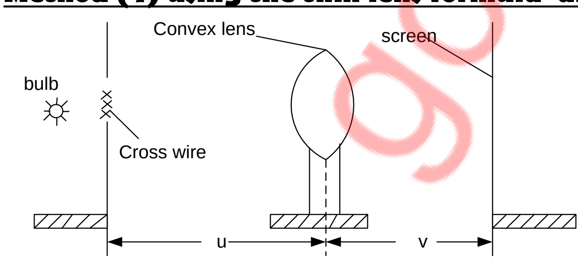
- A plane mirror M is placed on a table and the a biconvex lens is placed on the mirror.
- A pin is clamped horizontally on a retort stand with the apex along the axis of the lens.
- Move the pin up or down to locate the position where the pin coincides with its image using the method of no parallax.
- The distance from the pin O to the lens is measured and this is the focal length ,f, of the lens.

Method (3) using a plane mirror and an illuminated object



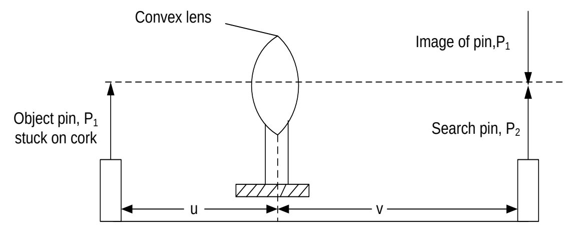
- A lens mounted in a holder is placed between a screen with cross wires and a plane mirror as shown above.
- The lens is moved in between, until a sharp image of the cross-wire is formed on the screen besides the object.
- The distance **f** of the lens from the screen is then the focal length of the lens, which can thus be measured.

Method (4) using the thin lens formula and an illuminated object.



- An illuminated object **O** is placed at a distance **u** in front of a mounted convex lens.
- The position of the screen is adjusted until a sharp image of **O** is formed on the screen at a distance **v** from the lens.
- The procedure is repeated for several values of u and the results are tabulated including values of uv and $u + v$.
- A graph of uv against $u + v$ is plotted and the slope **s** of such a graph is equal to the focal length **f** of the lens.

Method (5) using the thin lens formula and the method of no-parallax.



❖ An object pin $\mathbf{P_1}$ is placed at a distance **u** in front of a mounted convex lens so that its tip lies along the axis of the mirror.

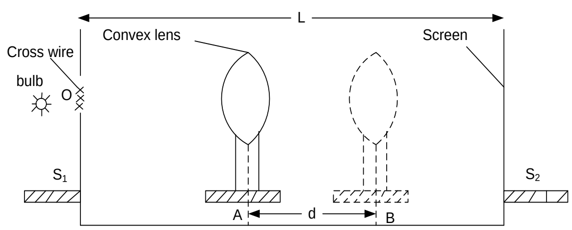
❖ A search pin $\mathbf{P_2}$ placed behind the lens is adjusted until it coincides with the image of pin $\mathbf{p_1}$ by no-parallax method.

❖ The distance **v** of pin $\mathbf{p_2}$ from the lens is measured.

❖ The procedure is repeated for several values of **u** and the results are tabulated including values of uv, and $u + v$.

❖ A graph of uv against $u + v$ is plotted and the slope **s** of such a graph is equal to the focal length **f** of the lens.

Method (6) using displacement method



❖ An illuminated object **O** is placed behind the hole in the screen S_1

❖ The convex lens is placed behind S_2 in a such position **A** so that a <u>sharp magnified image</u> is formed on the screen S_2.

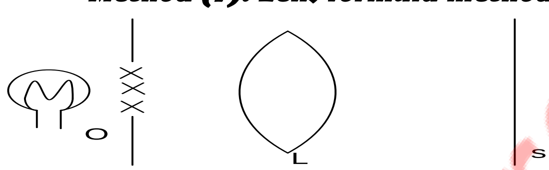
❖ Keeping the screen and the object fixed, the lens is then moved to position B suc that <u>sharp diminished image</u> is formed on the screen S_2.

❖ The distance d between A and B is measured

❖ The distance L between S_1 and S_2 is measured

❖ The focal length **f** of the lens can then be calculated from $f = \frac{l^2 - d^2}{4l}$

Method (7): Lens formula method



❖ The apparatus is arranged as above

❖ The wire gauze is illuminated with a bulb and the position of the lens L is adjusted until the <u>sharp image</u> of the wire gauze is formed on the object screen O

❖ The <u>distance OL</u> is measured and recorded as u

❖ The <u>distance LS</u> is measured and recorded as v

❖ The procedure is repeated for various values of u and v and tabulated including values of $\frac{1}{u}$ and $\frac{1}{v}$

❖ A graph of $\frac{1}{u}$ and $\frac{1}{v}$ is plotted

❖ The intercepts A and B are read from the graph and focal length, f is obtained from the equation $f = \frac{2}{A+B}$

NOTE:

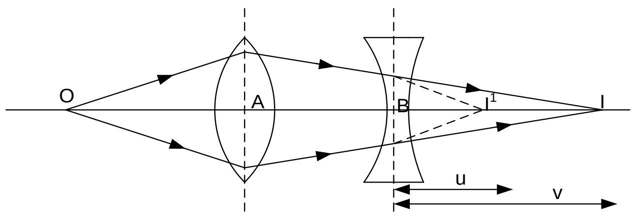
Since no measurements need be made to the surfaces of the lens in Method (5), then it is most useful when finding the focal length of:

(i) a thick lens

(ii) an inaccessible lens, such as that fixed inside an eye-piece or telescope tube.

MEASUREMENT OF FOCAL LENGTH OF A DIVERGING LENS

Method (1) using a converging lens



- An object **O** is placed in front of a converging lens of known focal length so as to obtain a real image on the screen at I[1]
- Measure the distance AI[1]

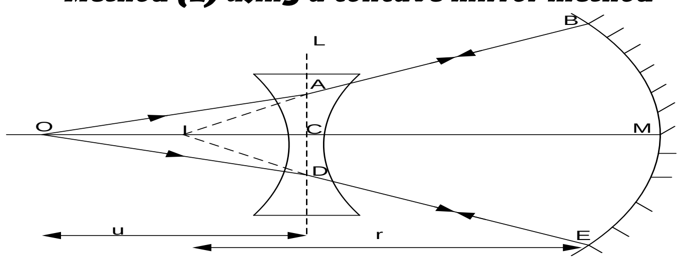
- A diverging lens whose focal length is required is placed between the screen and the converging lens.
- The screen is moved to obtain a new real image **I** on to it
- Measure the distance AB and BI
- The object distance u, from this lens is got from $u = -(AI^1 - AB)$.
- The focal length is calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where $v = BI$

NOTE:

When a diverging lens is placed in between **L_1** and **L_2** the image **I_1** formed by a converging lens acts as a virtual object for the diverging lens hence giving the final real image **I.**

A virtual object A collection of points which may be regarded as a source of light rays for a portion of an optical system but which does not actually have this function.

Method (2) using a concave mirror method



- An illuminate object O is placed <u>infront</u> of a diverging lens, L **arranged coaxially** with a concave mirror, M of known foval legth, f.

- The postion of object **O** is adjusted until it <u>coincides with its image.</u>
- Measure and record distances **OC** and **CM**
- focal length, f, can be calculated from

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Where $u = OC$ and $v = -(r - CM)$ **but** $r = 2f$

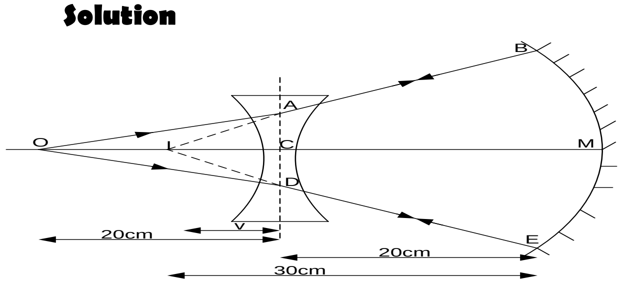
NOTE:

As the object and its image are coincident at **O,** the rays must be incident normally on the mirror and there fore retrace their own path through the centre of curvature of the mirror at **I** and this is the position of the virtual image.

Examples

1. An object is placed 20cm in front of a diverging lens that is coaxially with a concave mirror of focal length 15cm. when concave mirror is 20cm from the lens, the final image coincides with the object
   (i) draw a ray diagram to show how image is formed
   (ii) determine the focal length of the diverging lens

Solution



Consider the action of a diverging lens

$u = 20cm$ and $v = -(30 - 20)cm$

$v = -10cm$ 'virtual image'.

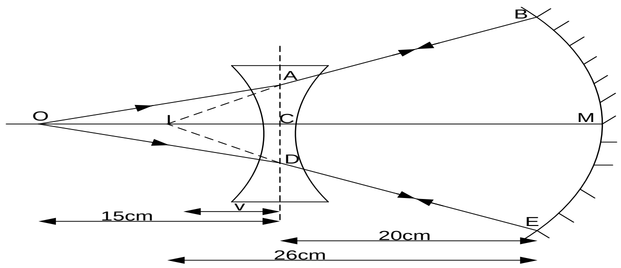
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{-10}$$

$$f = -20cm$$

2. An object is placed **15cm** in front of a diverging lens placed coaxially with a concave mirror of focal length **13cm.** When the concave mirror is **20cm** from the lens the final image coincides with the object.
   (i) draw a ray diagram to show how final image is formed
   (ii) determine the focal length of the diverging lens

Solution



Consider the action of a diverging lens

$u = 15cm$ and $v = -(26 - 20)cm$

$v = -6cm$ 'virtual image'.

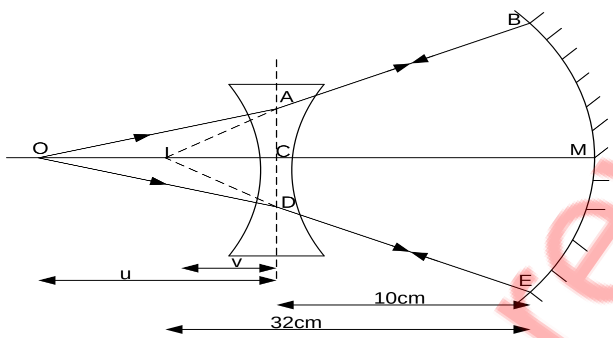
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{15} + \frac{1}{-6}$$

$$f = -10cm$$

3. A concave lens of focal length 20cm is placed 10cm in front of concave mirror of focal length 16cm. calculate the distance from the lens at which an object will coincide with its image

Solution



Consider the action of a diverging lens

$f = -20cm$ and $v = -(32 - 20)cm$

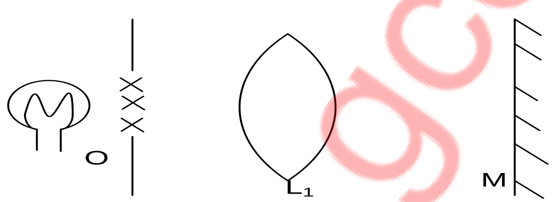
$v = -22cm$ 'virtual image'

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-20} = \frac{1}{u} + \frac{1}{-22}$$

$$v = -220cm$$

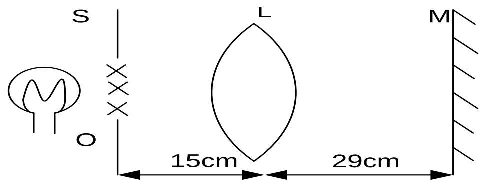
Method (3) Using a plane mirror, converging lens and an illuminated object



- The apparatus is arranged as above
- The wire gauze is illuminated with a bulb and the position of the lens L_1 is adjusted until a sharp image of the wire gauze is formed on the object screen O
- The distance OL_1 is measured and recorded as f_1
- The test lens L_2 is now cemmented on L_1 and again placed between O and M. The position of the combined lens is adjusted until asharp image of the wire gauze is formed at O
- The distance OL_2 is measured and recorded as f
- Focal length of the test lens is then calculated from $\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$

Examples

A convex lens **L**, a plane mirror **M** and a screen **S** are arranged as shown below so that a sharp image of an illuminated object **O** is formed on the screen **S**.

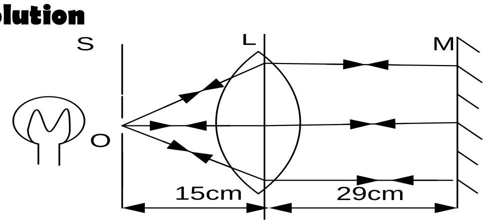


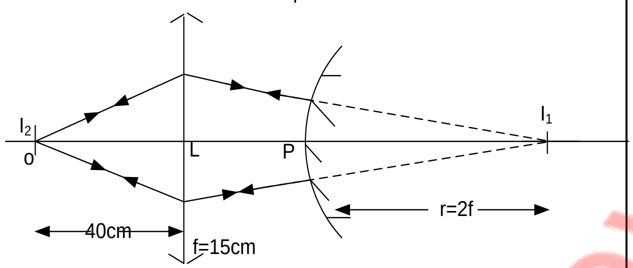
When the plane mirror is replaced by a convex mirror, the lens has to be moved **25cm** towards the mirror so as to obtain a sharp focused image on the screen.

(i) Illustrate the two situations by sketch ray diagrams.

(ii) Calculate the focal length of the convex mirror.

Solution





Consider the action of a convex lens

$u = 40cm$, and $f = 15cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{15} = \frac{1}{40} + \frac{1}{v}$$

$$v = 24cm$$

The radius of curvature $\boldsymbol{r} = (\mathbf{24} - \mathbf{4})\boldsymbol{cm}$

$$\boldsymbol{r} = \mathbf{20}\boldsymbol{cm}$$

Using the relation $\boldsymbol{r} = \mathbf{2}\boldsymbol{f}$

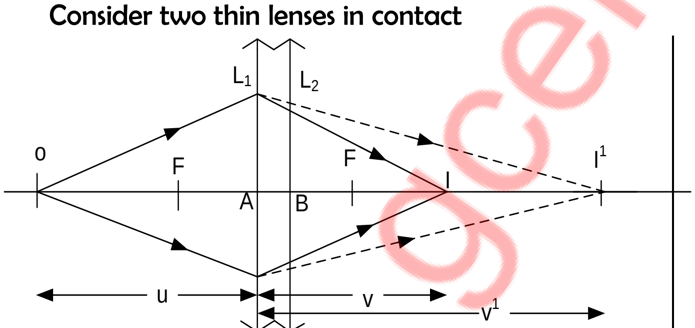
$\Rightarrow \quad 2f = 20cm$

$\therefore \quad f = 10cm$

Thus $\boldsymbol{f} = -\mathbf{10}\boldsymbol{cm}$ "The centre of curvature of a convex mirror is virtual"

Combined focal length of two thin lenses in contact

Consider two thin lenses in contact



With lens L_1 of focal length f_1

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v^1} \ldots\ldots\ldots\ldots\ldots (1)$$

The image I^1 forms the virtual object for lens L_1 of focal length f_2

$$\frac{1}{f_2} = -\frac{1}{v^1} + \frac{1}{v} \ldots\ldots\ldots\ldots\ldots (2)$$

equation (1) + (2)

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v^1} + \left(-\frac{1}{v^1} + \frac{1}{v}\right)$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$$

But $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where f is the focal length of the combined lens

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

NOTE:

(i) This formula for f applies for any two lenses in contact such as two diverging lenses or a converging and diverging lens.

(ii) When the formula is used the signs of the focal length must be considered as illustrated below.

Example

1. Suppose a converging lens of focal length 8cm is placed in contact with a diverging lens of focal length 12cm

Solution

$f_1 = +8cm$ and $f_2 = -12cm$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{8} + \frac{1}{-12}$$

$$f_2 = 24cm$$

The positive sign shows that the combination acts as a convex lens.

2. Suppose a thin converging lens of focal length 6cm is placed in contact with a diverging lens of focal length 10cm. what is the combined focal length.

Solution

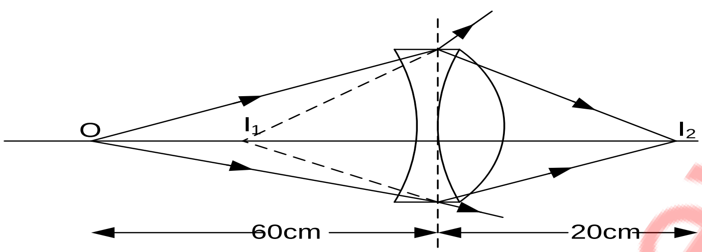
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{-10} + \frac{1}{6}$$

$$f_2 = 15cm$$

3. A small object is placed at a distance of 60cm to the left of a diverging lens of focal length 30cm.A converging lens is then placed in contact with the diverging lens. If a real image is formed at a distance of 20 cm to the right of the combined lenses; find the focal length of the converging lens.

Solution



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

For diverging lens: $u = 60cm, f = -30cm$

$$\frac{1}{-30} = \frac{1}{60} + \frac{1}{v}$$

$v = -20cm$ (virtual image)

For converging lens: virtual image becomes the real object of the converging lens

$u = 20cm, v = 20cm$

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{20}$$

$$f = 10cm$$

Or For combination of lenses:

$u = 60cm, v = 20cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{60} + \frac{1}{20}$$

$$f = 15cm$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

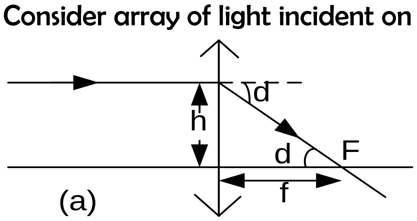
$$\frac{1}{15} = \frac{1}{-30} + \frac{1}{f_2}$$

$$f_2 = 10cm$$

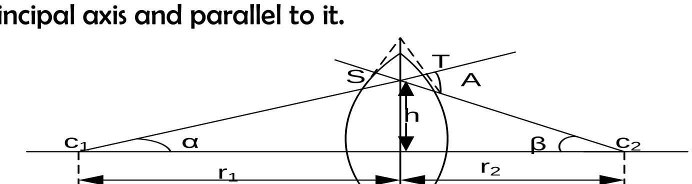
Full thin lens formula

Consider the parallel rays RS incident on a convex lens

Consider array of light incident on a lens close to its principal axis and parallel to it.



(a)



(b)

From figure (a) above deviation d is small and if it is measured in radians

$$d \approx \tan d = \frac{h}{f} \ldots \ldots \ldots \ldots (1)$$

for small angle prism d $= (n-1)\,A$

$$\frac{h}{f} = (n-1)\,A \ldots \ldots \ldots \ldots (2)$$

For figure (b), the normal at Sand T pass through the centre of curvature c_1 and c_2

$\alpha + \beta = A$ (using exterior < properties)

for α and β being small angles in radians,

$$\alpha \approx \tan\alpha = \frac{h}{r_1}$$

$$\beta \approx \tan\beta = \frac{h}{r_2}$$

Hence $\frac{h}{r_1} + \frac{h}{r_2} = A \ldots \ldots \ldots \ldots \ldots \ldots . . (3)$

Substitute equation (2) into (3)

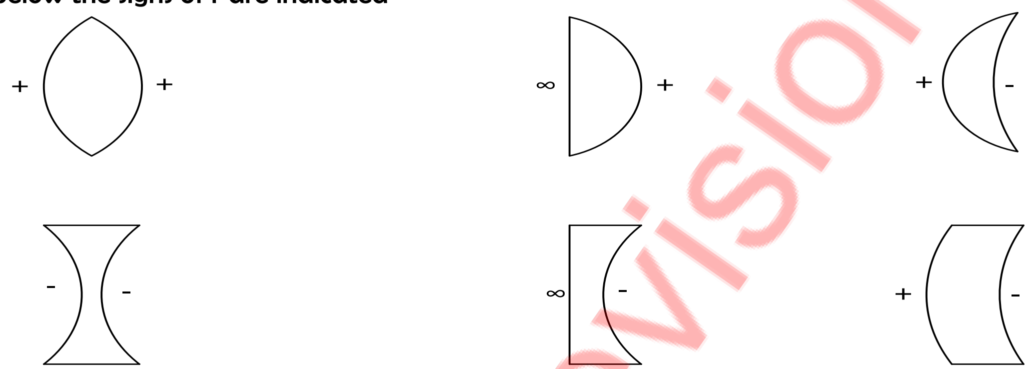
$$\frac{h}{r_1} + \frac{h}{r_2} = \frac{h}{f(n-1)}$$

$$\frac{1}{f(n-1)} = \frac{1}{r_1} + \frac{1}{r_2}$$

Hence $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$

Sign convention for radius of curvature

If the surface of the lens is convex, then the corresponding radii of curvature is positive, however if the surface is concave then its corresponding radius of curvature is negative. Thus for the type of lenses shown below the signs of r are indicated



NOTE:

In numerical work for a convex meniscus, the radius of curvature with the largest magnitude takes up the negative sign.

EXAMPLE:

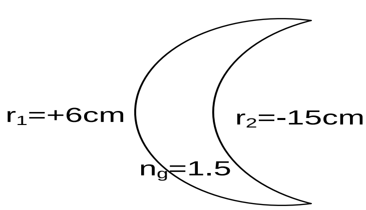
1. A converging meniscus with radii of curvature **15cm** and **6cm** is made of glass of refractive index **1.5.** Calculate its focal length when surrounded by:

(i) Air

(ii) a liquid of refractive index **1.2**

Solution:

(i)



$r_1 = 6cm, r_2 = -15cm$ and $n = 1.5$

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{6} + \frac{1}{-15}\right)$$

$$\boldsymbol{f = 20cm}$$

(ii)

$$\frac{1}{f} = (\frac{n_g}{n_l} - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (\frac{1.5}{1.2} - 1)\left(\frac{1}{6} + \frac{1}{-15}\right)$$

$$\boldsymbol{f = 40cm}$$

2. A thin lens with faces of radii of curvature 30cm is to be made from the glass with refractive index 1.6. what will be the focal length of the lens if it is;

(i) Biconvex

(ii) biconcave

Solution

(i)

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{30} + \frac{1}{30}\right)$$

$$\boldsymbol{f = 25cm}$$

(ii)

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{-30} + \frac{1}{-30}\right)$$

$$\boldsymbol{f = -25cm}$$

3. Calculate the focal length of converging meniscus with radii 25cmand 20cm whose refractive index is 1.5

Solution

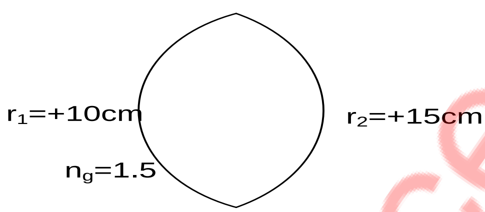
$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

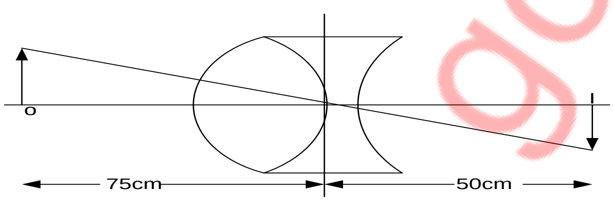
$$\frac{1}{f} = (1.5-1)\left(\frac{1}{25} + \frac{1}{-20}\right)$$

$$\boldsymbol{f = -200cm}$$

4. A thin concave lens is placed in contact with a convex lens made of glass of refractive index **1·5** and its surfaces have radii of curvature **10cm** and **15cm**. If an object placed **75cm** in front of the lens combination gives rise to an image on a screen at a distance **50cm** from the combination, calculate the focal length of the concave lens.

Solution





For length combination $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{f} = \frac{1}{75} + \frac{1}{50}$$

$$f = 30cm$$

For the convex lens

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{10} + \frac{1}{15}\right)$$

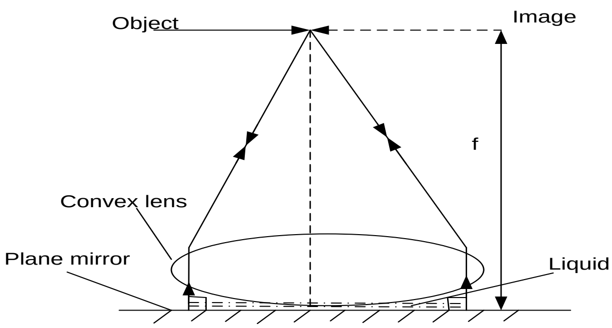
$$\boldsymbol{f = 12cm}$$

But $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\frac{1}{30} = \frac{1}{12} + \frac{1}{f_2}$$

$$f_2 = -20cm$$

Determining refractive index of a liquid using a convex lens and a plane mirror



- ❖ The focal length of the lens is first determined by placing it directly on a plane mirror.
- ❖ Clamp a pin horizontally on a retort stand with its apex along the axis of the lens.
- ❖ Move the pin up or down to locate the position where the pin coincides with its image using the method of the no parallax.
- ❖ Measure the distance of f_1 of the pin from the lens.
- ❖ Remove the lens and place a little quantity of the specimen liquid on the plane mirror.
- ❖ Replace the convex lens and then locate the new position where the pin coincides with the image.
- ❖ Measure the distance f_2 of the pin from the lens. If f_l is the focal length of the liquid then $\frac{1}{f_2} = \frac{1}{f_1} + \frac{1}{f_l}$ and $\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{-r}\right)$
- ❖ The refractive index of the liquid is got from $n_l = 1 - \frac{r}{f_2}$
  where r is the radius of curvature of the convex lens

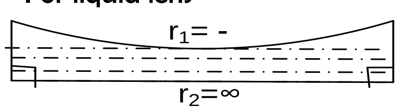
NOTE:
It can be seen from the experiment that the liquid lens is Plano concave type with its lower surface corresponding to the plane surface and the upper surface to the convex lens. There fore $r_1 = -ve$ is the radius of curvature of the upper surface and $r_2 = \infty$ is the radius of curvature of the lower surface.

Example

1. A converging lens is placed on top of a liquid of refractive index 1.4 and a glass slide. Using pin O, position is found where O coincides with it images. If both surfaces of the lens have radii of curvature of 15cm and refractive index of the lens 1.5. Determine the position of coincidence

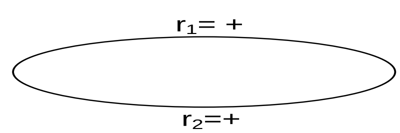
Solution

For liquid lens



$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{-15} + \frac{1}{\infty}\right)$$
$$\frac{1}{f_l} = -\frac{0.4}{15}$$

Glass lens



$$\frac{1}{f_g} = (1.5 - 1)\left(\frac{1}{15} + \frac{1}{15}\right)$$
$$\frac{1}{f_g} = \frac{1}{15}$$

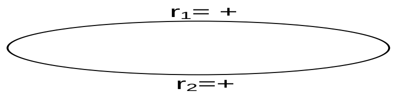
For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

$$\frac{1}{f} = -\frac{0.4}{15} + \frac{1}{15}$$
$$f = 25cm$$

2. A thin equiconvex lens of refractive index 1.5 whose surfaces have radius of curvature 24cm is placed on a horizontal plane mirror. When the space between the mirror and lens is filled with a liquid, a pin held 40cm vertically above the mirror is found to coincide with its own image. What is the refractive index of the liquid

Solution

Glass lens



$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_g} = (1.5 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$$

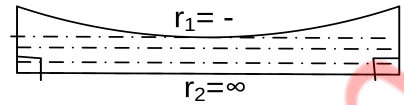
$$\frac{1}{f_g} = \frac{1}{24}$$

For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

$$\frac{1}{40} = \frac{1}{f_l} + \frac{1}{24}$$

$$\frac{1}{f_l} = -\frac{1}{60}$$

For liquid lens



$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$-\frac{1}{60} = (n_l - 1)\left(\frac{1}{-24} + \frac{1}{\infty}\right)$$

$$n_l = 1.4$$

3. The curved face of a Plano convex lens of refractive index 1.5 is placed in contact with a plane mirror. A pin placed at a distance 20cm coincides with its image. A film of a liquid is now introduced between the lens and the plane mirror. Then the coincidence of the pin and its image is found to be at a distance 100cm. Calculate the refractive index of the liquid.

Solution:

$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{20} = (1.5 - 1)\left(\frac{1}{24} + \frac{1}{\infty}\right)$$

$$r_2 = 10cm$$

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$$

$$\frac{1}{100} = \frac{1}{f_l} + \frac{1}{20}$$

$$\frac{1}{f_l} = -\frac{1}{25}$$

Consider the liquid lens.

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$-\frac{1}{25} = (n_l - 1)\left(\frac{1}{-10} + \frac{1}{\infty}\right)$$

$$n_l = 1.4$$

4. A small quantity of a liquid of refractive index 1.4 is poured on a horizontal plane mirror and a biconvex lens of focal length 30cm and refractive index 1.5 is then placed on top of the liquid. The pin is moved along the axis of the lens until no parallax between it and its image find the distance between the pin and the lens.

Solution

$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

For biconvex $r_1 = r_2 = r$

$$\frac{1}{30} = (1.5 - 1)\left(\frac{1}{r} + \frac{1}{r}\right)$$

$$r = 30cm$$

Consider the liquid lens.

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{-30} + \frac{1}{\infty}\right)$$

$$f_l = 75cm$$

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$$

$$\frac{1}{f} = \frac{1}{-75} + \frac{1}{30}$$

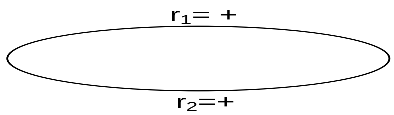
$$f = 50cm$$

The distance between the pin and the lens is **50cm.**

5. An equi-convex lens of refractive index 1.5 is placed on a horizontal plane mirror. A pin coincides with its own image when it is 0.6m above the lens. When the space between the mirror and lens is filled with a liquid, a pin has to be raised by 0.20m for coincidence to occur again. What is the refractive index of the liquid

Solution

Glass lens



$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{60} = (1.5 - 1)\left(\frac{1}{r} + \frac{1}{r}\right)$$

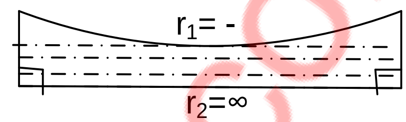
$$r = 60cm$$

For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

$$\frac{1}{80} = \frac{1}{f_l} + \frac{1}{60}$$

$$\frac{1}{f_l} = -\frac{1}{240}$$

For liquid lens



$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

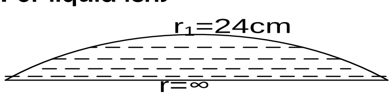
$$-\frac{1}{240} = (n_l - 1)\left(\frac{1}{-60} + \frac{1}{\infty}\right)$$

$$n_l = 1.25$$

6. A biconcave lens of radius of curvature 24cm is placed on a liquid film on a plane mirror. A pin clamped horizontally above the lens coincides with image at a distance of 40cm above the lens. If the refractive index of the liquid is 1.4 ,what is the refractive index of the material of the lens.

Solution

For liquid lens



$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{24} + \frac{1}{\infty}\right)$$

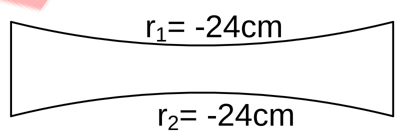
$$\frac{1}{f_l} = \frac{1}{60}$$

For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

$$-\frac{1}{40} = \frac{1}{f_g} + \frac{1}{60}$$

$$\frac{1}{f_g} = -\frac{1}{24}$$

Glass lens



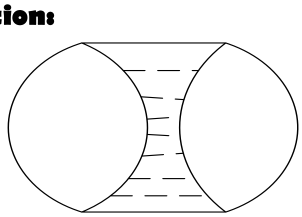
$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$-\frac{1}{24} = (n_g - 1)\left(\frac{1}{-24} + \frac{1}{-24}\right)$$

$$n_g = 1.5$$

8. Two equiconvex lenses of focal length **20cm** and made of glass of refractive index **1.6** are placed in contact and the space between them is filled with a liquid of refractive index **1.4**. Find the focal length of the lens combination.

Solution:



$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

For biconvex $r_1 = r_2 = r$

$$\frac{1}{20} = (1.6-1)\left(\frac{1}{r}+\frac{1}{r}\right)$$

$$r = 24cm$$

Consider the equi concave lens.

$$\frac{1}{f_l} = (n_l-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4-1)\left(\frac{1}{-24}+\frac{1}{-24}\right)$$

$$f_l = -30cm$$

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l}+\frac{1}{f_g}+\frac{1}{f_g}$$

$$\frac{1}{f} = \frac{1}{-30}+\frac{1}{20}+\frac{1}{20}$$

$$f = 15cm$$

focal length of the lens combination **15cm.**

9. In an experiment to determine the refractive index of paraffin the apparatus was first set up as shown using a convex lens of focal length f**:** Some water of refractive index $^4/_3$ was placed on the mirror and the lens on top. A pin placed at a height h_1 vertically above the lens coincides with its image. The experiment was repeated using paraffin instead of water and the new position of coincidence was found to be at a height h_2. Show that the refractive index n_p of paraffin is given by

$$n_p = 1+\frac{h_1(h_2-f)}{3h_2(h_1-f)}$$

Solution

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l}+\frac{1}{f_g}$$

$\frac{1}{h_1} = \frac{1}{f_l}+\frac{1}{f}$……………………(1)

But

$$\frac{1}{f_l} = (n_l-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = \left(^4/_3-1\right)\left(\frac{1}{-r_1}+\frac{1}{\infty}\right)$$

$$\frac{1}{f_l} = \frac{1}{-3r_1}$$

put into (1) $\frac{1}{h_1} = \frac{1}{f_l}+\frac{1}{f}$

$$\frac{1}{h_1} = \frac{1}{-3r_1}+\frac{1}{f}$$

$r_1 = \frac{h_1 f}{3(h_1-f)}$……………………(2)

Consider the paraffin-lens combination.

$\frac{1}{h_2} = \frac{1}{f_p}+\frac{1}{f}$……………………(3)

But

$$\frac{1}{f_p} = (n_p-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\frac{1}{f_p} = (n_p-1)\left(\frac{1}{-r_1}+\frac{1}{\infty}\right)$$

$$\frac{1}{f_p} = \frac{1-n_p}{r_1}$$

put into (3) $\frac{1}{h_2} = \frac{1}{f_p}+\frac{1}{f}$

$$\frac{1}{h_2} = \frac{1-n_p}{r_1}+\frac{1}{f}$$

$n_p = 1+\left(\frac{h_2-f}{h_2 f}\right)r_1$……………………(4)

Putting (2) into (4)

$$n_p = 1+\left(\frac{h_2-f}{h_2 f}\right)\left(\frac{h_1 f}{3(h_1-f)}\right)$$

$$n_p = 1+\frac{h_1(h_2-f)}{3h_2(h_1-f)}$$

DEFECTS IN IMAGES (ABERRATIONS)

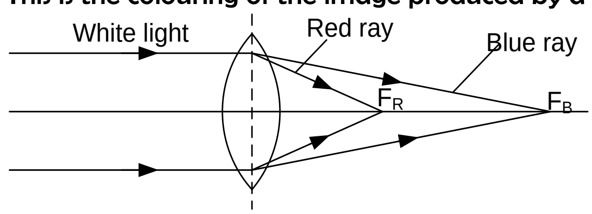
This is the distortion of images formed by either spherical mirrors or spherical lenses.

When mirrors and lenses under consideration are of large aperture, images formed by them can differ in shape and color from the object. Such defects are known as aberration or defects in images. There are two types of aberrations namely:

(i) Chromatic aberration.
(ii) Spherical aberration.

CHROMATIC ABERRATION

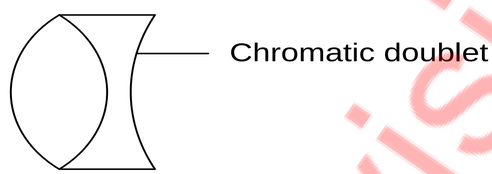
This is the colouring of the image produced by a lens



- When white light is incident on a lens, the different colour components are refracted by different amounts.
- Images corresponding to the different colours are formed in different positions along the principal axis of the lens.
- The image viewed has colored edges.

Minimizing chromatic aberration

Chromatic aberration can be reduced by using an achromatic doublet. This consists of a convex lens combined with a concave lens made from different glass materials. The convex lens deviates the rays while the concave lens nullifies the diversion.



NOTE:

The distance $F_r\ F_v$ **(i.e.** $F_r - F_v$**)** is the longitudinal chromatic aberration for the lens.

Conditions for chromatic doublet to work

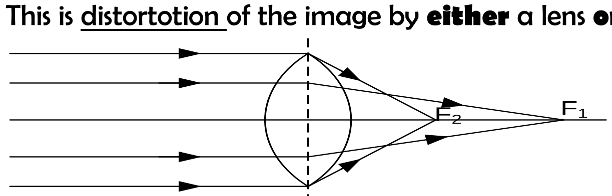
- Lenses should be of different glasses eg crown and flint glasses
- Ratio of their focal length should be equal to the ratio of their dispersive power
- If lenses are of same glass then should not be in contact with each other
  The separation between them should be equal to mean of their focal lengths

This is as a result of the marginal rays being converged nearer the lens or mirror than the paraxial rays.

In lenses, Spherical aberration can be reduced by using a circular stop to cut off marginal rays.

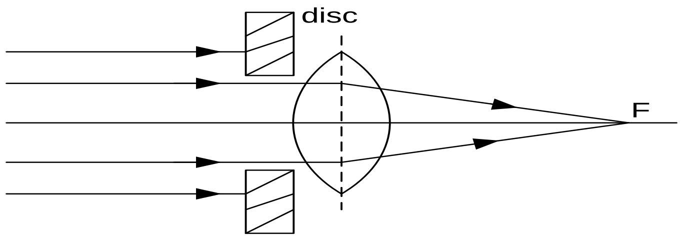
SPHERICAL ABERRATION

This is distortotion of the image by **either** a lens **or** a mirror of wide aperture.



When a wide beam of white light is incident on **either** a lens **or** a mirror of wide aperture, central rays are brought to converge far away from the lens. The rays which are far from the principal axis are brought to converge near the lens. The image formed is circular blurred due to a series of images of the same object

In lenses, Spherical aberration can be minimized using a stopper i.e. using an opaque disc with a central hole to cut off marginal rays.



The disadvantage with this method is light intensity is cut down and so the brightness of the image is reduced.

NOTE:

A circular stop is an opaque disc having a hole in the middle for allowing in only paraxial rays incident on the lens

We can also use plano convex lens with curved side facing the incident rays

In mirrors, spherical aberration can be minimized by using a parabolic mirror.

This because a parabolic mirror converges a wide parallel beam of light incident onto its surface to a single focus as shown.

COMPARISION OF NARROW AND WIDE APERTURE LENSES

Lenses of narrow aperture are widely used in optical instruments so as to avoid spherical aberration. This is because when a wide beam of light falls on a lens of narrow aperture, all rays are paraxial and are thus brought to a single focus to form a sharp image. However a lens with a wide aperture allows in both paraxial and marginal rays, which are thus brought to different focus to form a blurred image.

EXAMPLE:

1. The curved surface of a plane convex lens has a radius of curvature of **20cm** and is made of crown glass for which the refractive index of red and blue light respectively is $1{\cdot}5$ and 1·52 calculate the longitudinal chromatic aberration for the lens.

Solution

For red light $\frac{1}{f_r} = (n_r - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$$\frac{1}{f_r} = (1.5 - 1)\left(\frac{1}{20} + \frac{1}{\infty}\right)$$

$$f_r = 40cm$$

For blue light $\frac{1}{f_b} = (n_b - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$$\frac{1}{f_b} = (1.52 - 1)\left(\frac{1}{20} + \frac{1}{\infty}\right)$$

$$f_b = 38.5cm$$

Thus longitudinal aberration $= (F_r - F_v)$

$$= (40 - 38{\cdot}5)cm$$

$$= 1{\cdot}5cm$$

2. A convex lens of radius of curvature **24cm** is made of glass of refractive index for red and violet light of **1·6** and **1·8** respectively. A small object illuminated with white light is placed on the axis of the lens at a distance **45cm** from the lens. Find the separation of the images formed in the red and violet constituents of light.

Solution

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

For biconvex $r_1 = r_2 = 24$

For red light $\frac{1}{f_r} = (1.8 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$

$$f_r = 30cm$$

For violet light $\frac{1}{f_v} = (1.8 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$

$$f_v = 20cm$$

Also

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

For red light

$$\frac{1}{30} = \frac{1}{45} + \frac{1}{V_r}$$

$$V_r = 90cm$$

For violet light

$$\frac{1}{20} = \frac{1}{45} + \frac{1}{V_V}$$

$$V_V = 36cm$$

Image separation$= V_r - V_V$

$$= 90 - 36$$

$$= 54cm$$

EXERCISE:12

1. Describe how the focal length of a convex lens can be determined using a plane mirror and the non-parallax method.
2. You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a convex lens, a plane mirror, and a meter ruler. Describe an experiment to determine the focal length of a convex lens using the above apparatus.
3. Describe an experiment, including a graphical analysis of the results to determine the focal length of a convex lens using a no parallax method.
4. Describe an experiment to determine the focal length of a thick convex lens having inaccessible surfaces.
5. A convex lens is contained in a cylindrical tube such that its exact position in the tube is not accessible. Describe how you would determine the focal length of the lens without removing it from the tube
6. Describe how the focal length of a diverging lens can be determined using a convex lens.
7. Describe how the focal length of a concave lens can be obtained using a concave mirror.
8. Derive an expression for the focal length of a combination of two thin converging lenses in contact, in terms of their focal lengths.
9. **(a)** Show that the focal length **f** of a thin convex lens in air is given by
   $\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, Where **n** is the refractive index of the material of the lens, r_1 and r_2 are the radii of curvature of the surfaces of the lens.
   (b) The radii of curvature of the faces of a thin convex meniscus lens of glass of refractive index **1·75** are **8cm** and **12cm**. Calculate the focal length of the lens when completely surrounded by a liquid f refractive index **1·25.**[**Answer: f = 60cm**]
10. **(a)** Describe, giving the relevant equations, how the refractive index of a liquid can be determined using a convex lens of known radius of curvature.
    (b) A biconvex lens of radius of curvature **24cm** is placed on a liquid film on a plane mirror. A pin clamped horizontally above the lens coincides with its image at a distance of **40cm** above the lens. If the refractive index of the liqui is **1·4**, calculate the refractive index of the material of the lens. [**Answer: n = 1·5**]
11. **(a)** Differentiate between **chromatic** and **spherical aberrations.**
    (b) Explain how the defects in **10(a) above** can be minimized in practice.
    (c) Explain why lenses of narrow aperture are preferred to lenses of wide aperture in optical instruments.

(d) Draw a ray diagram showing the reflection of a wide beam of light by a concave mirror of wide aperture

10. A thin biconvex les is placed on a plane mirror. A pin is clamped above the lens so at it apex lies on the principal axial of the lens. The position of the pin is adjusted until the pin coincides with its image at a distance of 15cm from the mirror. When a thin layer of water of refractive index 1.33 is placed between the mirror ,the pin coincides with its image at a point 22.5cm from the mirror. When water is replaced by paraffin, the pin coincides with the image at a distance of 27.5cm from the mirror. Calculate the refractive index of paraffin. **An(1.45)**
11. A bi-convex lens of radius of curvature 24cm is placed on a liquid film on a plane mirror. Apin clamped horizontally above the lens coincides with its image at a distance of 40cm above the lens. If the refractive index of the liquid is 1.4. What is the refractive index of the material of the lens **An(1.5)**
12. An equi-convex lens is placed on a horizontal palne mirror and a pin held vertically above the lens is found to coincide with its image when positioned 20.0cm above the lens. When a few drops of liquid is placed between the lens and the mirro, the pin has to be raised bu 10.0cm to obtain coincidence with the image. If the refractive index of converging lens. What is the refractive index of the liquid **An(1.33)**
13. An illuminated object is placed on the 0cm mark of an optical bench. A converging lens of focal length 15cm is placed at the 22.5cm mark. A diverging lens of focal length 30cm and a plane mirror are placed at the 37.5cm and 77.5cm marks respectively. Find the position of the final image.(at 0cm mark). Illustrate your answer with a ray diagram
14. A converging lens of focal length 1.5cm is placed 29.0cm in front of another converging lens of 6.25cm.An object of height 0.1cm is placed 1.6cm a way from the first lens on the side remote from the second lens at right angles to the principal axis of the final image by the system. Determine the position and size and final image of the object.
15. An equi-convex lens A is made of glass of refractive index 1.5 and has a power of 5.0radm^{-1}. It is combined in contact with a lens B to produce a combination whose power is 1.0radm^{-1}. the surfaces in contact fits exactly. The refractive index of the glass in lens B is 1.6. What are the radii of the four surfaces? Draw a diagram to illustrate your.
16. A lens forms the image of a distant object on a screen 30cm away. Where should a second lens of focal length 30cm be placed so that the screen has to be moved 8cm towards the first lens for the new image to be in focus.
17. A convex lens of focal length 20cm, forms an image on a screen placed 40cm beyond the lens . A concave lens of focal length 40cm is then placed between a convex lens and a screen a distance of 20cm from the convex lens.
    (i) Where must the screen be placed in order to receive the new image?
    (ii) What is the magnification produced by the lens system?

Optical instruments.

These are instruments which work on the principle of reflection and refraction of light rays. They include telescopes, microscopes, prism binoculars, camera and projection lantern.

Near point

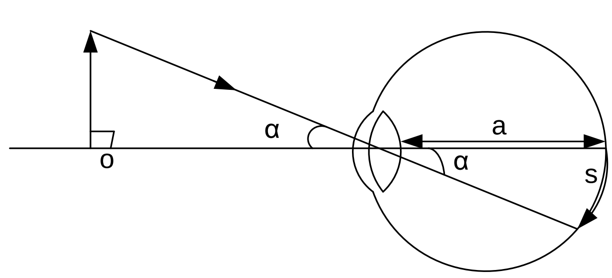
This is the point at which the eye is able to view the object in greatest detail

Least distance of distinct vision, $D \approx 25cm.$

It's the shortest distance of the eye from the object at which the eye can see clearly.

Virtual angle

Angle subtended by an object or image at the eye.



α −visual angle

arc length S = size of the image.

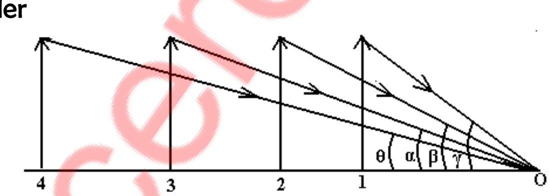
$$S = \alpha a$$

Where a is the distance between the eye lens and the retina.

∴ Size of the image formed at the retina is directly proportional to the visual angle.

Explain why the further vertical pole in line with others of equal height looks shorter.

The image size depends on the visual angle. The farthest pole subtends a small visual angle and hence image size is smaller



The poles that are nearer to the observer subtend bigger angles at the observer's eye than the furthest pole . Since the apparent height of the object is proportional to the angle it subtends at the eye, the furthest pole appears shortest.

Angular magnification (magnifying power).

It is the ratio of the angle subtended by the final image at the eye when instrument is being used to the angle subtended by the object at an unaided eye (naked eye)

i.e. angular magnification, $m = \frac{\alpha^1}{\alpha}$

Where α^1 is angle subtended at the eye by the image when instrument is used and

α is the angle subtended at an unaided eye by the object.

NOTE:

Unaided eye is when the object is viewed without using an instrument.

Microscopes

These are used to view near objects

Angular magnification of microscopes $m = \frac{\alpha^1}{\alpha}$

α is the angle subtended at the eye object at the near point when microscope is not used.

α^1 is the angle subtended at the eye by image when microscope is used.

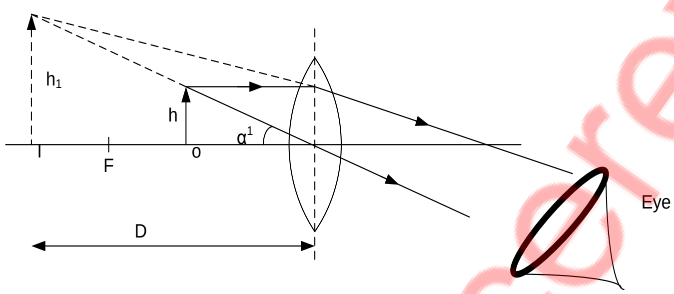
In normal adjustment or use, the microscope forms the image at the near point.

Simple microscope / magnifying glass

This consists of a single convex lens with the distance between the object and the lens less than or equal to the focal length of the lens.

Simple microscope with image at near point (normal adjustment)

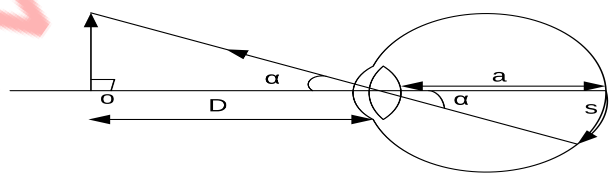
A simple microscope in normal adjustment consists of a converging lens set in such a way that it forms a virtual magnified erect image of an object placed between the principal focus and the optical centre of the lens at the least distance of distinct vision as shown.



For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_1}{D}$

If α is the angle subtended at the eye by the object at the near point

Before using a microscope, the object is first viewed at the near point of the eye by unaided eye as shown:



$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_1}{D}\right)}{\frac{h}{D}} = \frac{h_1}{h}$$

hence

$$\boxed{m = \frac{h_1}{h}}$$

but $m = \frac{v}{f} - 1$

There fore angular magnification, $\boxed{m = \frac{D}{f} - 1}$

where $v = D$

EXAMPLE:

Calculate the angular magnification produced by a magnifying glass of focal length **5cm** adjusted such that an image is formed at a distance of **25cm** in front of it.

Solution:

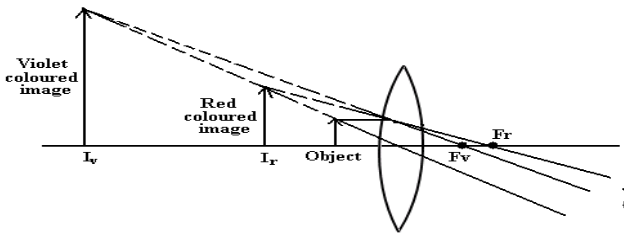
$m = \frac{D}{f} - 1$ but $D = -25cm$

$$m = \frac{-25}{5} - 1$$

$$m = -6$$

Thus the required angular magnification is **6**

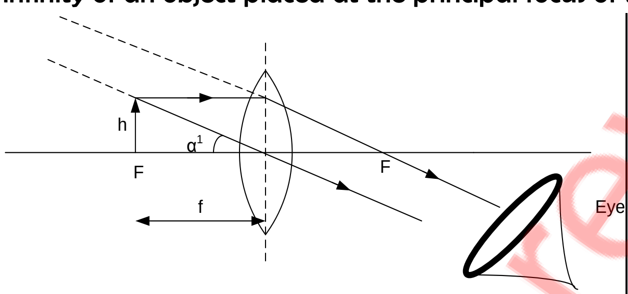
Explain why chromatic aberration is not experienced in magnifying glass



When an object **O** is viewed through a converging lens used as a magnifying glass, various coloured virtual images corresponding to say red and violet rays are formed at slightly different positions I_r and I_v respectively as shown. These images subtend the same angle at the eye and therefore appear superimposed. Thus the virtual image seen in a simple microscope is almost free from chromatic aberration.

Simple microscope with final image at infinity (not in normal adjustment)

This simple microscope consists of a converging lens which forms an erect virtual magnified image at infinity of an object placed at the principal focus of the lens as shown.



Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h}{f}$

$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h}{f}\right)}{\frac{h}{D}} = \frac{D}{f}$$

Hence angular magnification, $\boxed{m = \frac{D}{f}}$

Note: (i) Angular magnification is higher when a simple microscope forms the image at infinity

(ii) For higher magnification, use lenses of short focal length.

Example

1. A thin converging lens of focal length 10.0cm is used as a magnifying glass. In one instance it is required that the final image to be formed at infinity and the other to be formed at 30.0cm from the lens. Find;
   (i) Angular magnification when the image is at infinity
   (ii) Position of the object when the image is at 30cm from the lens and its angular magnification

Solution

(i) $m = \frac{D}{f}$

$$m = \frac{-25}{10} = -2.5$$

(ii) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{10} = \frac{1}{u} + \frac{1}{30}$$

$$v = 15cm$$

$$m = \frac{D}{f} - 1$$

$$m = \frac{-25}{10} - 1 = -3.5$$

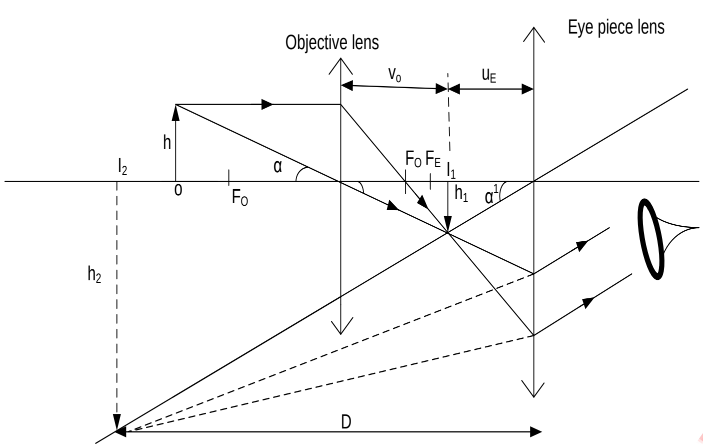
Compound microscopes

This is used to give a greater magnifying power than the simple microscope. It of two converging lenses, namely the objective (which is near the object) and the eye piece, near the eye.

Compound microscope in normal adjustment.

A compound microscope consists of two converging lenses of short focal lengths. This enables a high angular magnification to be obtained.

In normal adjustment, the objective of a compound microscope forms a real inverted image of the object at a point distance less than Fe from the eyepiece. This intermediate image formed acts as a real object for the eye piece which thus forms a virtual magnified image at a distance of distinct vision from the eye piece as shown.



The objective lens forms a real image I_1, of the object O . I_1 is formed at a point nearer the eye piece than the principal focus f_e of the eye piece.

The eye piece acts as a magnifying glass. It forms a virtual image I_2 of I_1. The observer's eyes should be taken to be close to the eye piece so that α' is the angle subtended at the eye by the final image I_2.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_2}{D}$

$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_2}{D}\right)}{\frac{h}{D}} = \frac{h_2}{h}$$

Multiplying h_1 and dividing by h_1

$$m = \frac{h_2}{h} x \frac{h_1}{h_1}$$

$$m = \frac{h_2}{h_1} x \frac{h_1}{h}$$

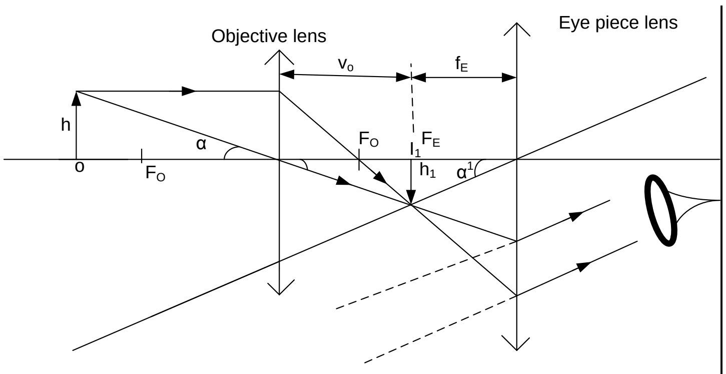
$$m = m_o x m_E$$

$$\boxed{m = \left(\frac{v_o}{f_O} - 1\right) x \left(\frac{D}{f_E} - 1\right)}$$

Note : For higher angular magnification, both the eye piece and the objective should have short focal lengths.

Compound microscope not in normal adjustment

The objective forms a real inverted image of the object at the principle focus Fe of the eye piece which thus forms a final virtual magnified image at infinity as shown.



The separation of the object and the eye piece is such that the object forms an image of the object at the principle focus Fe of the eye piece, hence the eye piece focuses the final image at infinity.

The angle α' subtended by the final image by the eye piece is

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_1}{f_E}$

$$\alpha \approx tan\alpha = \frac{h}{D}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_1}{f_E}\right)}{\frac{h}{D}} = \frac{D}{f_E} x \frac{h_1}{h}$$

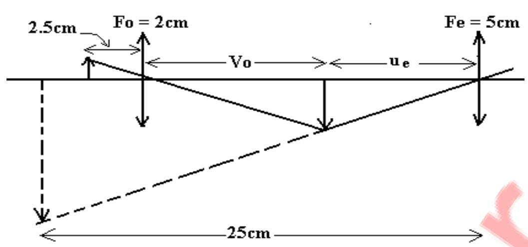
$$m = m_o x m_E$$

$$\boxed{m = \left(\frac{v_O}{f_O} - 1\right) x \left(\frac{D}{f_E}\right)}$$

EXAMPLES:

1. The objective of a compound microscope has a focal length of **2cm** while the eyepiece has a focal length of **5cm**. An object is placed at a distance of **2.5cm** in front of the objective. The distance of the eyepiece from the objective is adjusted so that the final image is **25cm** in front of the eyepiece. Find the distance between the lenses and the magnifying power of the microscope.

Solution:



Consider the action of the eyepiece

$V_E = -25cm$ and $f_E = 5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{5} = \frac{1}{u_E} + \frac{1}{-25}$$

$$u_E = 4.167cm$$

Consider the action of the objective

$u_o = 2{\cdot}5cm$ and $f_O = 2cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{2} = \frac{1}{2.5} + \frac{1}{v_O}$$

$$v_O = 10cm$$

$\therefore$ **The required lens Separation**

$$= v_O + u_E = (10 + 4 \cdot 2)\ cm = \mathbf{14{\cdot}2cm}$$

The required magnifying power

$$m = m_o x m_E$$

$$m = \frac{10}{2.5} x \frac{25}{4.167}$$

$$m = 24$$

Alternatively

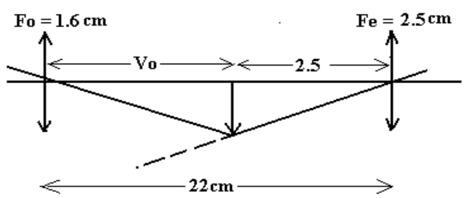
$m = \left(\frac{v_O}{f_O} - 1\right) x \left(\frac{D}{f_E} - 1\right)$ where **D = -25cm**

$$m = \left(\frac{10}{2} - 1\right) x \left(\frac{-25}{5} - 1\right)$$

$\Rightarrow$ $\therefore$ $M = -24$ Thus the required magnifying power **M = 24**

2. A compound microscope has an eyepiece of focal length **2·5cm** and an objective of focal length **1·6cm**. If the distance between the objective and the eye piece is **22cm,** calculate the magnifying power produced when the object is at infinity.

Solution



For the image to be at infinity, the object must be at the focal point of the eyepiece

Thus the image distance in the objective

$= (22 - 2{\cdot}5)\ cm = \mathbf{19{\cdot}5cm}$

$m = \left(\frac{v_o}{f_O} - 1\right) x \left(\frac{D}{f_E}\right)$ where $\boldsymbol{D} = \mathbf{-25cm}$

$$m = \left(\frac{19.5}{1.6} - 1\right) x \left(\frac{-25}{2.5}\right)$$

$\Rightarrow \quad \therefore \quad \boldsymbol{M} = -111{\cdot}875$

Thus the required magnifying power

$$\boldsymbol{M} = \mathbf{111{\cdot}875}$$

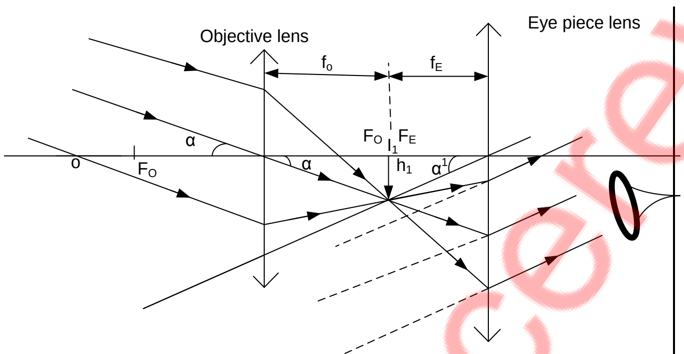
TELESCOPES

Telescopes are used to view distant objects. The angular magnification of a telescope is the ratio of the angle subtended by the final image at the aided eye to the angle subtended by the object at the un aided eye. In normal adjustment, the final image is at infinity.

REFRACTING ASTRONAMICAL TELESCOPE IN NORMAL ADJUSTMENT

A telescope is in normal adjustment when the final image of a distant object is formed at infinity.
An astronomical telescope consists of f two converging lenses; one is an objective of long focal length and the other an eyepiece of short focal length. This enables a high angular magnification to be obtained.

In normal adjustment, the objective forms a real inverted image of a distant object at its focal point $\mathbf{F_o}$ situated exactly at the principal focus $\mathbf{F_e}$ of the eyepiece. This intermediate image acts as a real object for the eyepiece to give rise to a final virtual image at infinity as shown.



In normal adjustment, the image of the distant object formed by the objective lens lies in the focal plane of both the objective and the eye piece.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_1}{f_E}$

$$\alpha \approx tan\alpha = \frac{h_1}{f_O}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_1}{f_E}\right)}{\left(\frac{h_1}{f_O}\right)} = \frac{f_O}{f_E}$$

$$\boxed{m = \frac{f_O}{f_E}}$$

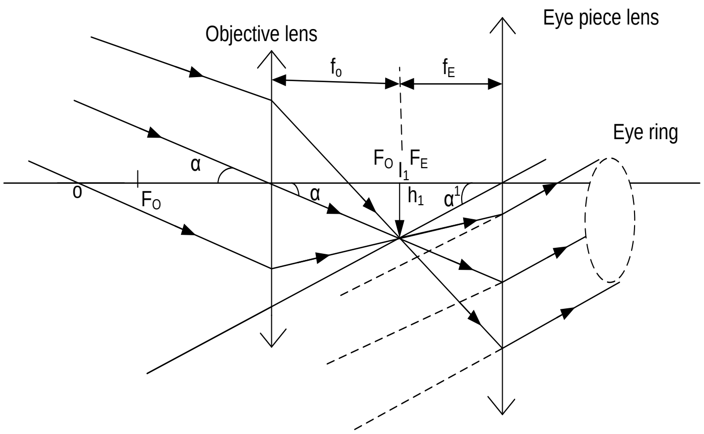
For a high magnifying power, the objective should have a long focal length and the eyepiece a short focal length.

Hence separation between the lenses $= f_o + f_E$

Eye ring/ Exit pupil

Eye ring is the best position for the eye when viewing an image through the instrument.

At the exit pupil, the eye receives a maximum amount of light entering the objective from out side so that its field of view is greatest



Note: when determining the eye ring, the separation is taken as the object distance and focal length of the eye piece is used in calculations. Hence from the above

hence ***v***, which is the eye ring can be obtained.

The eye ring, and relation to angular magnification

$u = f_o + f_E, f = f_E$

Then use $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f_E} = \frac{1}{f_o+f_E} + \frac{1}{v}$

$v = \frac{f_E}{f_o}(f_E + f_o)$

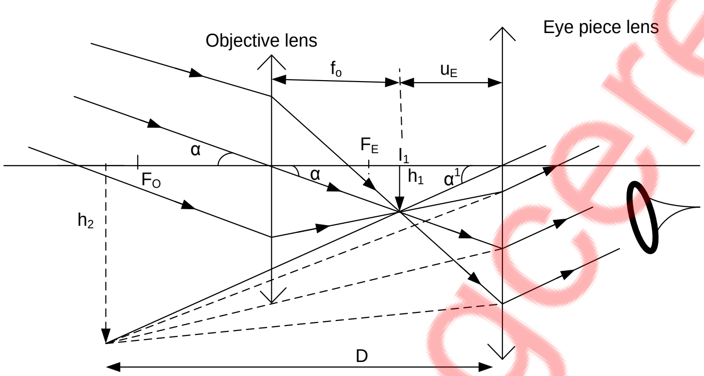
$$\frac{diameter\ of\ the\ eye\ ring}{diameter\ of\ the\ objective} = \frac{v}{u} = \frac{\frac{f_E}{f_o}(f_E + f_o)}{(f_o + f_E),} = \frac{f_E}{fo}$$

hence angular magnification,

$$m = \frac{diameter\ of\ the\ objective}{diameter\ of\ the\ eye\ ring} = \frac{f_o}{f_E}$$

The above expression for the magnifying power is only true for a telescope in normal adjustment with lens separation f_o+f_E.

Astronomical telescope with image formed at near point (not in normal adjustment)



The intermediate image should be formed in front of the focal point of the eye piece.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h_2}{D}$

$$\alpha \approx tan\alpha = \frac{h_1}{fo}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h_2}{D}\right)}{\left(\frac{h_1}{fo}\right)} = \frac{fo}{D} x \frac{h_2}{h_1}$$

$$m = m_o x m_E$$

$$\boxed{m = \left(\frac{D}{f_E} - 1\right) x \left(\frac{fo}{D}\right)}$$

The lens separation $= f_o + u_E$

DISADVANTAGES OF AN ASTRONOMICAL TELESCOPE

It forms an inverted final image.

NOTE:

The structure of an astronomical telescope can be modified to over come the above disadvantage by use of a terrestrial telescope which forms an erect image.

EXAMPLES:

1. An astronomical telescope has an objective and an eyepiece of focal length **75.0cm** and **2.5cm** respectively. Find the separation of the two lenses if the final image is formed at **25cm** from the eyepiece, calculate the:

Solution

Consider the action of the eyepiece

$v = -25cm$ and $f_E = 2.5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{2.5} = \frac{1}{u} + \frac{1}{-25}$$

$$u_E = 2.27cm$$

$\Rightarrow$ **The lens separation** $= \boldsymbol{f_o + u_E}$

$= (75 + 2 \cdot 27)\,cm = \boldsymbol{77 \cdot 27cm}$

2. An astronomical telescope has an objective and an eyepiece of focal length **100cm** and **5cm** respectively.
   (a) Find the angular magnification of the telescope if arranged in normal adjustment.
   (b) If the lenses are arranged in such a way that the final image is formed at **25cm** from the eyepiece, calculate the:
      (i) angular magnification of the telescope in this setting.
      (ii) separation of the objective and eyepiece.

Solution:

(a) In normal adjustment, **magnifying power** $m = \frac{f_o}{f_E} = \frac{100}{5} = 20$

(b) (i) With the final image at near point,

$$m = \left(\frac{D}{f_E} - 1\right) x \left(\frac{f_o}{D}\right)$$

$$m = \left(\frac{-25}{5} - 1\right) x \left(\frac{100}{25}\right)$$

$$m = -24$$

(ii) Consider the action of the eyepiece

$v = -25cm$ and $f_E = 5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{5} = \frac{1}{u} + \frac{1}{-25}$$

$$u = 3.57cm$$

$\Rightarrow$ **The lens separation** $= \boldsymbol{f_o + u}$

$= (100 + 3 \cdot 57)\,cm = \boldsymbol{103 \cdot 57cm}$

3. The objective of an astronomical telescope in normal adjustment has a diameter of **12cm** and focal length of **80cm**.
   (a) If the eyepiece has a focal length of **5cm,** find the:
      (i) magnifying power of the telescope in this setting.
      (ii) Position of the eye-ring
      (iii) diameter of the eye-ring

(b) State the advantage of placing the eye at the eye ring.

Solution

(a) (i) In normal adjustment, **magnifying power** $m = \frac{f_o}{f_E} = \frac{80}{5} = 16$

(ii) Consider the action of the eyepiece

$u = \boldsymbol{f_o} + \boldsymbol{f_E} = (80 + 5)$ = 85cm

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{5} = \frac{1}{85} + \frac{1}{-25}$$

$$v = 5.313cm$$

$\therefore$ The eye-ring is **5•313cm** from the eyepiece

(ii) In normal adjustment,

$$\frac{diameter\ of\ the\ objective}{diameter\ of\ the\ eye\ ring} = \frac{f_o}{f_E}$$

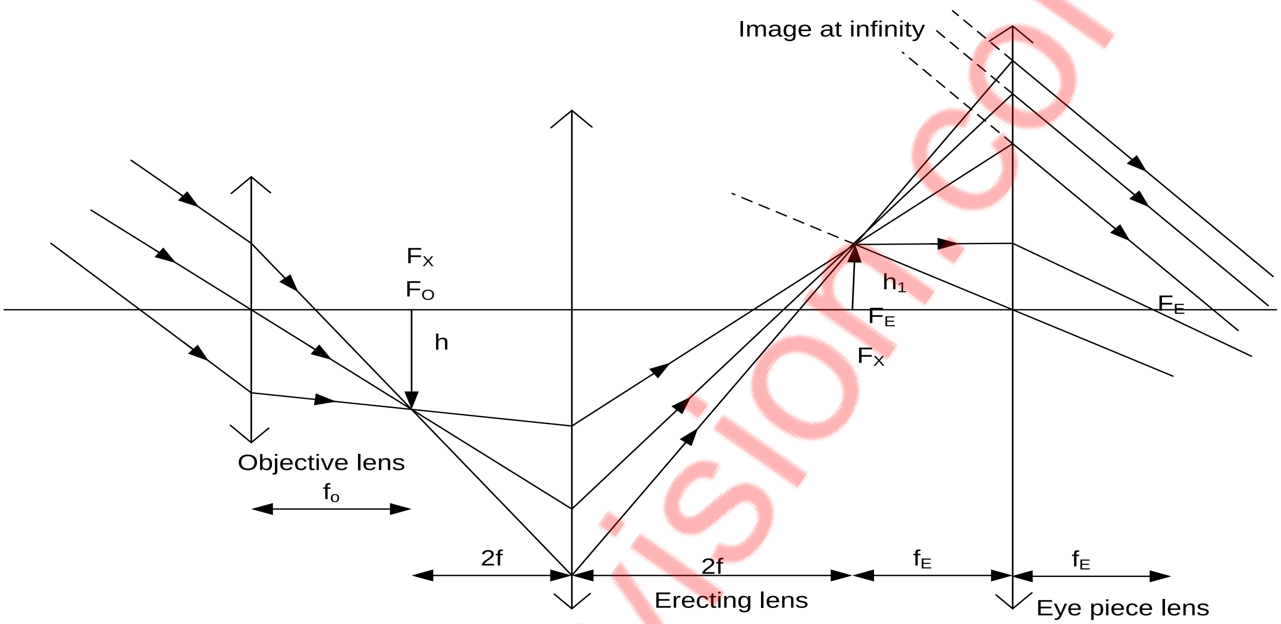
$$\frac{18}{diameter\ of\ the\ eye\ ring} = \frac{80}{5}$$

$$diameter\ of\ the\ eye\ ring = 1.125cm$$

(iii) The eye placed at the eye ring has a wide field of view since most of the light entering the objective passes through the eye ring.

Terrestrial telescope

It is a refracting telescope with an intermediate erecting lens of focal length **f**, which is place between the objective lens and the eyepiece. The erecting lens should be a distance **2f** after the principal focus of the objective lens and a distance **2f** before the principal focus of the eyepiece. The objective lens forms a real inverted image of a distant object at its focal point $\mathbf{F_o}$. This act as a real object for the erecting lens which forms a real erect image of the same size as the inverted image formed by the objective.



ADVANTAGE OF A TERRESTRIAL TELESCOPE

It forms an erect final image.

DISADVANTAGES OF A TERRESTRIAL TELESCOPE

(i) It is bulky since its length is increased by **4f** compared with an astronomical telescope.

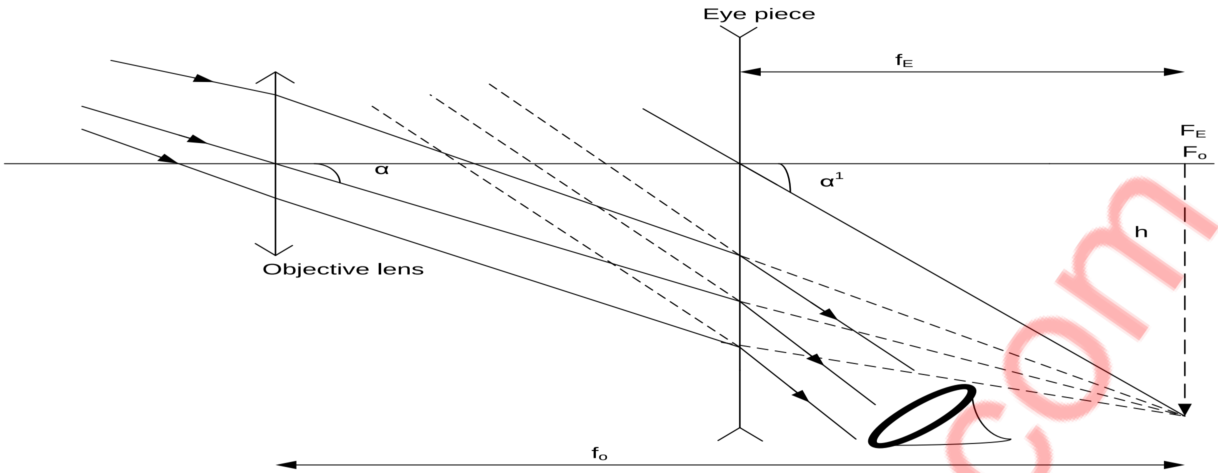
(ii) It reduces the intensity of light emerging through the eyepiece. This is due to light losses at several lens surfaces

GALILEAN TELESCOPE:

This telescope provides an erect image of a distant object with the aid of an objective which is a converging lens of long focal length and an eyepiece which is a diverging lens of short focal length.

GALILEAN TELESCOPE IN NORMAL ADJUSTMENT

A converging lens is arranged coaxially with a diverging lens such that their focal points are at the same point. The converging lens forms a real image of a distant object at its focal point $\mathbf{F_o}$ situated exactly at the principal focus $\mathbf{F_e}$ of the diverging lens. This image formed acts as a virtual object for the diverging lens which thus forms a final virtual image at infinity as shown.



Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h}{f_E}$

$$\alpha \approx tan\alpha = \frac{h}{f_O}$$

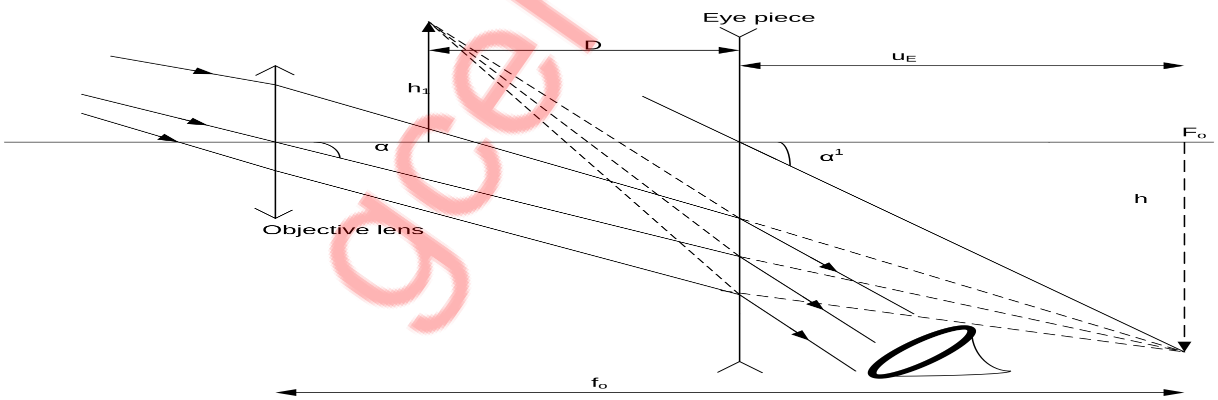
$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h}{f_E}\right)}{\left(\frac{h}{f_O}\right)} = \frac{f_O}{f_E}$$

$$\boxed{m = \frac{f_O}{f_E}}$$

$$Separation\ of\ the\ lens = f_O - f_E$$

GALILEAN TELESCOPE WITH FINAL IMAGE AT NEAR POINT

A converging lens arranged coaxially with a diverging lens forms a real image of a distant object at its focal point **F_o** situated a distance **u** beyond the diverging lens. This image formed acts as a virtual object for the diverging lens which thus forms a final erect virtual image between the converging lens and the diverging lens at an image distance **D** as shown.



Let h be the height of the image formed at F_o.

Angular magnification, $m = \frac{\alpha^1}{\alpha}$

For small angles in radians: $\alpha^1 \approx tan\alpha^1 = \frac{h}{U_E}$

$$\alpha \approx tan\alpha = \frac{h}{f_O}$$

$$m = \frac{\alpha^1}{\alpha} = \frac{\left(\frac{h}{U_E}\right)}{\left(\frac{h}{f_O}\right)} = \frac{f_O}{U_E}$$

$$\boxed{m = \frac{f_O}{U_E}}$$

Consider the action of the eyepiece

Then use $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$V = D$ and $f = f_E$

$$\frac{1}{f_E} = \frac{1}{U_E} + \frac{1}{D}$$

$$U_E = \frac{f_E D}{D - f_E}$$

$$m = \frac{f_O}{U_E}$$

$$m = \frac{f_O}{\left(\frac{f_E D}{D - f_E}\right)}$$

$$\boxed{m = \frac{f_O}{f_E}\left(1 - \frac{f_E}{D}\right)}$$

NOTE:

(i) There is need to consider the signs of **f_e** and **D** while using the above expression and are taken to be negatives.

Advantages of Galilean Telescope:

(i) It is shorter than astronomical telescope when in normal adjustment, hence it used for *opera glasses*

(ii) The final image is upright or erect.

Disadvantages of Galilean Telescope:

(i) it has a virtual eye ring not accessible to the observer.

(ii) it has a narrow field of view.

EXAMPLE

1. A Galilean telescope has a convex lens of focal length **50cm** and a diverging lens of focal length **5cm.**

(a) Find the angular magnification of the telescope if arranged in normal adjustment.

(b) If the lenses are arranged in such a way that the final image is formed at **25cm** from the eyepiece, calculate the:

(i) angular magnification of the telescope in this setting.

(ii) separation of the objective and eyepiece

Solution

(a) (i) In normal adjustment,

magnifying power $m = \frac{f_O}{f_E} = \frac{50}{5} = 10$

(b) (i) With the final image at near point**,**

magnifying power $m = \frac{f_O}{f_E}\left(1 - \frac{f_E}{D}\right)$

$$m = \frac{50}{-5}\left(1 - \frac{-5}{-25}\right) = -8$$

Thus the required angular magnification is 8

(ii)

The lens separation $= f_O - u$

Consider the action of the eyepiece

$v = -25cm$ and $f_E = -5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-5} = \frac{1}{u} + \frac{1}{-25}$$

$$u = -8.33cm$$

The required lens separation $= fo - u$

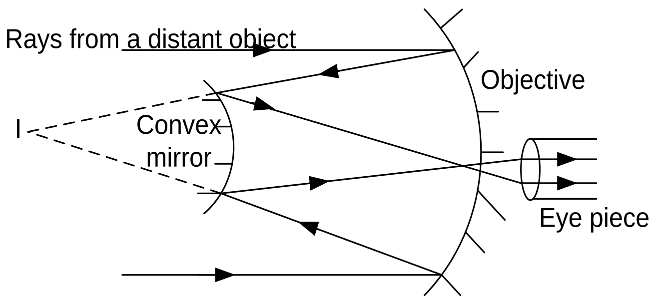
$= (50 - 8 \cdot 33)\ cm = 41 \cdot \mathbf{67cm}$

REFLECTING ASTRONOMICAL TELESCOPE

The objective of a reflecting telescope is a concave mirror with long focal length.
There are three types of reflector telescopes namely:

(i) Cassegrain Reflector Telescope
(ii) Newton Reflector Telescope
(iii) Coude Reflector Telescope

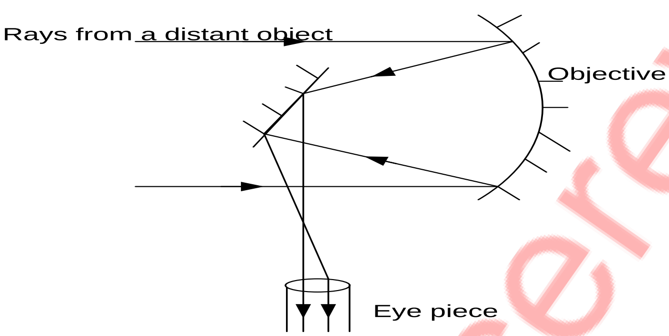
Case grain reflecting telescope



- The objective consists of a concave mirror with a long focal length
- Parallel rays of light from a distant object are first reflected at concave mrirror and then at a small convex mirror to form a real image **I** at a hole situated at the pole of the concave mirror
- The eyepiece is set such that **I** coincide with its principal focus thus forming a magnified virtual image at infinity

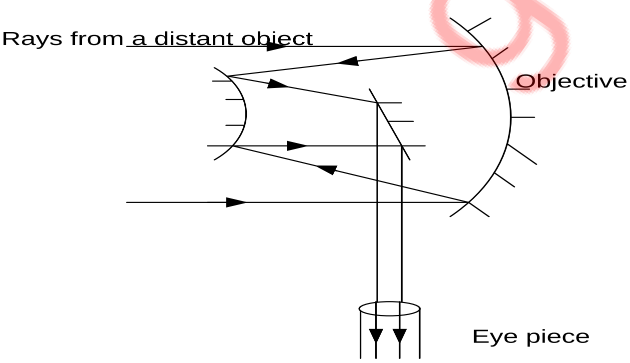
Newton's reflecting telescope

It consists of a concave mirror of long focal length as the objective instead of a convex lens, a plane mirror and a convex eye piece.



- Parallel rays of light form a distant object are first reflected at objective and then at a small slanting plane mirror to form a real image at **I**.
- The plane mirror helps to bring intermediate image to a more convenient focus
- The plane ,mirror is small so that it can not affect the effective focal length of the objective
- The eye piece is adjusted until the magnified virtual image is formed at infinity

Coude reflecting telescope



- The objective consists of a concave mirror of long focal length.
- Light from a distant object is reflected first at a concave mirror and then at a small convex mirror which then reflects it on to a slanting plane mirror to form a real image at I.
- The eyepiece is set such that **I** coincide with its principal focus thus forming a virtual magnified image at infinity as shown.

Advantages of reflecting telescope over refracting telescope

(i) There is no chromatic aberration since no refraction occurs at the objective which is a mirror.
(ii) Spherical aberration can be reduced easily by using a parabolic mirror as the objective.
(iii) It is much cheaper and easier to make a mirror than a lens since one surface requires to be grounded.
(iv) Images are brighter because it is easy to make mirrors of large aperture which collects a lot of light compared to making a lens with large aperture
(v) Resolving power is also greater (i.e. seeing different images as separate)

Disadvantage of reflecting telescope over refracting telescope

(i) they tarnish easily and absorb light resulting in dull images.

Differences between telescopes and microscopes

Telescopes	Microscopes
(1) views distant objects	views near objects
(2) objective lens has larger focal length	objective lens has smaller focal length
(3) smaller resolving power	larger resolving power
(4) Either reflecting or refracting	only refracting

EXERCISE;13

1. **(i)** Define the term **visual angle** as applied to optical systems**.**
(ii) Explain why the farthest vertical pole in line with others of equal height looks shorter.

2. **(i)** Define the term **angular magnification** of an optical instrument.
(ii) State the reason why the focal length of the objective of a telescope is always much longer than that of the eyepiece.
(iii) What is meant by the term **normal adjustment** as applied to a telescope?
(iv). Draw a ray diagram to show the action of an astronomical telescope in normal adjustment, and derive an expression for its magnifying power in terms of the focal lengths f_o and f_E of the objective and eyepiece respectively.
(v) Calculate the separation of the eyepiece and objective of an astronomical telescope in normal adjustment whose magnifying power is **20** and its eyepiece has a focal length of **5cm.**

[**Answer: 105cm**]

3. **(i)** What is meant by the term **exit pupil** as applied to a telescope?
(ii) What is the significance of the eye-ring of an astronomical telescope?
(iii) With the aid of a ray diagram, show that for an astronomical telescope in normal adjustment having its objective lens and an eyepiece of focal lengths f_o and f_E respectively, $\frac{diameter\ of\ the\ objective}{diameter\ of\ the\ eye\ ring} = \frac{f_o}{f_E}$.
(iv) Calculate the distance of the eye-ring from the eyepiece of an astronomical telescope in normal adjustment whose objective and eyepiece have focal lengths of **80cm** and **10cm** respectively.

[**Answer: 11·25cm**]

(v). Draw a ray diagram to show the formation of the final image by an astronomical telescope at near point, and derive an expression for its magnifying power.

(vi) State the disadvantage of using an astronomical telescope when viewing distant objects on earth. Describe how an astronomical telescope can be modified to overcome this disadvantage.

4. (i) The objective and eyepiece of an astronomical telescope have focal lengths f_o and f_E respectively. Derive an expression for the magnifying power of this telescope if the final image is a distance **D** in front of the eyepiece.

(ii) The objective and eyepiece of an astronomical telescope have focal lengths of **80cm** and **5cm** respectively. Calculate the magnifying power of this telescope and separation of its two lenses if arranged in normal adjustment. [**Answer: 16, 85cm**]

(iii) The objective and eyepiece of an astronomical telescope have focal lengths of **75cm** and **2·5cm** respectively. Calculate the magnifying power of this telescope and separation of its two lenses if the final image is a distance **25cm** in front of the eyepiece. [**Answer: 33, 77·273cm**]

(iv) An astronomical telescope has an objective and an eyepiece of focal lengths **80cm** and **5cm** respectively. If the lenses are arranged in such a way that the final image of a distant object which subtends an angle of **0·6°** at the objective is formed at a distance of **25cm** in front of the eyepiece, calculate the:

(a) angular magnification and the separation of the lenses in this setting

(b) size of the final image seen.

[**Answer: (a) 19·2, 84·17cm (b) 5·03cm**]

(v) Draw a ray diagram to show the action of a terrestrial telescope in normal adjustment. List one advantage and one disadvantage of this type of telescope.

5. (i) Draw a ray diagram to show the action of a Galilean telescope in normal adjustment, and derive an expression for its magnifying power in terms of the focal lengths f_o and f_E of the objective and eyepiece respectively.

(ii) Calculate the separation of the eyepiece and objective of a Galilean telescope in normal adjustment whose magnifying power is **20** and its eyepiece has a focal length of **5cm.**

[**Answer: 95cm**]

(iii) Draw a ray diagram to show the formation of the final image by a Galilean telescope at near point, and derive an expression for its magnifying power.

(iv) The objective and eyepiece of a Galilean telescope have focal lengths f_o and f_E respectively. Derive an expression for the magnifying power of this telescope if the final image is a distance **D** in front of the eyepiece.

(v) With the aid of a ray diagram, describe the action of a telescope made up of a converging and a diverging lens when used in normal adjustment. List one advantage and one disadvantage of this type of telescope.

6. (i) A convex lens of focal length **60cm** is arranged co-axially with a diverging lens of focal length **5cm,** to view a distant star.

(a) If the final image is at infinity, draw a ray diagram to show the formation of the Image of the star.

(b) Calculate the angular magnification obtained if the image of the star is formed at a distance of **25cm** in front of the eyepiece.

(c) List one advantage and one disadvantage of this type of arrangement over an astronomical telescope.

(ii) With the aid of a ray diagram, describe the structure and action of a reflecting telescope in normal adjustment.

(iii) State **two** advantages of a reflecting telescope over a refracting telescope.

[**Answer: (i) (b) 9·6**]

7. A small convex mirror is placed **100cm** from the pole and on the axis of a large concave mirror of radius of curvature **320cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

(ii) Suggest a practical application for the arrangement of mirrors in **a (i)** above**.**

(iii) Calculate the radius of curvature of the convex mirror

(b) If the distant object subtends an angle of $\mathbf{3 \times 10^{-3}}$ **radians** at the pole of the concave mirror, calculate the

(i) size of the real image that would have been formed at the focus of the concave mirror.

(ii) size of the image formed by the convex mirror

8. The objective of a compound microscope has a focal length of **1.0cm** while the eyepiece has a focal length of **5.0cm**. An object is placed at a distance of **1.1cm** in front of the objective. The distance of the eyepiece from the objective is adjusted so that the final image is **30cm** in front of the eyepiece. Find the distance between the lenses and the magnifying power of the microscope. **An(15.3cm, 69.8)**

9. A distant object subtending $6x10^{-3}$ radian is viewed with a reflecting telescope whose objective is a concave mirror of focal length **15m**. The reflected light is intercepted by a convex mirror placed **12cm** from the pole of the objective where there is a hole. The image is viewed with a convex lens of focal length **5cm** used as a magnifying glass producing a final image at infinity.

(a) Draw a ray diagram for this arrangement

(b) Calculate the:

(i) diameter of the real image that would be formed at the focus of the concave mirror.

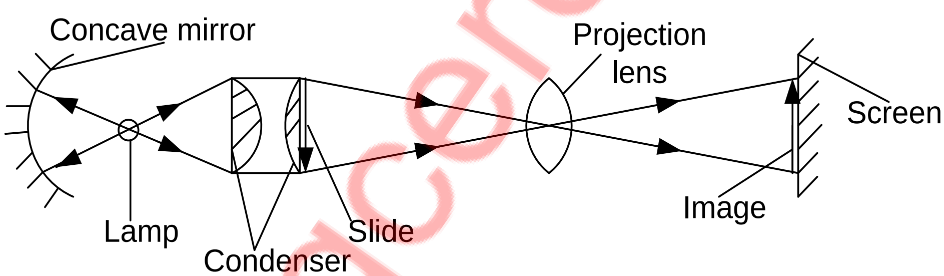
(ii) diameter of the image formed at the pole of the concave mirror.

(iii) angular magnification for the arrangement.

[**Answers: (a) (iii) 150cm (b) (i) 0·48cm (ii) 0·8cm**]

Projector Lantern

A projector is designed to throw on a screen a magnified image of a film or transparency. It consists of an illumination system and a projection lens.



A strong source of light is placed at the centre of curvature of the concave reflector. Light from the source is reflected back onto the condenser which concentrates it onto the slide. The projection lens forms a magnified real image of the slide on the screen.

Functions of the parts of the projector

(i) the concave reflector concentrates light from the source on to the condenser.

(ii) The condenser uniformly illuminates the object with light from the source. Or it concentrates light toward the object.

(iii) The slide bears the object to be projected to the screen
(iv) Projection lens focuses and magnifies an upright image on the screen
(v) Screen is where the image is viewed.

NOTE:

(i) A projector is used for showing a magnified image of the film or slides on the screen.

(ii) The film is the region where the slide is to be placed.

(iii) The magnification of the slide is given by $m = \frac{v}{u}$ or $m = \frac{v}{f} - 1$, where **v** and **u** are the respective screen and slide distances from the projection lens of focal length.**f.**

(iv) The above expression clearly shows that a high magnification can be produced by using a projection lens of short focal length **f** compared to distance **v.**

(v) For an enlarged image, $(linear\ scale\ factor)^2 = \frac{image\ Area}{Object\ Area}$

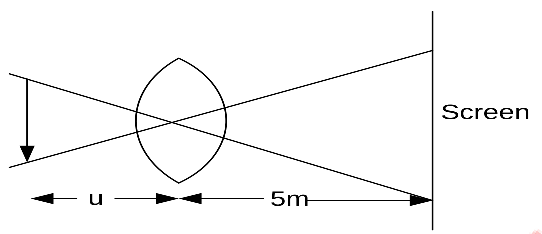
But Linear scale factor = magnification.

$\Rightarrow$ Magnification **m**= $\sqrt{\left(\frac{image\ Area}{Object\ Area}\right)}$

EXAMPLE:

1. A projector produces an image of area $1{\cdot}2m\ X 1{\cdot}8m$ onto a screen placed **5m** from the projector lens. If the area of the object slide is $2{\cdot}4cm\ X 3{\cdot}6cm$, Calculate the:
   (i) focal length of the projection lens.
   (ii) distance of the slide from the lens.

Solution:



(i) Image area = $1{\cdot}2m\ X 1{\cdot}8m = \mathbf{2{\cdot}16m^2}$

Object area = $2{\cdot}4cm\ X 3{\cdot}6cm$

$= 8{\cdot}64cm^2 = \mathbf{8{\cdot}64\ X 10^{-4} m^2}$

Image distance $v = \mathbf{5m}$

m= $\sqrt{\left(\frac{image\ Area}{Object\ Area}\right)}$

m= $\sqrt{\left(\frac{2.16}{8{\cdot}64\ X 10^{-4}}\right)} = \mathbf{50}$

$m = \frac{v}{f} - 1$

$50 = \frac{5}{f} - 1$

$\boldsymbol{f = 0.098m}$

$\therefore$ The required focal length $\boldsymbol{f = 0{\cdot}098m}$

OR $\boldsymbol{f = 9{\cdot}8cm}$

(ii) Using the relation $m = \frac{v}{u}$ gives

$$50 = \frac{5}{u}$$

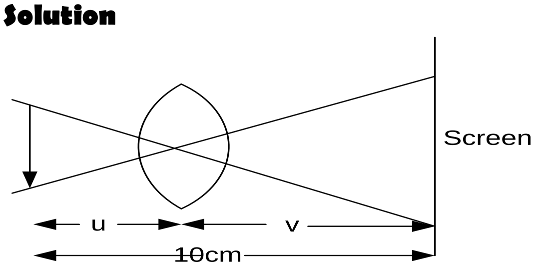
$$u = 0.1m$$

The distance of the slide from the lens **u = 0·1m**

OR u = 10cm

2. A projector is required to project slides which are **7·5cm** square onto a screen which is **4·2m** square. If the distance between the slide and the screen is **10m,** what focal length of the projection lens would you consider more suitable.

Solution



Object area = 75 cm square

Object area $= 7{\cdot}5\ X 7{\cdot}5cm^2$

$= \mathbf{7{\cdot}5\ X 7{\cdot}5\ X 10^{-4}} m^2$

Image area = $4{\cdot}2\ m\ square$

$= \mathbf{4{\cdot}2\ X 4{\cdot}2} m^2$

$\mathbf{m}=\sqrt{\left(\frac{image\ Area}{Object\ Area}\right)}$

$\mathbf{m}=\sqrt{\left(\frac{4{\cdot}2\ x4{\cdot}2}{7{\cdot}5\ x7{\cdot}5\ x10^{-4}}\right)} = 56$

$m = \frac{v}{u}$

$56 = \frac{v}{u}$

$\therefore \quad v = 56u$ ----------------------(i)

But $u + v = 10$ -------------------(ii)

Substituting equation **(i)** in **(ii)** gives

$u = 0{\cdot}1754m$ and $v = 9{\cdot}8246m$

$But \quad m = \frac{v}{f} - 1$

$56 = \frac{9.824}{f} - 1$

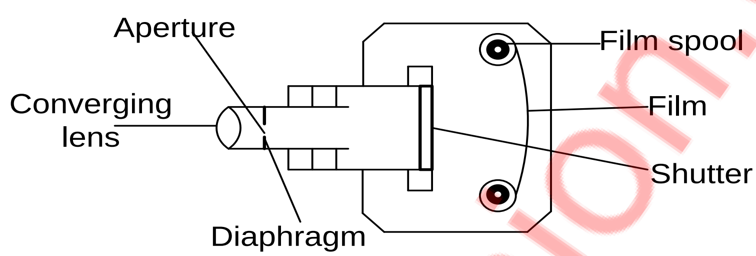
$f = 0.1724m$

$\therefore$ The required focal length $f = 0{\cdot}1724m$

OR $f = 17{\cdot}24cm$

The lens camera

A camera consists of a lens system, a light sensitive film at the back and a focusing arrangement D. The latter is used to adjust the distance of the lens for proper focusing.



The camera lens system refracts light from an object onto the film. The film is where the image is formed. The shutter cuts off light where necessary. The stopper regulates amount of light incident on the film. Diaphragm adjusts the size of the aperture which is an opening thru. Which light passes to reach the retina. Focusing ring focuses the image on the retina

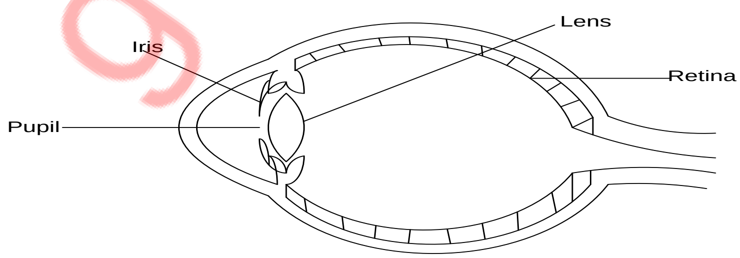
NOTE:

(i) Chromatic aberration in a photographic camera is minimized by use of an achromatic combination of two lenses one convex and the other concave to form an achromatic doublet.

(ii) Spherical aberration in a photographic camera is minimized by use of a shield with a small hole in the middle and lenses of small aperture.

The human eye

The eye is a light sensitive organ for vision in animals



Functions of parts of an eye

(i) Converging lens:

- Focuses the image of an object on to the retina

(ii) Iris

- ➢ Controls the amount of light entering the eye by controlling the size of the pupil

(iii) Pupil

- ➢ Circular opening through which light enters the eye

(iv) Retina

- ➢ Light sensitive part where image is formed

Similarities between the eye and the camera

- Both have a converging lens
- Both have light sensitive parts for image formation
- Both have openings that admits light
- Both have the ability to control the amount of light entering them

Differences between the eye and the camera

Eye	Camera
(i) Has natural lens with a variable focal length	(i) Has artificial lens with a fixed focal length
(ii) The image distance is fixed	The image distance is variable

Accommodation

Accommodation is the ability of the eye to focus an image of an object by altering the focal length of the lens.

Near point and far point

- ➢ The near point is the closest point at which the eye can focus clearly
- ➢ The far point is the furthest point at which the eye can focus clearly

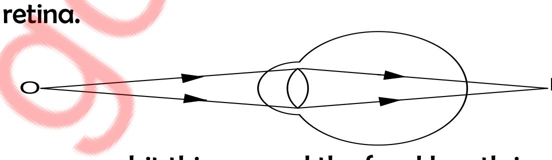
Note: for a normal eye the near point is 25cm and the far point is at infinity .

The eye defects and their corrections

Eye defects are caused by inability of accommodation.

(i) Long sightedness (hypermetropia)

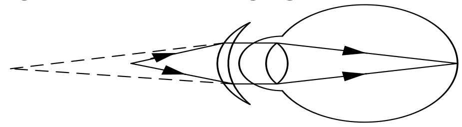
- ➢ A long sighted person sees only far objects clearly because their images are formed on the retina.
- ➢ A person with long sight defect can't see near objects clearly and this occurs when the image is formed beyond the retina.



- ➢ The lens of the eye becomes a bit thinner and the focal length increases and this causes all the images of nearer objects to fall behind the retina.

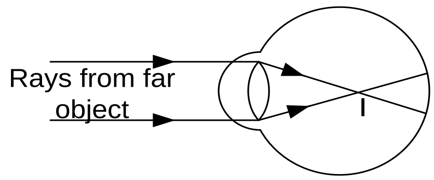
Correction of long sightedness

It is corrected by using a suitable converging lens called a converging meniscus



(ii) Short sightedness (myopia)

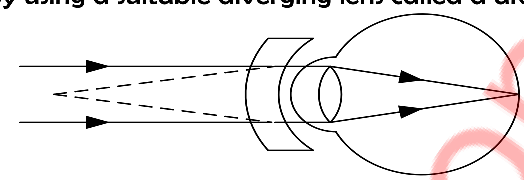
- A short sighted person sees only near objects clearly because their images are formed on the retina.
- A person with short sight defect can't see far objects clearly and this occurs when the image is formed in front of the retina.



- The lens of the eye becomes a bit thicker and the focal length decreases and this causes all the images of far objects to fall in front of the retina.

Correction of short sightedness

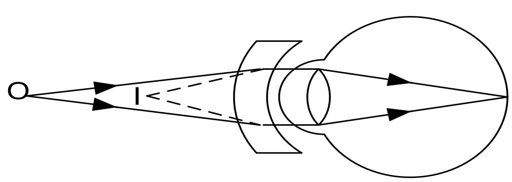
It is corrected by using a suitable diverging lens called a diverging meniscus



Example

1. A person with a normal near point distance of 25cm wears spectacles with a diverging lens of focal length 200cm in order to correct the far point distance to infinity. Calculate the near point distance when viewing using the spectacles

Solution



The object at O appears to be at I the near point for the eye.

$v = -25cm$ and $f = -200cm$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{u} = \frac{1}{-200} - \frac{1}{-25}$$

$$u - 28.6cm$$

EXERCISE:14

1. **(i)** Define the term **angular magnification** as applied to a microscope.

(ii) What is meant by the term **normal adjustment** as applied to a microscope?

(iii) Describe with the aid of a ray diagram, the action of a magnifying glass in normal adjustment. Hence derive an expression for its magnifying power in terms of the focal length **f** of the magnifying lens.

(iv) Describe with the aid of a ray diagram, the action of a magnifying glass in forming the final image at infinity. Hence show that in this case magnifying power $M = \frac{D}{f}$, where **f** is the focal length of the magnifying lens and **D** is the distance of most distinct vision.

(v) Explain why chromatic aberration is **not** observed in a simple microscope.

2. **(i)** State the reason why the focal lengths of the objective and eyepiece of a compound microscope are both small.

(ii). Describe with the aid of a ray diagram, the action of a compound microscope in normal - adjustment. Hence derive an expression for its magnifying power in terms of the focal lengths f_o and f_e of the objective and eyepiece respectively.

(iii). Describe with the aid of a ray diagram, the action of a compound microscope in forming the final image at infinity. Hence derive an expression for its angular magnification in terms of the focal lengths f_o and f_e of the objective and eyepiece respectively

(iv) State three differences between compound microscopes and telescopes.

3. **(i)** The objective of a compound microscope has a focal length of **5cm** while the eyepiece has a focal length of **4cm**. If the distance between them is **20cm** the final image of an object placed in front of the objective is formed **25cm** in front of the eyepiece. Calculate the position of the object and the magnifying power of the microscope.

(ii) The objective of a compound microscope has a focal length of **4·2cm**. An object placed **6cm** in front of the objective gives rise to a final image at a distance of **24cm** in front of the eyepiece and in the plane of the object when viewed with the eye close to the eyepiece. Calculate the:

(a) separation of the lenses.

(b) focal length of the eyepiece.

(c) angular magnification of the microscope.

[**Answers: (a) 18cm (b) 4·8cm (c) 14·6**]

(iii) An object of size 2·0mm is placed 3·0cm in front of the objective of a compound microscope. The objective of a compound microscope has a focal length of **2·5cm** while the eyepiece has a focal length of **5·0cm.** The microscope forms a virtual image of the object at the near point of the eye. Find the:

(a) size of the final image

(b) position of the eye-ring [**Answers: (a) 60mm (b) 6·85cm**]

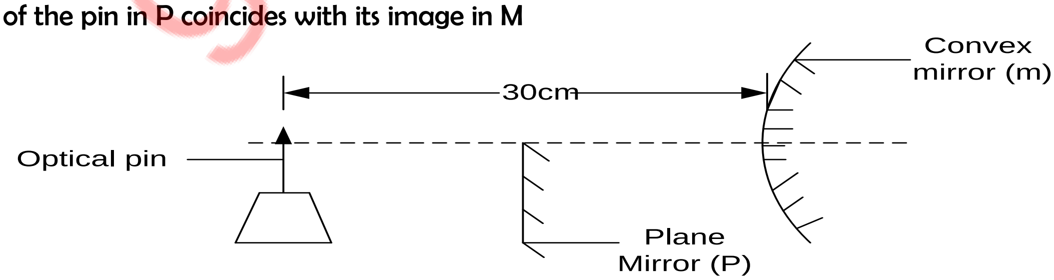
4. The objective of a compound microscope has a focal length of 2.0 cm while the eye piece has a focal length of 5.0 cm. An object is placed at a distance of 2.5 cm infront of the objective. The distance of the eyepiece from the objective is adjusted so that the final images is 25 cm infront of the eyepiece. Find the distance between the objective and eyepiece.

5. A projector is required to project slides which are **5.0cm** square onto a screen which is **5.0m** square. If the focal length of the projection lens is 0.1m. What should be the distance between the slide and the screen. **An(10.201m)**

Uneb 2016

1. (a) (i) Describe how the focal length of a convex mirror can be measured using a convex lens of a known focal length. (04marks)

(ii) The plane mirror, P in figure below is adjusted to a position 20cm from the optical pin, the image of the pin in P coincides with its image in M



Calculate the focal length of the convex mirror. (04marks)

(b) A pin is clamped horizontally above a concave mirror with its tip along the principal axis. When the pin is adjusted, it coincides with its image at a distance **R** from the mirror. When a small amunt of liquid of refractive index, n, is put in the mirror, the pin again coincides with its image at a distance R^1 from the mirror. Show that the refractive index, **n** is given by $n = \frac{R}{R^1}$ (04marks)

(c) (i) Explain the term **eye-ring** as applied to a telescope. (02marks)

(ii) Draw a ray diagram to show the formation of a final image in a Galilean telescope in normal adjustment. (04marks)

(iii) Explain **two** advantages and **one** disadvantage of the telescope in (c) (ii) (04marks)

2. (a) (i) When does light pass through a prism symmetrically (01mark)

(ii) Find the angle of incidence, i, on an equilateral prism of refractive index 1.5 placed in air, when light passes through it symmetrically (03marks)

(iii) Describe what happens to the deviation of light passing through the prism in (a) (ii) whe the angle of incidence is increased form a value less than i to a value greater than i. (02marks)

(b) Describe how the refracting angle of a prism can be determined using optical pins. (05marks)

(c) (i) Draw a sketch ray diagram showing formation of the image of a finite size real object by a concave lens. (02marks)

(ii) A concave lens of focal length 15.0cm is arranged cpaxially with a concave mirror of focal length 10.0cm, a distance of 4.0cm apart. An object is placed 20.0cm infornt if th lens, on the side remote from the mirror. Find the distance of the final image from the lens. (04marks)

(d) With the aid of a sketch ray diagram explain **spherical aberration** in concave lenses, and state how it is minimized. (04marks)

Uneb 2015

1. **(a)** Explain what is meant by **conjugate points .** (02marks)

(b) A converging lens forms an image of height h_1 on a screen, of an object O of height h. When the lens is displaced towards the screen, an image of height h_2 is formed.

(i) Sketch a ray diagram to show the formation of the images on the screen. (02marks)

(ii) Show that $h = h_1 h_2$ (04marks)

(c) Describe an experiment to determine the focal length of a diverging lens using a concave mirror of known focal length . (05marks)

(d) The objective of an astronomical telescope in normal adjustment has a diameter of 150mm and focal length of 3.0 m. the eye- piece has a focal length of 25.0mm. Calculate the;

(i) Positon of the eye- ring. (03marks)

(ii) Diameter of the eye-ring. (02marks)

(e) Give one advantage of placing the eye at the eye ring. (01mark)

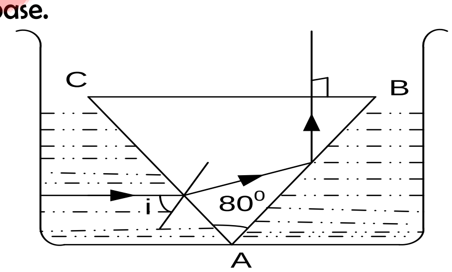
2. (a) Show that for a ray of light passing through layers of transparent media separated by parallel boundaries, $n \sin i = a$

Where **a** is a constant and **n** is the refractive index of the medium containing ***i*.** (04marks)

(b) (i) What is meant by critical angle . (01marks)

(ii) Describe an experiment to determine the critical angle for a water- air boundary.(05marks)

(c) Figure below shows an isosceles prism ABC of refractive index 1.51, dipped in a liquid with its refracting edge downwards. A ray of light incident on the prism at angle $i = 34.6°$ emerges perpendicularly through the base.



Calculate the refractive index of the liquid (04marks)

(d) Explain how an optical cable transmits light. (03marks)

(e) An optical pin held above a concave mirror containing water of refractive index 1.33, coincides with its image at a distance of 12.0cm aboeve the mirror. When the water is replaced by a little quantity of a cetain liquid, the point of coincidence of the object and the image become 13.3cm. Calculate the refractive index of the liquid. (02marks)

Uneb 2013

1. **(a)** distinguish between **a real image** and **a virtual image .** (02marks)

(b) Derive an expression relating the focal length, **f**, of a convex mirror to the object distance **u** and image **v**. (05marks)

(c) A concave mirror forms an image half the size of the object. The object is then moved towads the mirror until the image size is three quarters that if the object. If the image is mved by a distance if 0.6cm, calculate the

(i) focal length of the mirror. (03marks)

(ii) new positon of the object. (03marks)

(d) (i) What is **critical angles?.** (01mark)

(ii) Explain how a mirage is formed (04marks)

(e) State four applications of total internal reflection. (02marks)

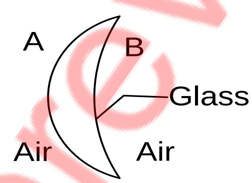
2. (a) State the **laws of refraction** (02marks)

(b) (i) The deviation, d, by a small angle prism of refracting angle A and refractive index,n, is given by $d = (n-1)A$.

Use this expression to show that the focal length, f, of a thin converging lens of refractive index, n, is given by $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Where r_1 and r_2 are radii of curvature of the lens surface (05marks)

(ii) Figure below a glass convex lens in air with surface A and B having radii of curvature 10cm and 15 cm respectively



If the refractive index if the glass material used is 1.50, calculate the power of the lens. (03marks)

(c) (i) With the aid of a ray diagram, describe the structure and action of a Galilean telescope in normal adjustment (05marks)

(ii) Derive an expression for the angular magnification of the telescope in (c) (i). (03marks)

(d) Explain the disadvantage of a Galilean telescope over the refracting type. (02marks)

Uneb 2012

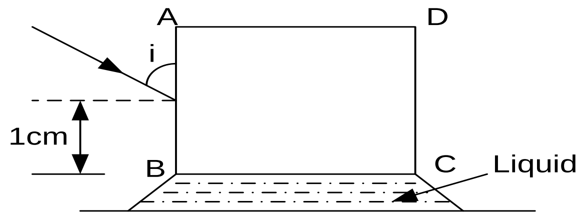
1. (a) (i) State the laws of refraction of light (02marks)

(ii) State the condition for total internal reflection to occur (02marks)

(b) (i) Describe an experiment to determine refractive index of a liquid using an air cell. (06marks)

(ii) Explain the difficulty encountered in the experiment describe in (b) (i) if white light is used. (02marks)

(C) A cube of glass of sides 3cm and refractive index 1.5 is palced on a thin film of liquid as shown below.



A ray of light in a vertical plane strikes side AB of the glass cube at an angle $i = 41°$. After refraction at X, the ray is reflected at the critical angle for the glass- liquid surface.

(i) Calculate the critical angle for the glass-liquid interface. (03marks)

(ii) Find the position from B where the ray strikes the glass-liquid interface. (02marks)

(d) Explain why the rays from the sun can still be seen shortly after sun set. (03marks)

2. (a) With the aid of a ray diagram, explain the following as applied to lenses.

(i) Conjugate points. (02marks)

(ii) Spherical aberration. (02marks)

(b) An object, **O**, placed in-front of a converging lens forms a real image, **I** on the screen. The distance between the object and its real image is, **d**, while that of the image from the lens is **x**. Derive the expression for the least distance between the object and its real image. (05marks)

(c) Give the properties of the lenses in an achromatic combination. (03marks)

(d) A compound microscope consists of two converging lenses of focal length 1.0 **cm** and **5.0cm** respectively . An object is placed at a distance of **1.1cm** in front of the objective. The microscope is adjusted so that the final image is **30cm** from the eyepiece. Calculate the;

(i) Separation of the lenses. (03marks)

(ii) Magnifying power of the lenses. (02marks)

(e) State two differences between a compound microscope and an astronomical telescope. (01mark)

Uneb 2011

1. (a) Define the following terms applied to a concave lens.

(i) Principal focus. (01marks)

(ii) Radii of curvature. (01marks)

(b) A pin object is placed a distance U infront of a diverging lens of focal length, f, to form an image at a distance V form the lens. Derive an expression relating, **u**, v and f. (04marks)

(c) Describe an experiment to determine the focal length of a concave lens using a palne mirror, a converging lens and an illuminated object. (04marks)

(d) What is meant by a

(i) Visual angle?. (01mark)

(ii) Near point?. (01mark)

(e) A person with a normal near point distance of 25cm wears spectacles with a diverging lens of focal length 200cm in order to correct the far point distance to infinity. Calculate the near point distance when viewing using the spectacle. (03marks)

(f) (i) Draw a ray diagram to show the formation of an image of a distant object in a terrestrial telescope in normal adjustment. (03marks)

(ii) State two disadavntages of the terrestrial telescope (02marks)

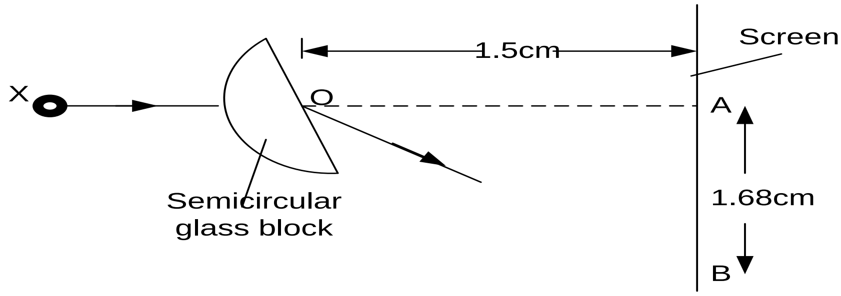
2. (a) What is meant by the term.

(i) Refraction?. (01marks)

(ii) Absolute refractive index?. (01marks)

(b) Describe an experiment to determine the refractive index of a liquid using a travelling microscope. (04marks)

(c) The figure below shows monochromatic light x incident towards a vertical screen



When the semi circular glass block is placed across the path of light with its flat face parallel to screen, bright spot is formed at A. when the glass block is rotated about horizontal axis through O, the bright spot moves downwards from A towards B then just disappears at B a distance 1.68cm from A.

(i) Find the refractive index of the material of glass block. (04marks)

(ii) Explain whether AB will be longer if the block of glass of higher refractive index was used . (02marks)

(d) (i) A ray of monochromatic light is incident at a small angle of incidence on a small angle prism in air. Obtain the expression $d = (n-1)A$ for the deviation, **d**, of light by the prism where **A** is the refracting angle of the prism and **n** the refractive index. (04marks)

(ii) Calculate the minimum deviation produced by a 60° glass prism if the refractive index of the glass is 1.50. (03marks)

(iii) State **two** applications of total internal reflection. (01mark)

Uneb 2010

1. (a) (i) Define principal focus of a concave lens. (01marks)

(ii) Draw a ray diagram to show formation of an image of a finite object by a concave lens. (02marks)

(iii) Describe the image formed in (a) (ii). (01marks)

(b) A concave mirror of radius of curvature 20cm s arranged coaxially with a concave lens of focal length 15cm, placed 10cm from the mirror. An object, 3cm tall is palced in front of the concave lens and its image is formed on a screen 440cm away form the lens.

(i) Find the position of the object. (07marks)

(ii) What is the height oof the image formed (03marks)

(iii) Explain what would happen if th lens was replaced with a similar one but of a much smaller focal length. (03marks)

(c) Explain how spherical aberration is minimized in a photographic camera. (03marks)

2. (a) Define **refractive index**. (01mark)

(b) (i) Describe with the aid of a diagram, how the refractive index of a liquid can be determined using an air cell.?. (05marks)

(ii) Derive the expression used to obtain the refractive index of the liquid in (b) (i) (03marks)

(c) A prism of refractive angle 60° has refractive index 1.515 and 1.529 for red and violet light respectively. When white light is incident on one face of the prism, red light undergoes minimum deviation. Calculate;

(i) Incidence for white light (04marks)

(ii) Emergence for violet light (03marks)

(d) Describe the adjustment that have to be made before a spectrometer can be used. (04marks)

Uneb 2009

1. (a) (i) Show that the effective focal length, f of two thin

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ where f_1 and f_2 are focal length of individual lenses. (04marks)

(ii) A compound lens consists of two lenses in contact having power of 12.5D and -2.5D. Find the position and nature of the image of an object placed 15.0cm from the compound lens.

(03marks)

(b) (i) Define **refractive index**. (01mark)

(ii) An equi-convex lens is placed on a horizontal plane mirror and a pin held vertically above the lens is found to coincide with its image when positioned 20.0cm above the lens. When a few drops of liquid is placed between the lens and the mirror, the pin had to be raised 10.0cm to obtain coincidence with the image. If the refractive index of the convex lens is 1.5, find the refractive index of the liquid. (05marks)

(C) (i) What is meant by **magnifying power** of an optical instrument. (01mark)

(ii) Derive an expression for the magnifying power of a compound microscope in normal adjustment (05marks)

(iii) Why should the objective and eye piece of a compound microscope have short focal lenght. (01mark)

2. (a) What is meant by the following terms as applied to 0ptics

(i) **Refraction?** (01mark)

(ii) **Critical angle?** (01mark)

(b) Show that the refractive index, n of a medium is given by

$$n = \frac{real\ depth}{apparent\ depth}$$ (04marks)

(c) A scratch is made at the bottom of a thick glass container which is filled with water. The scratch appears displaced by 0.5 cm when viewed from above the water. If the refractive indices of water and glass are 1.33 and 1.50 respectively, find the apparent displacement when water is removed and the scratch is again observed from above. (05marks)

(d) A ray of light incident at angle, i, on a prism of angle, A, passes through it symmetrically;

(i) Write the expression for the deviation, d, of the ray interms of i and A (01marks)

(ii) Find the value of d, if the angle of the prism is 60° and the refractive index of the glass is 1.48 (03marks)

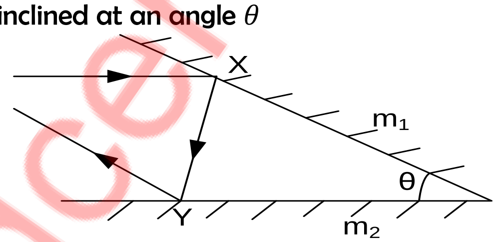
(e) Describe how you would determine experimentally the angle of minimum deviation produced by a prism. (05marks)

Uneb 2008

1. (a) (i) Distinguish between **a real image** and **a virtual image .** (02marks)

(ii) Describe how the position of an image in aplane mirror can be located. (03marks)

(b) The diagram below shows a ray of light undergoing two successive eflections at points X and Y in two mirrors M_1 and M_2 inclined at an angle θ



Show that the ray is deviated through an angle 2θ (05marks)

(c) (i) What is **radius of curvature** of a convex mirror. (01mark)

(ii) Describe the experiment to determine the focal length of a convex mirror using a plane mirror. (05marks)

(d) A small convex mirror is placed 0.60m from the pole and on the axis of a large concave mirror of radius of curvature 2.0m. The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole. Calculate the radius of curvature of the convex mirror. (04marks)

(e) State **four** advantages of reflecting telescope over the refracting type. (02marks)

2. (a) (i) Defien the term **linear** magnification and **angular** magnififcation as applied to a lens. (02marks)

(ii) Derive an expression for the magnifying power of a magnifying glass when the final image is formed at the near point. (04marks)

(b) An object is placed at a distance $f + x$ from a converging lens of focal length, f. The lens produces an image at a distance, $f + y$ form the lens. Show that $f^2 = xy$. (03marks)

(c) (i) Describe with the aid of a labelled diagram, the structure and operaton of a simple projection lantern (04marks)

(ii) The slide of a projection latern has dimensions 36mm by 24mm. What focal length of the projection lens is required to project an image 1.44m by 0.98m on a screen placed 4.0m from the lens. (04marks)

(d) Distinguish between **achromatic** and **spherical** aberration . (03marks)

Uneb 2007

1. (a) (i) State the **laws of reflection .** (02marks)

(ii) Show that the image formed in a plane mirror is as far behnd the mirror as the object is in front. (04marks)

(b) (i) Draw a ray diagram to show how a concave mirror forms a real image of a real object placed perpendicular to its principal axis. (01mark)

(ii) Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using the No- parallax method. (06marks)

(c) A concave mirror M of focal length 20.0cm is placed 90cm in front of a convex mirror, N of focal length 12.5cm. An object is placed on the common axis of M and N at a point 25.0cm in front of M.

(i) Determine the distance from N of the image formed by reflection, first in M and then in N (05marks)

(ii) Find the magnification of the image formed in (c) (i) above. (02marks)

2. (a) (i) What is meant by reversibility of light as applied to formation of a real image by a convex lens. (02marks)

(b) (i) Draw a ray diagram to show the action of an astronomical telescope in normal adjustment. (03marks)

(ii) Derive an expression for the magnifying power of the telescope in (b) (i) above in terms of the focal length f_o and f_e of objective and eye piece respectively (03marks)

(iii) The objective and eye piece of an astronomical telescope have focal length of 75.0cm and 2.5cm respectively. Find the separation of the two lenses if the final image is 25cm from the eyepiece. (04marks)

(c) (i) What is the significance of the eye-ring of an astronomical telescope. (02marks)

(ii) State **two** advantage of a reflecting telescope over a refracting telescope. (02marks)

(d) Explain why **achromatic** aberration is not observed in a simple microscope. (04marks)

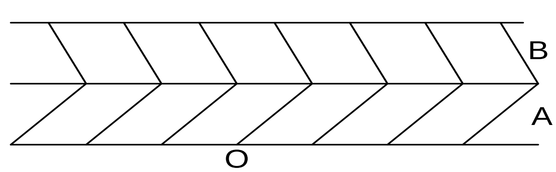
Uneb 2006

1. (a) (i) What is meant by **refraction of light.** (01marks)

(ii) State the **laws of reflection** (02marks)

(b) Describe how the refractive index of a liquid can be determined using a concave mirror.(05marks)

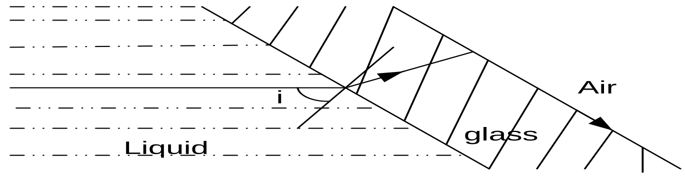
(c)



Two parallel sided blocks A and B of thickness 4.0cm and 5.0cm respectively are arranged such that A lies on an object O as hsown above. Calculate the apparent displacement of O when observed from directily above, if the refractive indices of A and B are 1.52 and 1.66 respectively. (05marks)

(d) (i) State **two** applications of total internal reflection. (02marks)

(ii)



In figure above, a parallel sided glass slide is in contact with a liquid on one side and air on the other side. A ray of light incident on the glass slide from the liquid emerges in air along the glass-air interface. Derive an expression for the absolute refractive index, n_l of aliquid in terms of absolute refractive index, n of glass and angle of incidence i (02marks)

2. (a) (i) Define **angular magnification** of a compound microscope. (01marks)

(ii) Draw a labelled ray diagram to show two converging lenses can be used to make a compound microscope in normal adjustment. (03marks)

(b) An object of size 2.0mm is placed 3.0cm in front of the objective of a compound microscope. The focal length of the objective is 2.5cm while that of the eye-piece is 5.0cm. The microscope forms a virtual image of the object at the near point of the eye. Find the;

(i) size of the final image (05marks)

(ii) position of the eye-piece (04marks)

(c) (i) With the aid of a labelled diagram, describe the essential parts of a photographic camera. (05marks)

(ii) Explain how **achromatic** and **spherical** aberration are minimized in the photographic camera. (02marks)

SECTION B: WAVES AND WAVE MOTION

A wave is any disturbance from an equilibrium position that travels with time from one region to another.

Wave motion is a means of transfer of energy from one point to another without the transfer of matter between the points

Waves are classified into two namely;

(i) Electromagnetic waves

(ii) Mechanical waves

Mechanical waves

These are waves that are produced by the disturbances of a material medium and are transmitted by the particles of the medium oscillating to and fro

Mechanical waves require a material medium for their transmission. They can be felt and seen

Examples of mechanical waves

- Sound waves
- Water waves
- Waves on compressed springs
- Waves on stretched string

Electromagnetic waves

These are waves produced by disturbances of varying electric and magnetic fields

They do not require a material medium for their transmission and travel in a vacuum.

Examples of electromagnetic waves

- Light waves
- $\gamma - rays$
- Radio waves
- All other electromagnetic band waves

Note:

All electromagnetic waves travel at a speed of light $ie\ 3x10^8 m/s$

Properties of electromagnetic waves

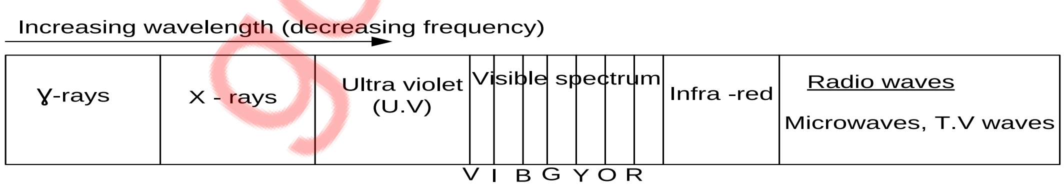
(i) Electromagnetic waves travel in a vacuum and therefore do not require a material medium for their transmission.

(ii) Electromagnetic waves travel at a speed of light $ie\ 3x10^3 ms^{-1}$

(iii) They are made of varying electric and magnetic vibration.

(iv) They vibrate with a high frequency.

(v) They have no charge.



Differences between mechanical and electromagnetic waves

Mechanical waves	Electromagnetic waves
❖ Need a material medium for their transmission	❖ Can propagate in vacuum
❖ Propagate at relatively low speeds	❖ Propagate at high speeds
❖ Have longer wavelength	❖ Have shorter wavelength
❖ Are due of vibrations of particles in the transmitting medium	❖ Are due of vibrations of particles in the transmitting medium

Types of waves

There are two types of wave motion namely

- ❖ Transverse waves
- ❖ Longitudinal waves

Transverse waves

These are waves in which displacement of the particles in the medium is perpendicular to the direction of wave travel.

Transverse waves are characterized by crests and troughs

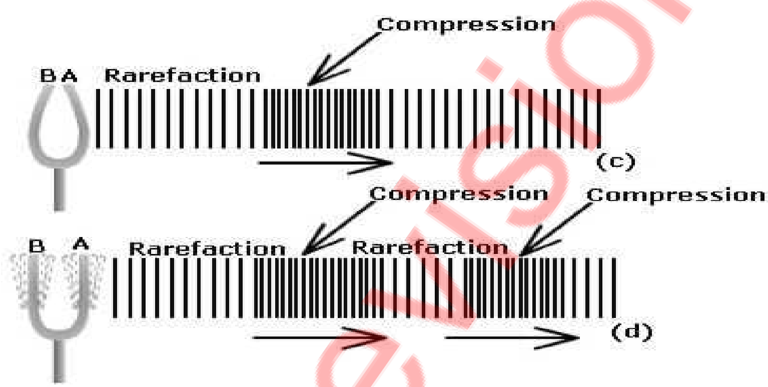
- ❖ Crest is the part of the wave above the line of zero disturbance
- ❖ Trough is the part of the wave below the line of zero disturbance

Examples

- Water waves
- Waves due plucked strings
- Light waves
- All electromagnetic waves ($eg\ \gamma - rays, X - rays$)

Longitudinal waves

These are waves in which the displacement of the particles is parallel to the direction of travel of the wave.



Longitudinal waves are characterized by compressions and rare factions

- ❖ Compressions are regions of high particle density in wave
- ❖ Rare factions are regions of low particle density in wave

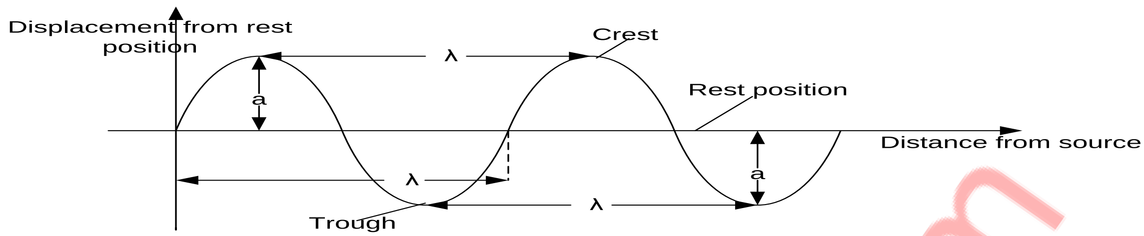
Examples

- Sound waves
- Waves on a compressed spring

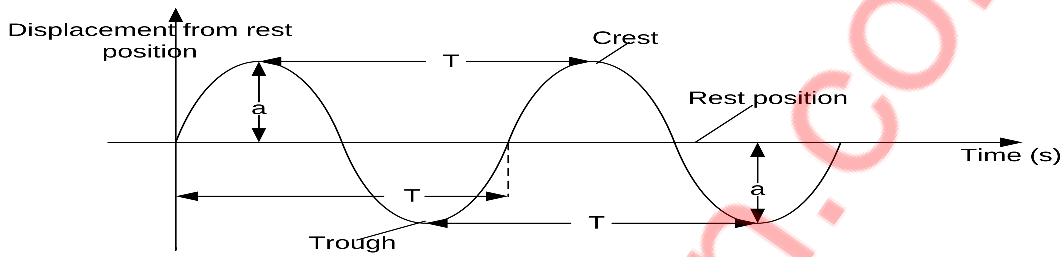
Differences between transverse and longitudinal waves

Transverse waves	Longitudinal waves
Particles vibrate at right angles to the direction of travel of the wave	Particles vibrate along the direction of travel of the wave
Transverse waves are represented by crests and troughs	longitudinal waves are represented by compression and rare faction regions

Representation of a wave



A displacement time graph can also be drawn



Terms used

(1) Amplitude (a)

This is the maximum displacement of a particle of a medium from its equilibrium position.

(2) Wave length (λ)

This is the distance between two successive particles in phase.
Wave length of **a transverse** wave is the distance between two successive crests or successive troughs.

(3) Oscillation or cycle

This is a complete to and fro movement of a wave particle in a medium

(4) Period (T)

This is the time taken for one particle to under one complete oscillation.
Or the time taken for a wave to travel a distance of one wavelength

$$\boxed{T = \frac{1}{f}}$$

Period T is measured in seconds

(5) Frequency (f)

The number of complete oscillations a wave particle makes in one second

$$\boxed{f = \frac{1}{T}}$$

The S.I unit of frequency is Hertz (Hz)

(6) Phase

Particles are in phase when they are exactly at the same point in their paths and are moving in the same direction

(7) Wave front

Is any section through an advancing wave in which all the particles are in the phase.

(8) **A ray.** This is the direction of an advancing wave

(9) Speed (V) of the wave

This is the linear distance travelled by a wave per unit time

$$V = \frac{linear\ distance}{time\ taken}$$

Since one complete wave is produced in time **T** and the length of one complete wave is λ

$$V = \frac{\boldsymbol{\lambda}}{T}$$

$$V = \frac{\mathbf{1}}{T} x\ \boldsymbol{\lambda}$$

But $f = \frac{1}{T}$

$$\boxed{V = f\ \boldsymbol{\lambda}}$$

EXAMPLES

1. Sanyu Fm broadcasts at a frequency of $88.2MHz$. Calculate the wavelength of the radio waves.

Solution

Note: All electromagnetic waves eg radio waves travel at a speed of light $3x10^8 m/s$

$f = 88.2MHz = 88.2x10^6 Hz,$

$v = 3x10^8 m/s$

$v = f\ \lambda$

$3x10^8 = 88.2x10^6\ \lambda$

$\lambda = \frac{3x10^8}{88.2x10^6}$

$\lambda = 3.4m$

2. A vibrator produces waves which travel a distance of $12m$ in $4s$. If the frequency of the vibrator is $2Hz$, what is the wavelength of the wave?

Solution

$f = 2Hz, t = 4s, distance = 12m$

$v = \frac{distance}{time} = \frac{12}{4}$

$v = 3m/s$

$v = f\ \lambda$

$3 = 2\ \lambda$

$\lambda = \frac{3}{2}$

$\lambda = 1.5m$

3. A vibrator has a period of 0.02s and produces circular waves of water in a tank. If the distance between any two consecutive crests is 3cm, what is the speed of the wave?

Solution

$T = 0.02\ s$,

But $f = \frac{1}{T}$

$f = \frac{1}{0.02}$

$f = 50Hz$

$\lambda = 3cm$, $\lambda = 0.03m$

$v = f\ \lambda$

$v = 50x\ 0.03$

$v = 1.5m/s$

4. Water waves are produced at a frequency of 50Hz and the distance between 10 successive troughs is 18cm. Calculate the velocity of the waves.

Solution

$f = 50Hz$

$9\ \lambda = 18\ cm, \lambda = \frac{18}{9} cm$

$\lambda = 2cm, \lambda = 0.02\ m$

$v = f\ \lambda$

$v = 50x\ 0.02$

$v = 1m/s$

Properties of waves

All waves can be;

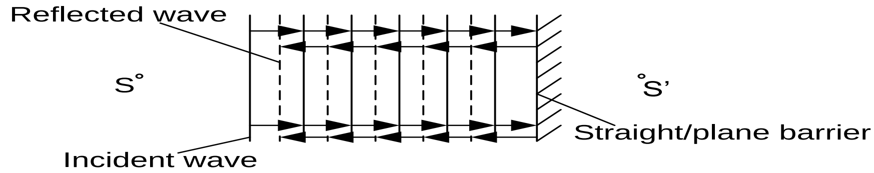
1. Reflected
2. Refracted
3. Diffracted
4. Interfered

1. REFLECTION OF WAVES

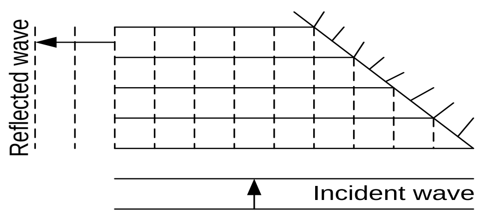
This is the bouncing back of waves when they meet a barrier

a) Plane reflector

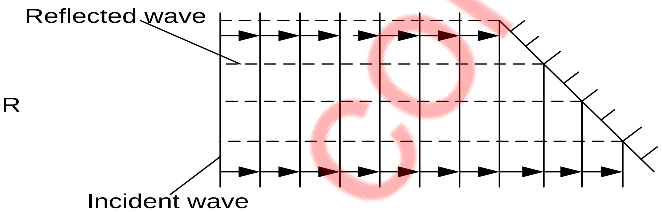
(i) Straight waves incident on a plane reflector



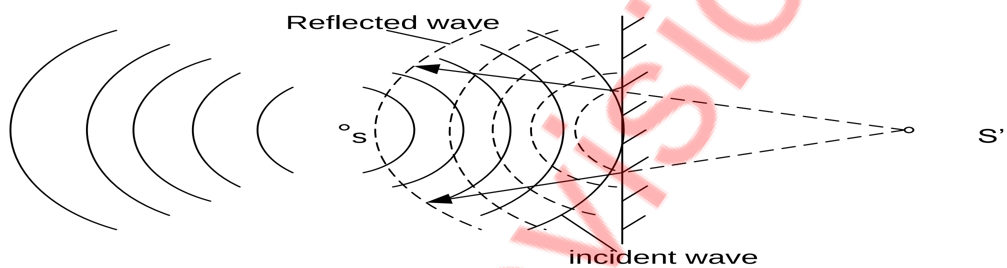
(ii) Straight waves incident on an inclined Plane surface



OR

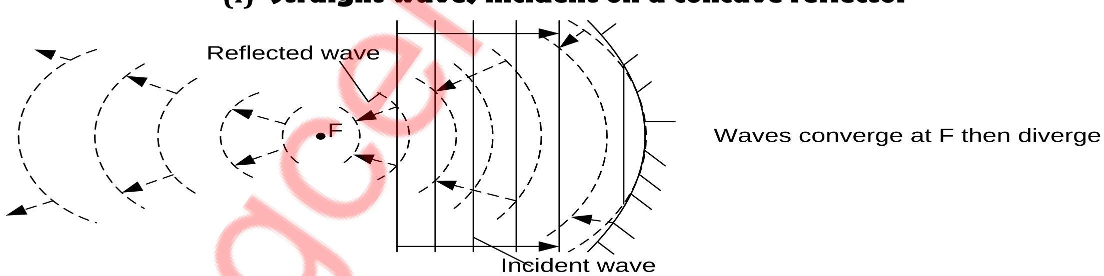


(iii) Circular waves incident on a plane reflector

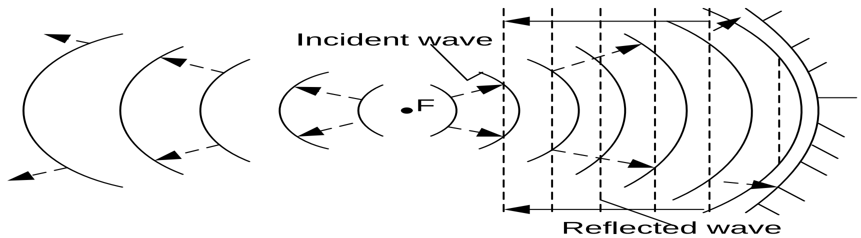


b) Concave reflector

(i) Straight waves incident on a concave reflector

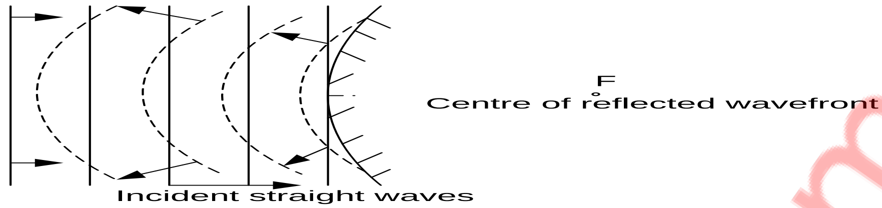


(ii) Circular waves on a concave reflector

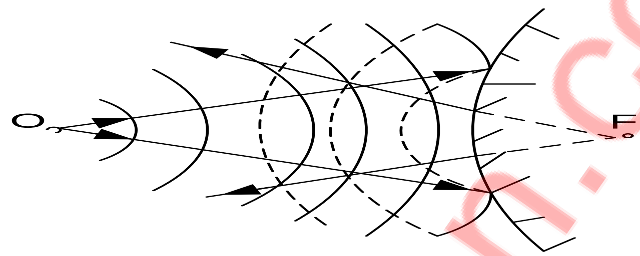


c) Convex reflector

(i) Plane waves incident on a convex reflector



(ii) Circular waves incident on a convex reflector



PROGRESSIVE WAVES

It is a wave in which the disturbance moves from the source to the surrounding places and energy is transferred from one point to another along the wave form

Examples

- Water waves
- All electromagnetic waves

Note: All transverse and longitudinal waves are progressive and the amplitude of a progressive wave is constant

Energy transmitted by a wave

In a progressive wave energy propagates through the medium in the direction in which the wave travels. So every particle in the medium possesses energy due to vibrations. This energy is passed on to the neighboring particles so for any system vibrating in form of simple harmonic motion, the energy of the vibrating particle changes from kinetic to potential energy and back but the total mechanical energy on the wave remains constant.

$$k.e = \frac{1}{2}mv^2$$

But $v = \omega A$

$$k.e = \frac{1}{2}m(\omega A)^2$$

$$\omega = 2\pi f$$

$$k.e = \frac{1}{2}m(2\pi f A)^2$$

$$\boxed{K.E = 2\pi^2 f^2 A^2 m}$$

Where f- frequency

A- Amplitude

M- mass of vibrating particle

$$Also\ energy\ per\ unit\ volume = \frac{E}{V}$$

$$\frac{E}{V} = \frac{2\pi^2 f^2 A^2 m}{\left(\frac{m}{\rho}\right)}$$

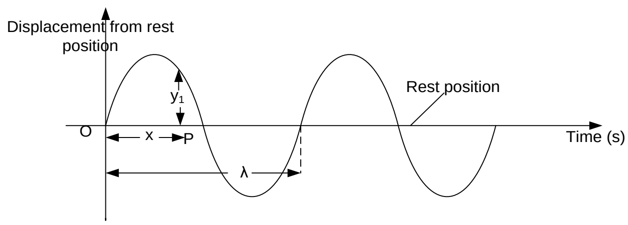
$$\boxed{Energy\ per\ unit\ volume = 2\pi^2 f^2 A^2 \rho}$$

Intensity of wave

This is the rate of flow of energy through an area of 1 m^2 perpendicular to the path of travel of wave

Equation of a progressive wave

Consider a wave form below



if the oscillation of the particle at O is simple harmonic with frequency f and angular velocity ω then its displacement y with time is given by

$$\boxed{y = asin\omega t} \ldots\ldots\ldots\ldots\ldots (1)$$

Suppose the wave generated travels towards the right, the particle at P a distance x form O will lag behind by a phase angle φ

$$\boxed{y_1 = asin(\omega t - \varphi)} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From the figure above, the phase angle of $2\pi = \lambda$ and phase angle $\varphi = x$

$2\pi = \lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

$\varphi = x \ldots\ldots\ldots\ldots\ldots (2)$

$$\frac{\varphi}{2\pi} = \frac{x}{\lambda}$$

$$\boxed{\varphi = \frac{2\pi x}{\lambda}}$$

Equation 2 will become

$$y_1 = asin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

But $\omega = 2\pi f = \frac{2\pi}{T}$

$$y_1 = asin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

$$y_1 = asin\ 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Generally for a wave travelling to the right the equation of a progressive wave is $\boxed{y = asin\ 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)}$

Note;

If the wave is travelling to the left it arrives at P before O. This makes the vibration at P to lead the vibrations at O and its equation is given by $\boxed{y = asin\ 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)}$

Examples

1. A displacement of travelling wave in the direction x- direction is given by $y = asin\ 2\pi\left(\frac{t}{0.5} - \frac{x}{0.2}\right) m$

Find the speed of the wave

Solution

$$y = asin\ 2\pi\left(\frac{t}{0.5} - \frac{x}{0.2}\right)$$

Compare with $y = asin\ 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$T = 0.5s$

$\lambda = 0.2m$

$v = f\lambda$

$v = \frac{1}{0.5} x 0.2$

$v = 0.4m/s$

2. A sound wave propagating in the x- direction is given by $y = 0.4sin\left[10\left(200t - \frac{x}{100}\right)\right] m$

Find the speed of the wave

Solution

$y = 0.4sin\left[10\left(200t - \frac{x}{100}\right)\right]$ Compare with

$y = asin\, 2\pi\left(ft - \frac{x}{\lambda}\right)$

$2\pi ft = 10x200t$

$f = 318.5Hz$

$\frac{10x}{100} = \frac{2\pi x}{\lambda}$

$\lambda = 62.8m$

$v = f\lambda$

$v = 318.5x62.8$

$v = 2.0x10^4 m/s$

3. The displacement y in meters of a plain pThe displacement y in meters of a plain progressive wave is given by $y = asin\, 2\pi\left(100t - \frac{20x}{17}\right)$. Find the wavelength of the wave and the speed of the wave

Solution

$y = asin\, 2\pi\left(100t - \frac{20x}{17}\right)$.

Compare with $y = asin\, 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$\frac{1}{T} = 100$

$\frac{1}{\lambda} = \frac{10}{17}$

$\lambda = 1.7m$

$v = f\lambda$

$v = 100x1.7$

$v = 170m/s$

Exercise

1. The displacement of a particle in a progressive wave is
   $y = 2\, sin\, [2\pi(0.25\, x - 100\, t)]$, where x and y are in cm and t is in seconds. Calculate the :
   (i) wave length,
   (ii) velocity of propagation of the wave **An(**$\lambda = 4.0cm, V = 4ms^{-1}$**)**
2. The displacement y given of a wave travelling in the x- direction at time t is:
   $y = asin\, 2\pi\left(\frac{t}{0.1} - \frac{x}{2.0}\right)$ meter
   Find
   (i) the velocity of the wave
   (ii) the period of the wave **An(**$T = 0.1\, s, V = 20ms^{-1}$**)**
3. A plane progressive wave is given by
   $y = a\, sin\, (100\, \pi t - \frac{10}{9}\, \pi x\,)$ where x and y are millimetres and t is in seconds
   Calculate the :
   (i) wave length,
   (ii) velocity of propagation
   (iii) period T of the wave
4. The displacement in metres of a plane progressive wave is given by the equation
   $$y = 0.5sin\left[\pi\left(200t - \frac{20x}{17}\right)\right]$$
   Find
   (i) wavelength and
   (ii) speed, of the wave
5. A progressive wave is represented by the equation $y = 0.1sin\left(100\pi t - \frac{10\pi x}{9}\right)$mm. find
   (i) Amplitude of the wave. **An(0.1mm)**
   (ii) Frequency **An(50Hz)**
   (iii) Wavelength **An(1.8mm)**
   (iv) Speed of the wave **An(0.09m/s)**
   (v) Speed of particles in the wave motion **An(use** $v = \omega a$**)**
   (vi) Phase difference between a point 0.245m from O and point 1.10m from O.
   An(use $\varphi = \frac{2\pi x}{\lambda}$ **)**

6. A progressive and a stationary wave each has a frequency of 240 Hz and a speed of 80 ms $^{-1}$. Calculate
   (i) phase difference between two vibrating points in the progressive wave which are 6 cm apart
   (ii) distance between modes in the stationary wave
7. A wave of amplitude 0.2m, wavelength 2.0m and frequency 50Hz. If the initial displacement is zero at point x = 0
   (i) write the expression for the displacement of the wave at any time t.
   (ii) find the speed of the wave
8. Two waves of frequencies 256 Hz and 280 Hz respectively travel with a speed of 340 m s – 1 through a medium. Find the phase difference at a point 2.0m from where they were initially in phase

STATIONARY WAVE / STANDING WAVE

This a wave formed as a result of superposition of two progressive waves of equal amplitude and frequency but travelling at same speed in opposite direction.
Therefore in a stationary wave, energy does not move along with the wave.
Stationary waves are characterized by node (N) and antinodes (A)

Formation of a stationary wave

Stationary waves are formed when two waves of equal frequency and amplitude travelling at same speed in opposite direction are supposed resulting into formation of node and antinode
At antinodes, waves meet in phase and the amplitude is maximum. At nodes, the wave meet antiphase and amplitude is minimum.

Condition for stationary waves to be formed

- Waves must be moving in opposite direction.
- Waves must have the same speed, same frequency and equal amplitude.

Equation of a stationary wave

Consider a progressive wave travelling to the right. The displacement of any particle of the medium is given by

$$y_1 = asin(\omega t - \varphi)$$

When this wave is reflected, it travels to the left. The displacement of any particle of medium will be

$$y_1 = asin(\omega t + \varphi)$$

When the two waves superpose, the resultant displacement I given by $y = y_1 + y_2$

$$y = asin(\omega t - \varphi) + asin(\omega t + \varphi)$$

$$\boxed{y = 2acos\varphi\ sin\omega t}$$

Where amplitude of vibration is $2acos\varphi$

Where $\varphi = \frac{2\pi x}{\lambda}$

Note: amplitude of a stationary wave varies with x hence its not constant

Principle of super position of waves

It states that for two wave travelling in the same region, the total displacement at any point is equal to the vector sum of their displacement at that point when the two waves over lap

Examples

A plane progressive wave is given by

$y = a\ sin\ (100\ \pi t - \frac{10}{9}\ \pi x\ )$ where x and y are millimetres and t is in seconds

(i) write the equation of the progressive wave which would give rise to a stationary wave if superimposed on the one above (1 mark)
(ii) find the equation of the stationary wave and hence determine its amplitude of vibration (3 marks)
(iii) determine the frequency and velocity of the stationary wave (4 marks)

solution

(i) $y_2 = a \sin(100\pi t + \frac{10}{9}\pi x)$

(ii) $y = y_1 + y_2$

$$y = a \sin\left(100\pi t - \frac{10}{9}\pi x\right) + a \sin(100\pi t + \frac{10}{9}\pi x)$$

$$y = 2a \cos(\frac{10}{9}\pi x)\sin(100\pi t)$$

Amplitude of vibration $is\ 2a \cos(\frac{10}{9}\pi x)$

(iii) $2\pi f = 100\pi$

$$f = 50Hz$$
$$v = 0ms^{-1}$$

Differences between stationary waves and progressive waves

Stationary waves	Progressive waves
1. Amplitude of the particles in the medium varies with position along the wave	1. All particles in the transmitting medium oscillate with the same amplitude.
2. Wave energy is not transferred but confined to a particular section of a wave	2. Wave energy is transferred form one point to another along the wave
3. Distance between any two successive nodes or antinodes is equal to $\frac{\lambda}{2}$	3. Distance between any two successive crests or troughs is equal to λ
4. Has nodes and antinodes	4. Doesn't have nodes and antinodes

MECHANICAL OSCILLATION

There are three types of oscillation i.e.

a) Free oscillation b) Damped oscillation c) Forced oscillation

a) Free oscillations

These are oscillations in which the energy of the system remains constant and is not lost to the surrounding. The amplitude of oscillation remains constant with time.

b) Damped oscillations

These are oscillations in which energy of oscillating system loses energy to the surrounding as a result of dissipative forces acting on it. Amplitude of oscillation decreases with time.

C) FORCED OSCILLATIONS

These are oscillations where the system is subjected to a periodic force which sets the system into oscillation. When the periodic force has the same frequency as the natural frequency of the oscillating system then resonance occurs.

SOUND WAVES

Sound is any mechanical vibration whose frequency lies within the audible range.
Sound waves propagate in air by series of compressions and rare factions.

Explain why sound propagates as an adiabatic process

Sound waves propagate in air by series of compressions and rare factions. In compressions the temperature of air rises unless heat is withdrawn. In rare factions, there is a decrease in temperature. The compressions and rare factions occur so fast that heat does not enter or leave the wave. Hence the process is adiabatic.

Characteristic of sound

a) Pitch

This is the characteristic of sound by which the ear assigns a place on a musical scale.
Pitch depends on the frequency of vibration of the sound waves ie it increases as the frequency of sound increases.

b) Loudness

This is the magnitude of the auditory sensation produced by sound.
Or Amount of sound energy entering the ear per second.

Factors that affect Loudness

- Sound intensity
- Amplitude of sound.

ECHOES

An echo is a reflected sound.
The time that elapses between hearing the original sound and hearing the echo depends on;

a) The distance away from the reflecting surface.
b) The speed of sound in the medium.

REVERBERATION

When sound is reflected from a hard surface close to the observer, the echo follows the incident sound so closely that the observer may not be able to distinguish between the two. Instead the observer gets an impression or hears a prolonged original sound. This effect is referred to as reverberation

Briefly explain why reverberation is necessary while making speeches

Too short a reverberation time makes a room sound dead but if it is too long, confusion results. For speeches half a second is acceptable. Reverberation time is made the same irrespective of the size of the audience b lining the walls with a soft material so that there is reduced reflection of sound

Refraction of sound

This is the change in the speed of sound waves as they move from one medium to another of different optical densities.

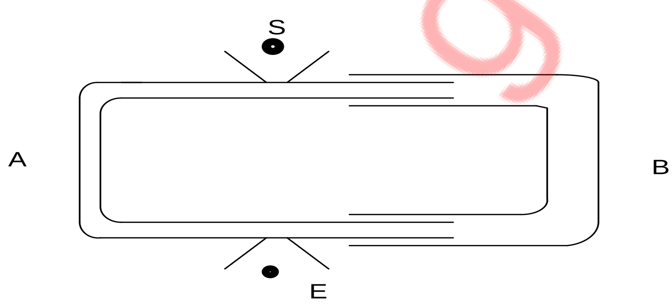
Explain why sound is easily heard at night than during day time

Distant sounds are more audible at night than day because the speed of sound in warm air exceeds that in the cold air and refraction occurs. At night the air is usually colder near the ground than it is higher up and refraction towards the earth occurs. During the day, the air is usually warmer near the ground than it is higher up

Interfernce of sound

Interference of waves is the superposition of waves from different two coherent soruces resulting into alternate regions of maximum and minimium intensity.

Experiment to show interference of longitudinal waves



- Tube A is fixed while B is free to move.
- A note is sounded at S and detected at E.
- Tube B is then **pulled out slowly**. It is noted that the sound detected at E increases to a maximum and reduces to a minimu in intensity at **equal intervals of length** of the tube.
- The alternate maximum and minimum intensity of sound are interference patterns

Differences between sound and light waves

Sound waves	Light waves
- They cant travel through a vacuum	- They can travel through a vacuum
- They travel at a low speed i.e 330m/s	- They travel at a high speed i.e $3x10^8$m/s
- Require a material medium for their transmission	-Do not require a material medium for their transmission
- They cant eject electrons from a metal surface	- They can eject electrons from a metal surface by photo electric emission
- They are longitudinal waves	- They are transverse waves

BEATS

A beat is aperiodic rise and fall in the intensity of sound heard when two notes of nearly equal frequency but similar amplitude are sounded together.

Formation of beats

When two waves of nearly equal frequency and similar amplitude are sounded together they superpose.
When they meet in phase constructive interference takes place and a loud sound is heard. When they meet out of phase destructive interference takes place and a soft sound is heard.
A periodic rise and fall in intensity of sound is heard which is called beats

Beat frequency

Its defined as the number of intense sounds heard per second

Derivation of Beat frequency

Let f_1 and f_2 be frequencies of two sound notes.
Suppose a note of frequency f_1 makes one cycle more than other in time T.
The number of cycles of frequency $f_1 = f_1T$
The number of cycles of frequency $f_2 = f_2T$

$$f_1T - f_2T = 1$$

$$(f_1 - f_2)T = 1$$

$$\frac{1}{T} = (f_1 - f_2)$$

But $\frac{1}{T} = f$

$\boxed{f = f_1 - f_2}$ This is called beat frequency

Uses of frequency

- Used in measurement of frequency of a note
- Determination of frequency of a musical note
- Tuning an instrument to a given note

Measurement of frequency of a note

- A note is sounded together with a tuning fork of known frequency, f_T
- The number of beats, n in t seconds are counted and the beat frequency, $f_b = \frac{n}{t}$ calculated

- One prong of the tuning fork is loaded with plasticine and then the experiment repeated. The new beat frequency $f_b{}^1$ is determined
- If $f_b{}^1 < f_b$ then the frequency of the test note f_n is calculated from $f_n = f_T + f_b$
- If $f_b{}^1 > f_b{}^1$ then the frequency of the test note f_n is calculated from $f_n = f_T - f_b$

Examples

1. Two tuning forks of frequency 256Hz and 250Hz respectively are sounded together in air. Find the number of beats per second produced

   Solution

   $f = f_1 - f_2$ | $f = 256 - 250$ | $\boldsymbol{f = 6\ Hz}$

2. Two sources, one with known freq 224Hz and the other unknown are sounded together .the beat freq recorded is 6Hz. When the unknown source is sounded again together with another known source of 250Hz the beat freq is 20Hz find the unknown freq of the 2nd source

   Solution

   In 1st case possible freq for 2nd source is either 224 – 6 =218 or 224 + 6 = **230**Hz

   In case 2 possible freq for 2nd source is either 250 – 20 = **230** or 250 +20 = 270Hz

   The common answer in both cases is 230Hz which is the freq of the 2nd source

3. Two whistles are sounded simultaneously. The wavelength sound emitted are 5.5m and 6.0m. find; (speed of sound in air 330m/s)
   (a) Beat frequency
   (b) Beat period

   Solution

   $f = f_1 - f_2$

   $f = \frac{v}{\lambda_1} - \frac{v}{\lambda_1}$

   $f = \frac{330}{5.5} - \frac{330}{6}$

   $f = 60 - 55$

   $\boldsymbol{f = 5\ Hz}$

   Beat period $= \frac{1}{f}$

   Beat period $= \frac{1}{5} = 0.2s$

4. Two sources of sound are vibrating simultaneously with frequency of 200Hz and 240Hz. If the speed of sound in air is 340m/s
   (i) How many beats are heard
   (ii) What is the distance between successive locations of maximum intensity

   Solution

   (i) $f = f_1 - f_2$

   $f = 240 - 200$

   $\boldsymbol{f = 40\ Hz}$

   (ii) $\lambda = \frac{v}{f}$

   $\lambda = \frac{v}{f}$

   $\lambda = \frac{340}{40}$

   $\lambda = 8.5m$

5. Two tuning forks x and y are sounded in air to produce beats of frequency 8Hz. Fork x has a known frequency of 512Hz. When y is loaded with a small piece of plasticine, beats of frequency 2Hz are heard when the two forks are sounded together. Find the frequency of y when it is unloaded.

   Solution

   $f = f_x - f_y$

   $8 = 512 - f_y$

   $\boldsymbol{f_y = 504\ Hz}$

   or

   $f = f_y - f_x$

   $8 = f_y - 512$

   $\boldsymbol{f_y = 520\ Hz}$

   When the fork is unloaded, the frequency of y is 520Hz, since beat frequency reduces

6. Two tuning forks A and B produce three beats per second when sounded together. If fork A has a frequency of 512Hz.
   (a) What is the possible frequency of B
   (b) Explain how you determine the actual frequency of tuning fork B

Solution

(a) $f = f_x - f_y$

$3 = 512 - f_y$

$f_y = \mathbf{509\ Hz}$

or

$f = f_y - f_x$

$3 = f_y - 512$

$f_y = \mathbf{515Hz}$

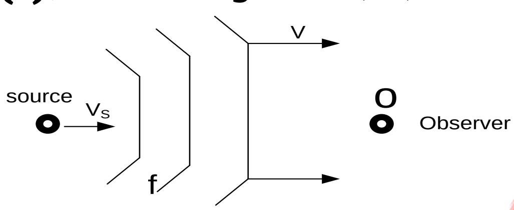
Attach a piece of plasticine on B and determine the new beat frequency by sounding A and B together. If the beat frequency increases, then the actual frequency of B is 509Hz, if the beat frequency decreases then the actual frequency of B is 515Hz

DOPPLER EFFECT

This is the apparent change in frequency and wave length of a wave when there is relative motion between the source of the waves and the observer
Doppler Effect takes place in both sound and light

Doppler Effect in sound
Case 1: source of sound in motion but observer fixed

(a) Source moving towards a stationary observer



$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source= $v - v_s$

Apparent change in wavelength, $\lambda_a = \frac{v - v_s}{f}$

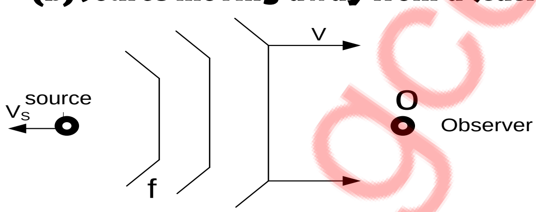
Velocity of wave relative to observer= $v - 0 = v$

Apparent change in frequency, $f_a = \frac{v}{\lambda_a}$

$$f_a = \left(\frac{v}{v - v_s}\right) f$$

Since $v - v_s < v$ then the apparent frequency appears to increase when the source moves towards an observer

(b) Source moving away from a stationary observer



$v_s - velocity\ of\ the\ source$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source= $v + v_s$

Apparent change in wavelength, $\lambda_a = \frac{v + v_s}{f}$

Velocity of wave relative to observer= $v - 0 = v$

Apparent change in frequency, $f_a = \frac{v}{\lambda_a}$

$$f_a = \left(\frac{v}{v + v_s}\right) f$$

Since $v + v_s > v$ then the apparent frequency appears to decrease when the source moves away from an observer.

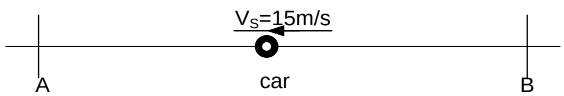
Note:

When the source is in motion, only wavelength and frequency change but the speed of the sound waves is not affected

Examples

1. A car sounds its horn as it travels at a steady speed of 15m/s along a straight road between two stationary observers A and B. Observer A hears a frequency of 538Hz while B hears a lower frequency. Calculate the frequency heard by B if the speed of sound in air is 340m/s

Solution



Since B receives sound of lower frequency, the car is moves away from B

Towards A: $f_A = \left(\frac{v}{v-v_s}\right)f$

$$538 = \left(\frac{340}{340-15}\right)f$$

$f_A = 514.265Hz$

Away from B: $f_B = \left(\frac{v}{v+v_s}\right)f$

$f_B = \left(\frac{340}{340+15}\right)x514.265$

$f_B = 492.54Hz$

2. A stationary observer notices that the pitch of the racing car changes in a ratio 4:3. The velocity of sound in air is 320m/s. Calculates the speed of the car.

Solution

Source moving towards:

$$f_1 = \left(\frac{v}{v-v_s}\right)f$$

$f_1 = \left(\frac{320}{320-v_s}\right)f.\,....1$

Source moving away from:

$$f_2 = \left(\frac{v}{v+v_s}\right)f$$

$f_2 = \left(\frac{320}{320+v_s}\right)f\,.........2$

$$f_1 : f_2 = 4:3$$

$$3f_1 = 4f_2$$

$$3\left(\frac{320}{320-v_s}\right)f = 4\left(\frac{320}{320+v_s}\right)f$$

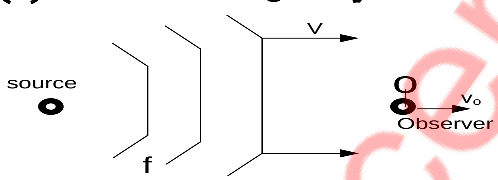
$$v_s = 45.71m/s$$

3. A car sounds its horn as it travels at a steady speed of 20m/s along a straight road between two stationary observers X and Y. Observer X hears a frequency of 560Hz while Y hears a lower frequency. Calculate the frequency heard by Y if the speed of sound in air is 330m/s.

An(495.9Hz)

Case2: observer in motion while the source is stationary

(a) observer moving away from a stationary source



$v_o - velocity\ of\ the\ observer$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source$= v - o = v$

Apparent change in wavelength,$\lambda_a = \frac{v}{f}$

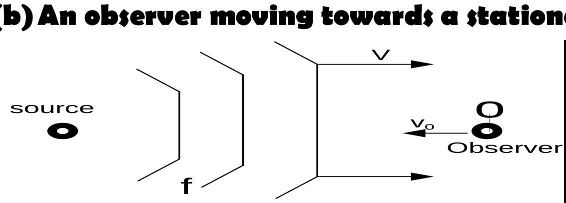
Velocity of wave relative to observer$= v - v_o$

Apparent change in frequency,$f_a = \frac{v-v_o}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v-v_o}{v}\right)f}$$

Since $v - v_o < v$ then the apparent frequency appears to decrease when the observer moves away from the source

(b) An observer moving towards a stationary source



$v_o - velocity\ of\ the\ observer$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source$= v - o = v$

Apparent change in wavelength,$\lambda_a = \frac{v}{f}$

Velocity of wave relative to observer$= v + v_o$

Apparent change in frequency, $f_a = \frac{v+v_o}{\lambda_a}$

$$f_a = \left(\frac{v+v_o}{v}\right)f$$

Since $v + v_s > v$ then the apparent frequency appears to increase when the observer approaches the source

Note:

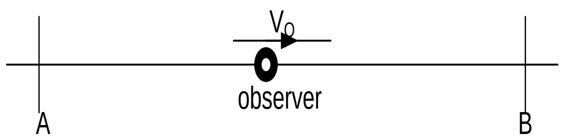
The motion of the observer has no effect on the wavelength of the sound but it affects the relative velocity of sound

Examples:

1. An observer moving between two stationary sources of sound along a straight line joining them hears beats at a rate of $4s^{-1}$ at what velocity is the observer moving if the frequency of the sources is 50Hz and speed of sound in air is 340m/s

Solution

let observer move away from A



$4 = f_B - f_A$..........................(1)

Away from A:

$$f_A = \left(\frac{v-v_o}{v}\right)f$$

$$f_A = \left(\frac{340-v_o}{340}\right)x50 \ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Towards B:

$$f_B = \left(\frac{v+v_o}{v}\right)f$$

$$f_B = \left(\frac{340+v_o}{340}\right)x50 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3)$$

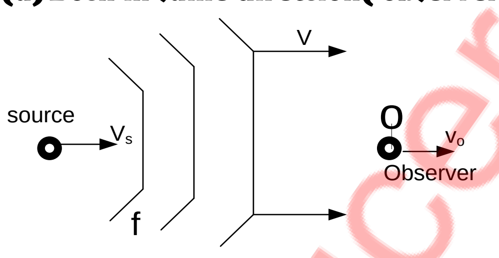
$$4 = \left(\frac{340+v_o}{340}\right)x50 - \left(\frac{340-v_o}{340}\right)x50$$

$$v_o = 13.6m/s$$

2. An observer moving between two identical stationary sources of sound along a straight line joining them hears beats at a rate of $5.0s^{-1}$. At what velocity is the observer moving if the frequency of the sources is 600Hz and speed of sound in air is 330m/s. **An(1.38m/s)**

Case3: observer and source in motion

(a) Both in same direction(observer ahead of source)



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source $= v - v_s$

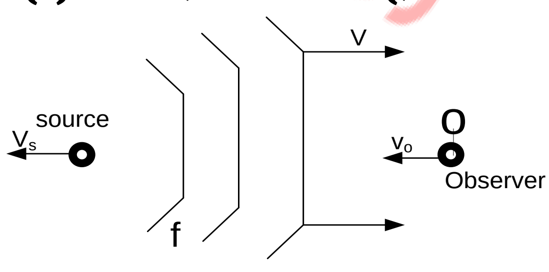
Apparent change in wavelength, $\lambda_a = \frac{v-v_s}{f}$

Velocity of wave relative to observer $= v - v_o$

Apparent change in frequency, $f_a = \frac{v-v_o}{\lambda_a}$

$$f_a = \left(\frac{v-v_o}{v-v_s}\right)f$$

(b) Both in same direction(source ahead of observer)



$v_o -$ *velocity of the observer*

$v_s -$ *velocity of the source*

$v -$ *velocity of sound*

$f -$ *frequency of the sound waves*

Velocity of wave relative to source $= v + v_s$

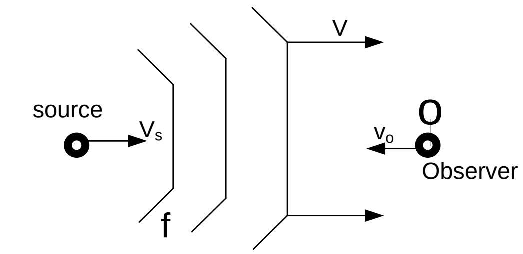
Apparent change in wavelength, $\lambda_a = \frac{v+v_s}{f}$

Velocity of wave relative to observer $= v + v_o$

Apparent change in frequency, $f_a = \frac{v+v_o}{\lambda_a}$

$$f_a = \left(\frac{v+v_o}{v+v_s}\right)f$$

(c) Both moving towards each other



v_o – velocity of the observer

v_s – velocity of the source

v – velocity of sound

f – frequency of the sound waves

Velocity of wave relative to source $= v - v_s$

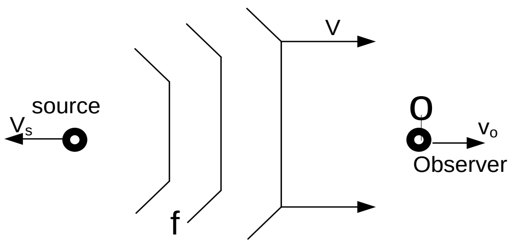
Apparent change in wavelength, $\lambda_a = \frac{v-v_s}{f}$

Velocity of wave relative to observer $= v + v_o$

Apparent change in frequency, $f_a = \frac{v+v_o}{\lambda_a}$

$$f_a = \left(\frac{v+v_o}{v-v_s}\right)f$$

(d) Both moving away from each other



v_o – velocity of the observer

v_s – velocity of the source

v – velocity of sound

f – frequency of the sound waves

Velocity of wave relative to source $= v + v_s$

Apparent change in wavelength, $\lambda_a = \frac{v+v_s}{f}$

Velocity of wave relative to observer $= v - v_o$

Apparent change in frequency, $f_a = \frac{v-v_o}{\lambda_a}$

$$f_a = \left(\frac{v-v_o}{v+v_s}\right)f$$

Generally

$$f_a = \left(\frac{v \pm v_o}{v \pm v_s}\right)f$$

Examples

1. A car A travelling at 36km/h has a horn of frequency 120Hz. A second car B travelling at 54km/h approaching the first car. What is the apparent frequency of the horn to an observer in the second car given that speed of sound in air 320m/s

Solution

$f_a = \left(\frac{v \pm v_o}{v \pm v_s}\right)f$

Since the observer is being approached the apparent frequency increase, therefore the numerator should be maximum and denominator minimum

$$f_a = \left(\frac{v+v_o}{v-v_s}\right)f$$

$$f_a = \left(\frac{320+15}{320-10}\right)x120$$

$$f_a = 129.68Hz$$

2. A cyclist and train approach each other with a speed of 10m/s and 20m/s respectively. A train sounded siren at 480Hz. Calculate the frequency of the note heard by the cyclist.
(Speed of sound in air is 340m/s)
   (a) Before the train passes him
   (b) After the train has passed him

Solution

(a) Before train passes him

$f_a = \left(\frac{v \pm v_o}{v \pm v_s}\right)f$

Since the train approaches the observer, apparent frequency increases so numerator should be maximum and denominator minimum

$$f_a = \left(\frac{v+v_o}{v-v_s}\right)f$$

$$f_a = \left(\frac{320+10}{320-20}\right)x120$$

$$f_a = 525Hz$$

(b) After passing him

Since the train recedes away from the observer, apparent frequency decreases so numerator should be minimum and denominator maximum

$$f_a = \left(\frac{v - v_o}{v + v_s}\right) f$$

$$f_a = \left(\frac{340 - 10}{340 + 20}\right) x120$$

$$f_a = 440Hz$$

3. A train approaching a hill at 36km/h sounds a whistle of 580Hz. Wind is blowing at 72km/h in the direction of motion of the train. Calculate the frequency of the whistle as heard by an observer on the hill. (Speed of sound in air is 340m/s)

Solution

Apparent frequency $f_a = \left(\frac{v^1+v_o}{v^1-v_s}\right) f$

But resultant speed of sound $v^1 = v + v_w$

$$f_a = \left(\frac{[v + v_w] + v_o}{[v + v_w] - v_s}\right) f$$

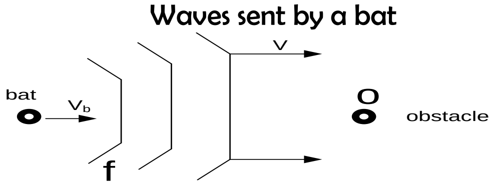
$$f_a = \left(\frac{340 + 20 - 0}{340 + 20 - 10}\right) x580$$

$$f_a = 596.57Hz$$

4. A bat can locate an obstacle by emitting a high frequency sound wave through detecting the reflected waves. A bat flying at a steady speed of 5m/s emits sound waves of frequency 7800Hz and is reflected back.
   (a) Derive the equation of the frequency of sound waves reaching the bat after reflection
   (b) Calculate the frequency of the sound received by the bat given that speed of sound in air is 340m/s. **An(80328.36Hz)**

Solution

Waves sent by a bat



$v_b - velocity\ of\ the\ bat$

$v - velocity\ of\ sound$

$f - frequency\ of\ the\ sound\ waves$

Velocity of wave relative to source $= v - v_b$

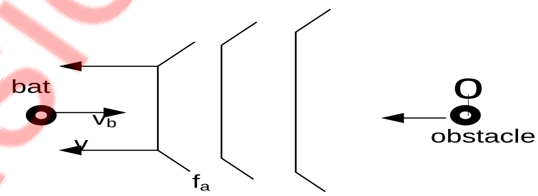
Apparent change in wavelength, $\lambda_a = \frac{v-v_b}{f}$

Velocity of wave relative to observer $= v - 0 = v$

Apparent change in frequency, $f_a = \frac{v}{\lambda_a}$

$$\boxed{f_a = \left(\frac{v}{v - v_b}\right) f}$$

Waves reflected by the obstacle



$v_b - velocity\ of\ the\ bat$

$v - velocity\ of\ sound$

$f_a - frequency\ of\ the\ reflected\ sound\ waves$

Velocity of wave relative to source $= v$

Apparent change in wavelength, $\lambda_a{}^1 = \frac{v}{f_a}$

Velocity of wave relative to observer $= v + v_b$

Apparent change in frequency, $f_a{}^1 = \frac{v+v_b}{\lambda_a{}^1}$

$$\boxed{f_a{}^1 = \left(\frac{v + v_b}{v}\right) f_a}$$

$$f_a{}^1 = \left(\frac{v + v_b}{v}\right)\left(\frac{v}{v - v_b}\right) f$$

$$f_a{}^1 = \left(\frac{v + v_b}{v - v_b}\right) f$$

Applications of Doppler Effect

(i) Used in radar speed traps
(ii) Measurement temperature of hot gases
(iii) Used in measurement of speed of the star

Speed traps

- Microwaves of frequency f_o from a stationary radar set are directed towards a motor vehicle moving with speed u
- Microwaves reflected from the moving car are detected at the radar
- The reflected signal mixes with the transmitted signals to obtain beats
- The beat frequency Δf which is equal to the difference between the frequency of the received and transmitted signal is determined
- The speed of the vehicle is $u = \frac{v\Delta f}{2f_o}$

Measurement of plasma temperature

- The broadening $\Delta\lambda$ of a spectral line emitted by the plasma is determined using a diffraction grating
- $\frac{\Delta\lambda}{\lambda_o} = \frac{2u}{c}$
- Assume $u = v_{rms}$
- $\frac{1}{2}mu^2 = \frac{3}{2}RT$ where m= $molar\ mass$

  $u = \left(\frac{3RT}{m}\right)^{\frac{1}{2}}$
- $T = \frac{m}{12R}\left(\frac{\Delta\lambda}{\lambda_o}\right)^2 C^2$

Speed of star

- The wavelength , λ of light emitted by the star is measured
- The absorption spectrum of an elemen known to be in the star is examined.
- The wavelenght λ^1 of the missing line is measured
- Doppler shift $= |\lambda^1 - \lambda|$

  $\Rightarrow \left|\frac{\lambda^1-\lambda}{\lambda}\right| = \frac{u_s}{C}$

  $\Rightarrow u_s = \left|\frac{\lambda^1-\lambda}{\lambda}\right| C$

Exercise

1. A car travelling at 72 kmh^{-1} has a siren which produces sound of frequency 500Hz. Calculate the difference between the frequency of sound heard by an observer by the roadside as the car approaches and recedes from the observer . [Speed of sound in air = 320 m s^{-1}]. **An(62.7Hz)**
2. An observer moving between two identical stationary sources of sound along a straight line joining them hears beats at the rate of 4.0. At what velocity is the observer moving if the frequencies of the sources are 500Hz and the velocity of sound when the observer makes the observation is 340 m s^{-1}?
3. a) Explain what is meant by Doppler effect (2)

   b) Deduce an expression for the frequency heard by an observer when:

   (i) He is stationary and a source of sound is moving towards him. (3)

   (ii) He is moving towards a stationary source of sound. (3)

   c) A bat flying at a speed of 30ms^{-1} towards an obstacle emits sound waves of frequency $2.5x10^8$ HZ. The bat hears an echo 0.5 s later. If the speed of the sound in air is 340ms-^{1}, how far is the obstacle from the bat when the bat hears the echo?. Find the apparent freq. Of the echo received by the bat (4)
4. (a) A source of sound moving with velocity u_s approaches an observer moving with velocity u_o, in the same direction. Derive the expression for the frequency of sound heard by the observer. (05 marks)

   (b) Explain what happens to the pitch of the sound heard by the observer in (b) above when the

   (i) observer moves faster than the source (02 marks)

   (ii) observer's velocity is equal to that of the sound (02 marks)

   (c) (i) What is meant by Doppler effect?

   (ii) A car sounds its horn as it travels at a steady speed of 15 ms^{-1} along a straight road between two stationary observers A and B. The observer A hears a frequency of 538 Hz while B hears a lower frequency. Calculate the frequency heard by B, assuming the speed of Sound in air is 340ms $^{-1}$ (4 marks)

(d) (i) Explain how beats are produced

(ii) An observer moving between two identical stationary sources of sound along a straight line joining them hears beats at the rate of 4.0. At what velocity is the observer moving if the frequencies of the sources are 500Hz and the velocity of sound when the observer makes the observation is 340 m s $^{-1}$?

5. (a) (i) A police car sounds a siren of 1000 Hz as it approaches a stationary observer. What is the apparent frequency of the siren as heard by the observer if the speed of sound in air is 340?

(ii) Give any three applications of the Doppler effect

(b) An observer standing by the roadside hers sound of frequency 600 Hz coming from the horn of an approaching car. When the car passes, the frequency appears to change to 560 Hz. Given that the speed of sound in air is 320 ms $^{-1}$, calculate the speed of the car. (5 marks)

6. (a) Describe briefly one application of the Doppler effect (2 marks)

(b) (i) Derive an expression for the frequency of sound observed by a stationary observer infront of a source moving with a velocity U m/s and emitting f pulse each second given that speed of sound on the day is C m/s

(ii) A police car travelling at 108 km/hr is chasing a lorry which is travelling at 72 km/hr. The police car given Emits sound of frequency 400 Hz as it approaches the lorry. Calculate the apparent frequency of the note heard by the lorry driver.

7. (a) (i) Define Doppler effect as applied to sound (1)

(ii) Explain briefly how Doppler effect can be used to measure the star (3)

(iii) A stationery police car by the roadside emits a siren of frequency f_s infront of an approaching taxi moving at a speed of vm/s. Find the expression for frequency received by the taxi driver if the speed of sound on that day was C m/s (3)

(b) A police car operating its siren of frequency 384 Hz travels at 90km/hr. Another car travels at 72 km/h. Given that the speed of sound on the day is 340 m/s, calculate the apparent frequency of the siren as heard by the occupants of the second car if they are travelling away from the police car. (4)

8. (a) Show that if a dog is to chase little Tamale,

(i) the apparent frequency of sound heard by Tamale is $f_T{}^1 = \left(\frac{V-U_T}{V-U_D}\right) f_D$ (5 marks)

(ii) the apparent frequency of sound heard by the dog is $f_D{}^1 = \left(\frac{V+U_D}{V+U_T}\right) f_T$ (3 marks)

Where V – speed of sound in air
U_D – speed of the dog
U_T – speed of Tamale
f_D – frequency of sound from dog
f_T – frequency of sound from Tamale

(b) (i) What is meant by the Doppler effect?

(ii) A police car sounds a siren of 1000 Hz as it approaches a stationary observer. What is the apparent frequency of the siren as heard by the observer if the speed of sound in air is 340ms $^{-1}$ (3 marks)

(iii) State one application of the Doppler effect (1 mark)

9. (a) (i) Explain Doppler effect (2 marks)

(ii) A car travelling at 10ms $^{-1}$ sounds its horn that sends sound waves of frequency 500Hz and this is heard in another car which is travelling behind the first one in the same direction with a velocity of 20 ms $^{-1}$. What frequency will be heard by

(i) the driver of the second car? (3 marks)

(ii) an observer standing some distance ahead of the first car (velocity of sound in air = 330 ms $^{-1}$) (3 marks)

(b) (i) Define Doppler effect as applied to sound (1)

(ii) Explain briefly how Doppler effect can be used to measure the star (3)

(iii) A stationery police car by the roadside emits a siren of frequency f_s infront of an approaching taxi moving at a speed of vm/s. Find the expression for frequency received by the taxi driver if the speed of sound on that day was C m/s

(C) (i) Derive an expression for the frequency of sound observed by an observer moving with a velocity μo m/s towards the stationary source emitting fs pulse each second given that speed of sound on the day is C m/s

(ii) A police car operating its siren of frequency 384 Hz travels at 90km/hr. Another car travels at 72 km/h. Given that the speed of sound on the day is 340 m/s, calculate the apparent frequency of the siren as heard by the occupants of the second car if they are travelling away from the police car. (4)

(d) (i) What is meant by **Doppler effect?** (1 mark)

(ii) A car sounds its horn as it travels at a steady speed of $20ms^{-1}$ along a straight road between two stationary observers X and Y. Observer X hears a frequency of $560H_z$ while Y hears a lower frequency.Calculate the frequency heard by Y assuming the speed of sound in air is $330ms^{-1}$

(iii) A man moving at 10m/s while blowing a whistle at freq of 1.5kHz towards a vertical tall wall. Calculate the apparent freq. of an echo received by the man

RESONANCE

This is a condition obtained when a system is set to oscillate at its own natural frequency as a result of impulses received form another system vibrating at the same frequency

Other terms

Fundamental frequency

This is the lowest possible frequency that an instrument can produce.

Overtones

These are note of higher frequencies than the fundamental frequency produ ed by an instrument.

Harmonic

These is one of the frequencies that can be produced by a particular instrument

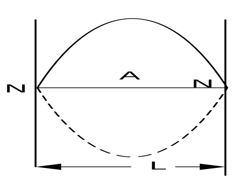
WAVES ON A STRETCHED STRING

When a stretched string is plucked, a progressive wave is formed and it travels to both ends which are fixed and these waves are reflected back to meet the incident wave. The incident and reflected waves both have the same speed, frequency and amplitude and therefore when they superimpose a stationary wave is formed.

Modes of vibration

When a string is plucked in the middle, the wave below is produced

(a) 1st harmonic (fundamental frequency)



$$l = \frac{\lambda}{2}$$
$$\lambda = 2l$$
$$v = f\lambda$$
$$\boxed{v = 2lf_o}$$

A is antinodes;

These are points on a stationary wave where particles have maximum displacement.

N is nodes;

This is a point on a stationary wave in which particles are always at rest (zero displacement)

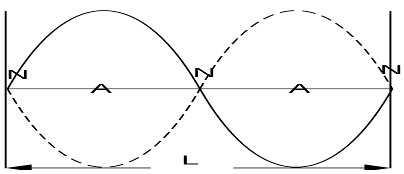
Note:

- The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$ where λ is wavelength.
- When a stationary wave is produced, the distance between the source and reflector is a multiple of $\frac{1}{2}\lambda$.

$$\boxed{distance = n\frac{\lambda}{2}}$$

Where n is the number of loops ie n is 1,2,3 … … … …

(b) 2nd harmonic (1st overtone)



$l = \lambda$

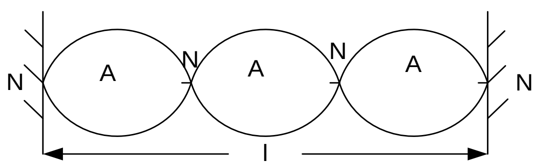
$v = f\lambda$

$v = lf_1$

$2lf_o = lf_1$

$\boxed{f_1 = 2f_o}$

(c) 3rd harmonic (2nd overtone)



$\lambda = \frac{2}{3}l$

$v = f\lambda$

$2lf_o = \frac{2}{3}lf_2$

$\boxed{f_1 = 3f_o}$

$l = \frac{3}{2}\lambda$

Generally $\boxed{f_n = nf_o}$

$\boxed{f_n = \frac{n\,v}{2\,l}}$ $n = 1,2,3,4,5,6 \quad n^{th} - harmonic$

Velocity of a transverse wave along a stretched string

The velocity of a wave on the sting depends on the following

(i) Tension T

(ii) Mass m

(iii) Length l

V ∝ $T^x m^y l^z$

V = $k\,T^x m^y l^z$……….(x)

[V] = $[K][T]^x[m]^y[l^z]$

K is a dimensionless constant

$LT^{-1} = (MLT^{-2})^x (M)^y (L)^z$

For powers of T

$-2x = -1$……………….(1)

$x = \frac{1}{2}$

For powers of M,

$0 = x + y$……………….(2)

$0 = \frac{1}{2} + y$

$y = -\frac{1}{2}$

For powers of L,

$1 = x + z$

$z = \frac{1}{2}$

$V = k\, T^x m^y l^z$

$V = k\, T^{\frac{1}{2}} m^{\frac{-1}{2}} l^{\frac{1}{2}}$

$$\boldsymbol{V = \sqrt{\frac{Tl}{m}}}$$

Examples

1. A string of length 0.5m has a mass of 5g. The string is stretched between two fixed points and plucked. If the tension is 100N, find the frequency of the second harmonic

Solution

$$v = \sqrt{\frac{Tl}{m}}$$

$$v = \sqrt{\frac{100x0.5}{5x10^{-3}}}$$

$$v = 100m/s$$

$$v = f_0\lambda$$

$$v = 2lf_0$$

$$100 = 2x0.5f_0$$

$$f_0 = 100Hz$$

$$f_2 = 2\, f_0$$

$$f_2 = 2x100$$

$$f_2 = 200Hz$$

Alternatively

$$v = \sqrt{\frac{Tm}{l}}$$

$$v = \sqrt{\frac{100x5x10^{-3}}{0.5}}$$

$$v = 100m/s$$

$$f_n = \frac{n\,v}{2l}$$

$$f_2 = \frac{2x\,100}{2x0.5}$$

$$f_2 = 200Hz$$

2. A wire under a tension of 20N is plucked at the middle to produce a note of frequency 100Hz. Calculate the;
   (i) diameter of the wire if its length is 1m and has a density of $600kgm^{-3}$
   (ii) frequency of the first overtone **An(200Hz)**

solution

$$v = \sqrt{\frac{Tl}{m}}$$

$$2lf_0 = \sqrt{\frac{Tl}{m}}$$

$$2x1x100 = \sqrt{\frac{20x1}{m}}$$

$$m = 0.0005kg$$

$$\rho = \frac{m}{volume}$$

$$volume = \frac{0.0005}{600} = 8.33x10^{-7}m^3$$

$$volume = \pi r^2 l$$

$$r = \sqrt{\left(\frac{8.33x10^{-7}}{\pi}\right)}$$

$$r = 5.15x10^{-4}m$$

$$d = 2r = 2x5.15x10^{-4}$$

$$= 1.03x10^{-3}m$$

3. A stretched wire of length 0.75m, radius 1.36 mm and density 1380 kg m $^{-3}$ is clamped at both ends and plucked in the middle. The fundamental note produced by the wire has the same frequency as the first overtone in a pipe of length 0.15 m closed at one end.
   (i) Sketch the standing wave pattern in the wire
   (ii) Calculate the tension in the wire

   [The speed of sound along the stretched wire is $\sqrt{\left(\frac{T}{\mu}\right)}$ where T is the tension in the wire and μ is the mass per unit length. Speed of sound in air = 330m s $^{-1}$]

   Solution

For the wire at fundamental note

$$V = \sqrt{\left(\frac{T}{\mu}\right)}$$

$$2lf_0 = \sqrt{\frac{Tl}{m}}$$

But $\boldsymbol{m} = \rho\pi r^2 l$

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\rho\pi r^2}}$$

$$f_0 = \frac{1}{2x0.75}\sqrt{\frac{T}{\pi(1.36x10^{-3})^2x1380}}$$ ....(i)

For a closed at first overtone

$v = f_1\lambda$

$330 = f_1\frac{4}{3}l$

$f_1 = \frac{330x3}{4x0.15}$ ....(ii)

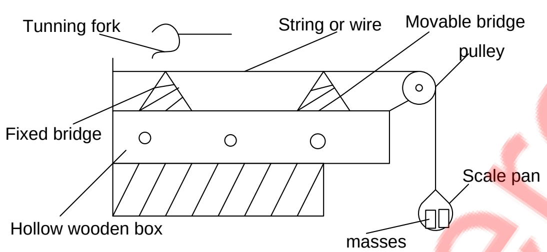
$$\frac{1}{2x0.75}\sqrt{\frac{T}{\pi(1.36x10^{-3})^2x1380}} = \frac{330x3}{4x0.15}$$

$$T = 4.91x10^4N$$

4. The wire of a guitar of length 50cm and mass per unit length $1.5x10^{-3}kgm^{-1}$ is under a tension of 173.4N. The wire is plucked at its mid-point. Calculate the;
   (i) frequency **An(340Hz)**
   (ii) wavelength of the tfundamental note **An(1.0m)**
5. A string of length 50cm vibrates in a fundamental mode. Find fundamental frequency of vibration. **An(330Hz)**
6. A wire of length 0.60m and mass $9x10^{-4}kg$ is under a tension of 135N. The wire is plucked such that it vibrates in its third harmonic. Calculate
   (iii) Fundamental frequency **An(250Hz)**
   (iv) Frequency of the third harmonic **An(750Hz)**
7. A wire of length 0.4m and mass $1.2x10^{-3}kg$ is under a tension of 120N. The is plucked in the middle. calculate
   (v) Fundamental frequency **An(250Hz)**
   (vi) Frequency of the third harmonic **An(750Hz)**

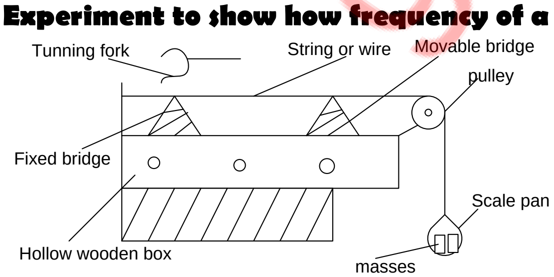
Factors on which frequency of a stretched string depends

Experiment to investigate the variation of frequency of a stretched string with length



- The experiment is set up as shown above
- Pluck the string in the middle and place a sounding tuning fork near it
- Move the bridge B towards A until when a loud sound is heard
- The distance l between the bridges is measured and recorded together with the frequency of the tuning fork
- Repeat the above procedures using different tuning forks of different frequency
- Tabulate your results including values of $\frac{1}{l}$
- Plot a graph of against $\frac{1}{l}$
- It's a straight line passing through the origin implying that f $\propto \frac{1}{l}$

Experiment to show how frequency of a stretched string varies with tension



- The experiment is set up as shown above
- The length l between two bridges is kept constant
- A suitable mass m is attached to the free end of a string (scale pan)
- Pluck the string in the middle and a tuning fork of known frequency f is sounded near it
- Vary the mass on the scale pan until when a loud sound is heard
- Record the mass of the corresponding frequency f in a suitable table including values of f^2

- Repeat the above procedures using different tuning forks of different frequency
- Plot a graph of f^2 against m
- It's a straight line passing through the origin implying that $f^2 \propto m$
- Since $T = mg$, it implies $f \propto \sqrt{T}$ hence frequency increases with increase in frequency

Resonance of air in pipes

When air is blown in a pipe, a longitudinal wave is formed. This wave travels along the pipe and if the pipe is closed the wave will be reflected back. The incident and reflected wave both have the same speed, same frequency and same amplitude. This results into formation of a stationary wave.

These are two type of pipe for air vibrations.

(i) Open pipes

This is one that has both ends open eg trumpet, a flute

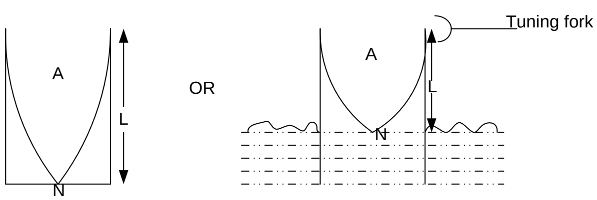
(ii) Closed pipes

It is one in which one end is open, while the other is closed eg a long drum.

a) Modes of vibration in closed pipes

For closed pipes, a node is formed at a closed end and an antinode at the open end

First harmonic / fundamental note



The length of the air column is L

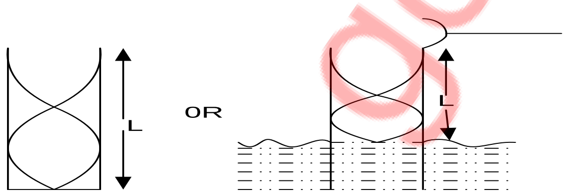
$$L = \frac{1}{4}\lambda$$
$$\lambda = 4\,l$$
$$v = f_0\lambda$$
$$\boxed{v = 4lf_0}$$
$$f_0 = \frac{v}{4l}$$

f_0 is the fundamental frequency

Describe the motion of air I a tube closed at one end and vibrating in its fundamental frequency

Air at end A vibrates with maximum amplitude. The amplitude of vibration decreases as end N is approached. Air at N is stationary. End N is node while end A is antinode

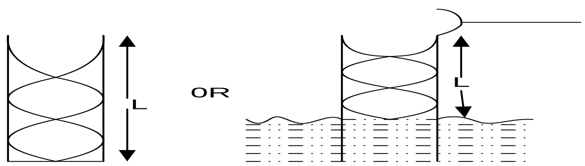
First overtones / third harmonic



The length of the air column is L

$$L = \frac{3}{4}\lambda$$
$$\lambda = \frac{4}{3}l$$
$$v = f_1\lambda$$
$$4lf_0 = f_1\frac{4}{3}l$$
$$f_1 = 3\,f_0$$

Second overtones/ fifth harmonic



The length of the air column is L

$$L = \frac{5}{4}\lambda$$

$$\lambda = \frac{4}{5}\, l$$
$$v = f_2\lambda$$
$$4lf_0 = f_2 x \frac{4\, l}{5}$$
$$f_2 = 5\, f_0$$

Note: In closed pipes, harmonics produced must be with frequencies $f_0, 3f_0, 5f_0, 7f_0 \ldots\ldots\ldots\ldots\ldots$

This implies that only <u>odd harmonics</u> are produced can be produced by closed pipes

$\boxed{f_n = \frac{n\, v}{4\, l}}$ $n = 1,3,5,7,9 \ldots\ldots$ $\quad n^{th} - harmonic$

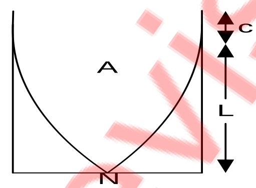
Variation of pressure with displacement of air in a closed pipe

At the mouth of the pipe, the air is <u>free to move</u> and therefore the displacement of air molecules is <u>large</u> and <u>pressure is low</u>. At the closed end the molecules are <u>less free</u> and the displacement is <u>minimal</u> and the <u>pressure is high</u>

END CORRECTIONS

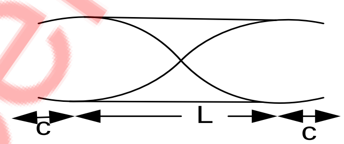
An antinode of stationary wave in a pipe is not formed exactly at the end of the pipe. Instead it is displaced by a distance, c. This distance is called the end correction

The effective length of a wave in the closed pipe of length l is $l + c$



$$l + c = \frac{\lambda}{4}$$

The effective length of a wave in an open pipe of length l is $l + 2c$



$$l + 2c = \frac{\lambda}{2}$$

Note:

C is related to the radius of the pipe by an equation $c = 0.6r$ implying that the end correction is more significant for wide pipes

Examples

1. A cylindrical pipe of length 30cm is closed at one end. The air in the pipe resonates with a tuning fork of frequency 825Hz sounded near the open end of the pipe. Determine the mode of vibration of air assuming there is no end correction. Take speed of sound in air as 330m/s.

Solution

$$f_n = \frac{n\,v}{4\,l}$$
$$825 = \frac{n\,x330}{4\,x0.3}$$
$$n = 3$$
But $n = 1,3,5,7,9 \ldots \ldots$
Mode of vibration is third harmonic

2. A cylindrical pipe of length 29cm is closed at one end. The air in the pipe resonates with a tuning fork of frequency 860Hz sounded near the open end of the pipe. Determine the mode of vibration of air and end correction. Take speed of sound in air as 330m/s.

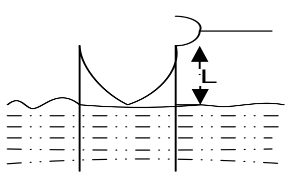
Solution

$$f_n = \frac{n\,v}{4\,l}$$
$$860 = \frac{n\,x330}{4\,x0.29}$$
$$n = 3.02$$
$n = 1,3,5,7,9 \ldots \ldots$

Mode of vibration is fifth harmonic
$$f_n = \frac{n\,v}{4\,(l+c)}$$
$$c = \frac{5x330}{4\,x860} - 029 = 0.1897m$$

3. A long tube is partially immersed in water and a tuning fork of 425Hz is sounded and held above it. If the tube is gradually raised, find the length of the air column when resonance first occurs. [speed of sound in air is 340m/s]

Solution



$f = 425Hz,\, v = 340ms^{-1}$
$v = f\lambda$
$340 = 425x\lambda$
$\lambda = 0.8\,m$

$$L = \frac{1}{4}\lambda$$
$$L = \frac{1}{4}x0.8$$
$$L = 0.2\,m$$

4. A tube 100cm long closed at one end has its lowest frequency at 86.2Hz. With a tube of identical dimensions but open at both ends, the first harmonic occurs at 171Hz. Calculate
   (i) The speed of sound
   (ii) End correction

Solution

(i) For closed pipe: $v = 4(l + c)f_0$
$v = 4(1 + c)86.2$..........(1)
For open pipe: $v = 2(l + 2c)f_0$
$v = 2(1 + 2c)171$..........(2)
Equating (1)and (2)

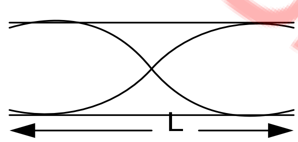
$$4(1 + c)86.2 = 2(1 + 2c)171$$
$$c = 8.25x10^{-3}m$$

(ii) $v = 4(1 + c)86.2$
$$v = 4(1 + 8.25x10^{-3})86.2$$
$$v = 347m/s$$

b) Modes of vibration in open pipes

In open pipes, antinodes are found at the two open ends of the pipe

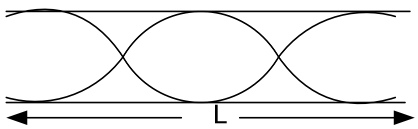
First harmonic / fundamental note



$$l = \frac{1}{2}\lambda$$

$$\lambda = 2\,l$$
$v = f_0\lambda$
$v = 2lf_0$
$$\boxed{f_0 = \frac{v}{2\,l}}$$
f_0 is the fundamental frequency

Second harmonic / first overtone



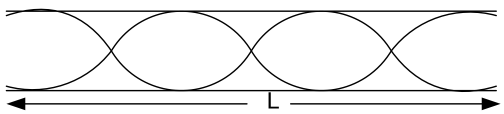
$l = \lambda$

$v = f_1 \lambda$

$2lf_0 = f_1 l$

$f_1 = 2 f_0$

Third harmonic / Second overtone



$l = \frac{3}{2}\lambda$

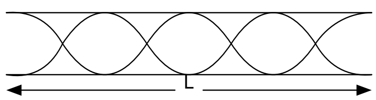
$\lambda = \frac{2}{3} l$

$v = f_2 \lambda$

$2lf_0 = f_2 x \frac{2l}{3}$

$f_2 = 3 f_0$

Fourth harmonic / Third overtone



$l = 2\lambda$

$\lambda = \frac{l}{2}$

$v = f_3 \lambda$

$2lf_0 = f_3 x \frac{l}{2}$

$f_3 = 4 f_0$

Note:

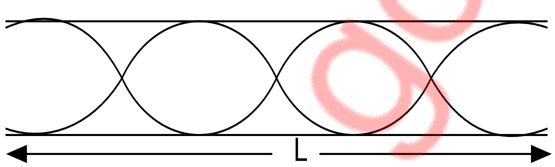
In open pipes, harmonics produced must be with frequencies $f_0, 2f_0, 3f_0, 4f_0, 5f_0 \dots\dots\dots\dots\dots$
Open pipes produce both odd and even harmonics and this is why open pipes are preferred as musical instruments.

$\boxed{f_n = \frac{nv}{2l}}$ $n = 1,2,3,4,5,6$ $n^{th} - harmonic$

Examples

1. The frequency of third harmonic in an open pipe is 660Hz, if the speed of sound in air is 330m/s. Find;
   (i) the length of the air column
   (ii) the fundamental frequency

Solution



i) $f = 660Hz$, $v = 330ms^{-1}$

$v = f\lambda$

$330 = 660x\boldsymbol{\lambda}$

$\lambda = 0.5\ m$

$L = \frac{3}{2}\boldsymbol{\lambda}$

$L = \frac{3}{2}x0.5$

$L = 0.75\ m$

$ii)$ $f_2 = 3 f_0$

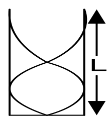
$f_0 = \frac{660}{3}$

$f_0 = 220Hz$

2. If the velocity of sound in air is 330m/s and the fundamental frequency is 110Hz in a closed tube.;
   (i) What is the approximate length of the tube if the tube is resonate at the first overtone
   (ii) What would be the fundamental frequency if the tube was open at both ends

Solution

i) $f_0 = 110Hz,\ v = 330m/s$



$f_1 = 3\,f_0$

$f_1 = 3\,x110 = 330Hz$

$v = \quad f_1\lambda$

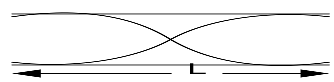
$\lambda = \dfrac{330}{330} = 1\,m$

$L = \dfrac{3}{4}\boldsymbol{\lambda}$

$L = \dfrac{3}{4}x1$

$L = 0.75\,m$

ii)



$L = \dfrac{1}{2}\boldsymbol{\lambda}$

$\boldsymbol{\lambda} = 2L$

$\boldsymbol{\lambda} = 2x0.75$

$\boldsymbol{\lambda} = \ 1.5m$

$v = \quad f_o\lambda$

$f_0 = \dfrac{330}{1.5}$

$f_0 = 220Hz$

3. Two organ pipes of length 50cm and 51cm respectively give beats of frequency 7Hz when sounding their fundamental notes together. Neglecting the end corrections calculate the velocity of sound in air.

Solution

$f_1 = \frac{v}{2\,l_1}$ and $f_2 = \frac{v}{2\,l_2}$

$f_b = \frac{v}{2\,l_1} - \frac{v}{2\,l_2}$

$7 = \frac{v}{2}\left(\frac{1}{0.5} - \frac{1}{0.51}\right)$

$\boldsymbol{v = 357m/s}$

4. Two organ open pipes of length 50cm and 51cm respectively give beats of frequency 6Hz when sounding heir fundamental notes together. Neglecting the end corrections calculate the velocity of sound air $\boldsymbol{An(306ms^{-1})}$

5. Two organ pipes of length 92cm and 93cm respectively give beats of frequency 3.0Hz when sounding their fundamental notes together. If the end corrections are 1.5cm and 1.8cm respectively. Calculate the velocity of sound in air.

Solution

$f_1 = \frac{v}{2(l_1+2\,C_1)}$ and $f_2 = \frac{v}{2(l_2+2\,C_2)}$

$f_b = \frac{v}{2(l_1+2\,C_1)} - \frac{v}{2(l_2+2\,C_2)}$

$3 = \frac{v}{2}\left(\frac{1}{0.92+2x0.015} - \frac{1}{0.93+2x0.018}\right)$

$\boldsymbol{v = 344.14m/s}$

8. A glass tube, open at the top, is held vertically and filled with water. A tuning fork vibrating at 264 Hz is held above the tube and water is allowed to flow out slowly. The first resonance occurs when the water level is 32.5 cm from the top while the second resonance occurs when the level is 96.3 cm from the top. Find:
   (i) the speed of sound in the air column
   (ii) the end correction

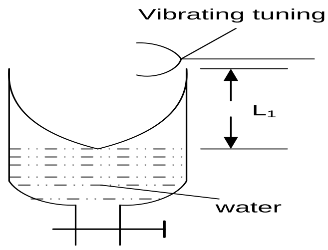
9. A uniform tube 50cm long, is held vertically and filled with water. A tuning fork vibrating at 512 Hz is held above the tube and water is allowed to flow out slowly. The first resonance occurs when the water level is 12 cm from the top while the second resonance occurs when the level is 43.3 cm from the top. Find the lowest frequency to which the air could resonate if the tube were empty.

Note:

Different instruments produce <u>different number</u> of overtones. The numbers of overtones produced affect the quality of the note played. Hence the quality of the notes produced by different instruments are different

Qn: Explain why a musical note played on one instrument sounds different from the same note played on another instrument

Experiment: To measure velocity of sound in air by Resonance tube and a tuning fork of a known frequency



- A glass tube which can be drained from the bottom is filled with water.
- A sounding tuning fork of known frequency f is brought to the mouth of tube.
- The water is then slowly drained until a loud sound is heard.
- The tap is closed and the length of the air column l_1 is measured.
- The tuning fork is sounded again at the mouth of the tube and water is drained further until a loud sound is heard.
- The tap is closed and the length of the air column l_2 is measured.
- Velocity of sound in air is obtained from $v = 2f(l_2 - l_1)$

Theory

$l_1 + c = \frac{1}{4}\lambda$…………………(1)

$l_2 + c = \frac{3}{4}\lambda$…………………(2)

Equation (2) – Equation (1)

$$(l_2 + c) - (l_1 + c) = \frac{3}{4}\lambda - \frac{1}{4}\lambda$$

$$l_2 - l_1 = \frac{1}{2}\lambda$$

$$\lambda = 2(l_2 - l_1)$$

But $v = f\lambda$

$$\boxed{v = 2f(l_2 - l_1)}$$

Example

1. A tuning fork of frequency 256Hz produces resonance in a tube of length 32.5cm and also in one of length 95cm. Calculates the speed of sound in air column of the tube.

Solution

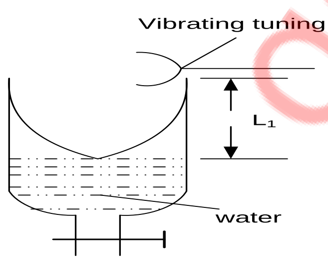
$v = 2f(l_2 - l_1)$

$v = 2 \times 256 \times \left(\frac{95 - 32.5}{100}\right)$

$v = 320ms^{-1}$

2. A uniform tube 50cm long is filled with water and a vibrating tuning fork of frequency 512Hz is sounded and held above the tube. When the level of water is gradually lowered, the air column resonates with the tuning fork when its length is 12cm and again when it is 43.3cm. Calculate
   (i) Speed of sound $An(322.56ms^{-1})$
   (ii) The end corrections $An(3.67cm)$
   (iii) Lowest frequency to which the air can resonate if the tube is empty **An(281.27Hz)**

Experiment: To measure velocity of sound in air using Resonance tube and different tuning forks of a known frequencies



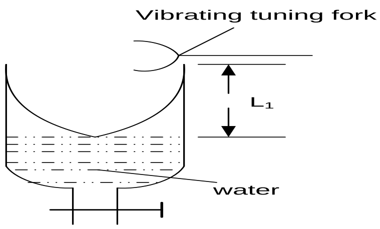
- A glass tube which can be drained from the bottom is filled with water.
- A sounding tuning fork of known frequency f is brought to the mouth of tube.
- The water is then slowly drained until a loud sound is heard.
- The tap is closed and the length of the air column l is measured.
- The experiment is repeated with other tuning forks and the value of l and f is recorded including values of $\frac{1}{f}$
- A graph of l against $\frac{1}{f}$ is plotted and slope s is obtained
- Velocity of sound in air is obtained from $v = 4s$

Theory

$$l + c = \frac{1}{4}\lambda$$
$$l = \frac{1}{4}\lambda - c$$

But $\lambda = \frac{v}{f}$
$$l = \frac{1}{4f}v - c$$

Experiment: To measure end corrections using Resonance tube and different tuning forks of a known frequencies



- A glass tube which can be drained from the bottom is filled with water.
- A sounding tuning fork of known frequency f is brought to the mouth of tube.
- The water is then slowly drained until a loud sound is heard.
- The tap is closed and the length of the air column $\boldsymbol{l}$ is measured.
- The experiment is repeated with other tuning forks and the value of l and f is recorded including values of $\frac{1}{f}$
- A graph of l against $\frac{1}{f}$ is plotted and the intercept c of the l axis determined from line graph
- The intercept c is the end corrections

VELOCITY OF SOUND IN GASES

Velocity of sound in gasses depends on the pressure and density of the gas

$$v \propto P\rho$$
$$v = kP\rho$$

Where k- constant

$[V] = [K]\,[P]^x[\rho]^y$

$$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y$$

For L: $1 = -x - 3y \ldots\ldots\ldots(1)$

For M: $0 = x + y \ldots\ldots\ldots(2)$

For T: $-1 = -2x \ldots\ldots\ldots(3)$

$$x = \frac{1}{2}$$
$$y = -x$$
$$y = -\frac{1}{2}$$

$$v = k\sqrt{\frac{P}{\rho}}$$

But if $k = \sqrt{\gamma}$

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

γ-ratio of molar heat capacity at constant pressure to molar heat capacity at constant volume

Note: The speed of sound in air depends on pressure, density and temperature

Explanation

When temperature of air is increased, the pressure increases. If the air is not restricted in volume it expands leading to a reduction in density. From the above expression a reduction in density leads to increase in velocity. Hence increase in temperature leads to increase in velocity of sound in air

VELOCITY OF SOUND IN SOLIDS

Velocity of sound in solids depends on the young's modulus E and density ρ of the solid

$$v \propto E\rho$$
$$v = kE\rho$$

Where k- constant

$[V] = [K]\,[E]^x[\rho]^y$

$$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y$$

For L: $1 = -x - 3y \ldots\ldots\ldots(1)$

For M: $0 = x + y \ldots\ldots\ldots(2)$

For T: $-1 = -2x \ldots\ldots\ldots(3)$

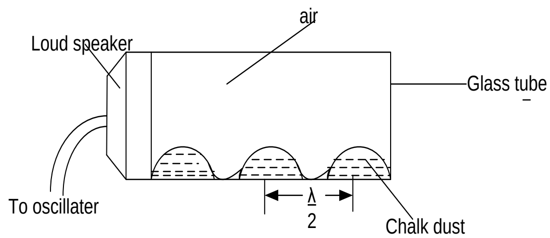
$$x = \frac{1}{2}$$
$$y = -x$$
$$y = -\frac{1}{2}$$

$$v = k\sqrt{\frac{E}{\rho}}$$

But if $k = 1$

$$\boxed{v = \sqrt{\frac{E}{\rho}}}$$

Measurement of speed of sound in air using Kundt's dust tube

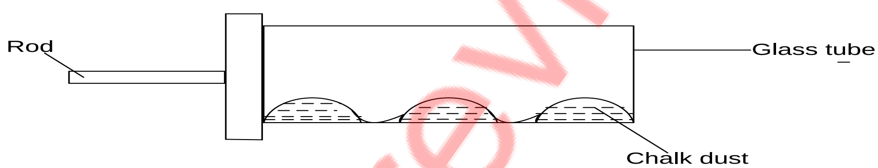


- A long glass tube is placed horizontally with chalk dust inside it
- The open end is fitted with a loud speaker which is connected to an oscillator of known frequency f
- When the oscillator is switched on, sound is produced and a stationary wave is formed in the glass tube which makes the power to settle into well-spaced heaps,
- Measure the distance l between the two consecutive heaps
- Wavelength of the wave generated is given by $\lambda = 2l$
- Speed of sound in air is got from $v = 2lf$

Note: Heaps are found at points where there are no vibrations (nodes)
Measurement of l from outside the tube may not be accurate hence a source of error

Examples

In an experiment to determine the speed of sound in air in a tube, chalk dust settled in heaps as shown in the diagram below;



If the frequency of the vibrating rod is 220Hz and the distance between three consecutive heaps is 1.50m, calculate the speed of sound in air

Solution

$\lambda = 1.50m$

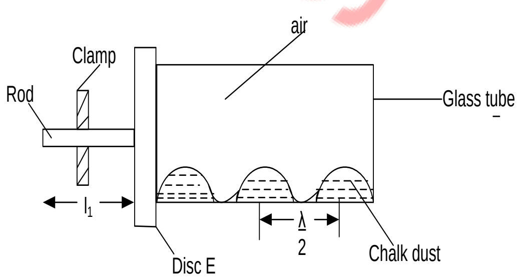
$v = \lambda f$

$v = 1.5x220$

$\boldsymbol{v = 330m/s}$

Measurement of speed of sound in a rod using Kundt's dust tube

- Sprinkle some chalk dust along the interior of the tube



- Sprinkle some chalk dust along the interior of the tube
- Clamp the rod at its mid-point with one end projecting into tube
- Connect disc E to the end of the tube such that it just covers the side of the tube
- Strike the rod using a piece of a cloth until when the [powder in the tube settles into heaps
- Measure the distance l_2 between two consecutive heaps and l_1 of the end

- Velocity of sound in the rod is obtained from $v_r = \frac{v_a l_1}{l_2}$

Where v_a is the velocity of sound in air which is in the tube

Theory

$v_r = 2f_r l_1 \ldots\ldots\ldots\ldots\ldots\ldots.(1)$ Where f_r is frequency of the rod

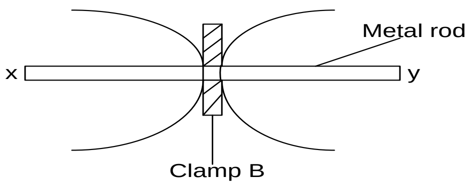
$v_a = 2f_a l_2 \ldots\ldots\ldots\ldots\ldots\ldots.(2)$

But $f_r = f_a$

$$\frac{v_r}{v_a} = \frac{2f_r l_1}{2f_a l_2}$$

$$v_r = \frac{v_a l_1}{l_2}$$

Measurement of speed of sound in a rod



- Rod xy is fixed a clamp B at its a middle point
- The rood is then stroked and a stationary wave is formed due to the vibration of the rod
- A node is formed at the mid-point of the node and antinode ta the free ends x and y
- Measure length l of the rod
- Wavelength of the wave generated by the rod is given by $\lambda = 2l$
- Speed of sound in the rod is got from $v = 2lf$

Uneb 2016

(a) What is meat by the following terms as applied to a waves
   (i) Resonance . (01mark)
   (ii) Frequency. (01mark)
(b) Explain with the aid f suitable diagrams, the terms **fundamental note** and **overtone** as applied to a vibrating air in a closed pipe. (05marks)
(c) Describe how you would determine the speed of sound in air using a resonane tube and several tunning forks. (05marks)
(d) (i) Explain the formation of beats (02marks)
   (ii) Derive the expression for beat frequency (03marks)
(e) Two observers **A** and **B** are provide with sources of sound of frequency 750Hz. If **A** remains stationary while **B** moves away at a velocity of $2.0ms^{-1}$, find the number of beats heard per second by **A**.
(velocity of sound in air = $330ms^{-1}$) (03marks)

Uneb 2015

(a) Distinquish between **progressive waves** and **stationary waves** . (03marks)
(b) (i) What are overtones?. (01mark)
   (ii) Expalin why a musical note played on one instrument sounds different from the same note played on another instrument. (03marks)
(c) A stretched string of length L, is fixed at both ends and then set to vibrate in its allowed mode. The wire is plucked such that it vibrates in its third harmonic. Calculate the frequency of the third harmonic. (04marks)
(d) A wire of length 0.6m and mass $9x10^{-4}kg$ is under tension of 135N. The wire is plucked ssuch that it vibrates in its third harmonic. Calculate the frequency of the third harmonic. (05marks)
(e) Describe the varation of pressure with displacement of air in a closed pipe vibrating with fundamental frequency. (04marks)

Uneb 2013

(a) (i) Distinquish between **free oscillations** and **damped oscillations.** (02marks)
   (ii) What is meant by **resonance** as applied to sound (01mark)

(b) Describe how you would determine the velocity of sound in air using a resonane tube and several tunning forks of different frequencies. (05marks)

(c) A uniform tube 80cm long is filled with water and a small loudspeaker connected to a signal generator is held over the open end of the tube. With the signal generator set at 600Hz, the water level in the tube is lowered until resonance is first obtained when the length of the air column is 69.8cm long, calculate the;

(i) Velocity of sound in air (04marks)

(ii) Fundamental frequency for the tube if it were open at both ends (03marks)

(d) (i) What is meant by **doppler effect** (01marks)

(ii) A motor cyclist and a police car are approaching each other. The motor cyclist is moving at 10m/s and the police car at 20m/s. if the police siren is sounded at 480Hz, calculate the freqeucny of the note heard by the cyclist after the police car passes by. (03marks)

(iii) Give two applications of Doppler effect. (01marks)

Uneb 2012

(a) What is meat by the following terms as applied to sound

(i) Resonance . (01mark)

(ii) Fundamental Frequency. (01mark)

(b) Describe an experiment to determine the end corrections of a resonance tube. (05marks)

(c) A wire of length 50cm, density 8.0gcm^{-3} is stretched between two points. If the wire is set to vibrate at a fundamental frequency of 15Hz, calculate;

(i) The velocity of the wave along the wire. (03marks)

(ii) The tension per unit length of cross-section of the wire. (03marks)

(d) (i) Explain using the principle of superposition of waves the formation of;

(i) beats (04marks)

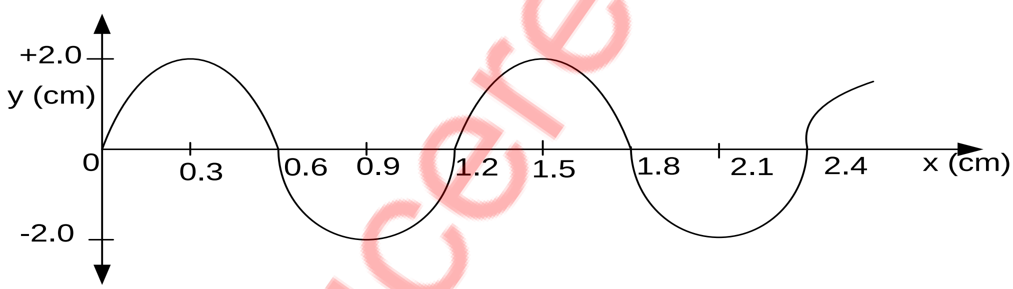
(ii) Stationary wave. (03marks)

Uneb 2011

(a) (i) Define the terms **wave front** and **a ray** in reference to a progressive wave. (02marks)

(ii) Draw a sketch diagram showing reflection of a circular wave by a plane reflector. (02marks)

(b) Figure shows a wave travelling in the positive x-direction away from the origin with a velocity 9ms^{-1}



(i) What is the period of the wave. (03marks)

(ii) Show that the displacement equation for the wave is $y = 2\ sin\frac{5}{3}\pi(9t - x)$. (03marks)

(c) What is meant by **Doppler effect**. (01mark)

(d) One species of bats locates obstacles by emitting high frequency sound waves and detecting the reflected waves. A bat flying at a steady speed of 5ms^{-1} emits sound of frequency 78.0kHz and is reflected back to it.

(i) Derive the equation for the frequency of the sound waves reaching the bat after reflection. (05marks)

(ii) Calculate the frequency of the sound received by the bat given that the speed of sound in air is 340ms^{-1}. (02marks)

(e) (i) What is meant by **intensity** of a sound note. (01marks)

(ii) distinguish between **loudness** and **pitch** of a sound note. (01marks)

Uneb 2010

(a) (i) Define the terms **amplitude** of a wave (01marks)

(ii) State two characteristics of a stationary wave. (02marks)

(iii) A progressive wave $y = asin(\omega t - kx)$ is reflected at a barrier to interfere with the in coming wave. Show that the resultant wave is a stationary one. (04marks)

(b) (i) What is meant by **beats**. (02marks)

(ii) Describe how you can determine the frequency of a musical note using beats. (05marks)

(c) Two open pipes of length 92cm and 93cm are found to give beat frequency of 3.0Hz when each is sounding in its fundamental note. If the end errors are 1.5cm and 1.8cm respectively, calculate the;

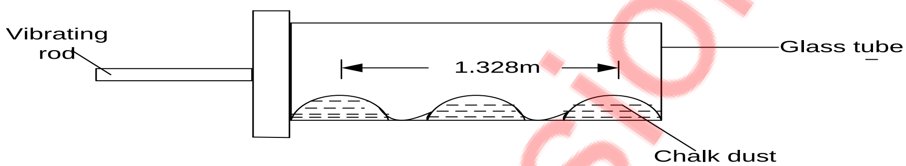
(i) Velocity of sound in air (04marks)

(ii) Frequency of each note. (02marks)

Uneb 2009

(a) (i) A progressive wave is represented by $y = asin2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ is reflected back along the same path. Show how the overlapping of the two waves will give rise to a stationary one. (03marks)

(ii) In an experiment to determine the speed of sound in air, a tube, chalk dust settled in heaps as shown below.



If the frequency of the vibrating rod is 252Hz and the distance between three consecutive heaps is 1.328m, calculate the speed of sound in air. (03marks)

(b) The speed of sound in air is given by $V = \sqrt{\frac{\gamma P}{\delta}}$ where p is pressure, δ is density and γ the ratio of the principal heat capacities of air.

Use this expression to explain the effect of temperatureon the speed of air. (03marks)

(c) (i) A train moving with uniform velocity, v_1, sounds its horn as it passes a stationary observer. Derive the expression for apparent frequency of the sound detected by the observer. (03marks)

(ii) If the frequency of thee sound detected by the observer after the train passes is 1.2times lower than the frequency detected in (c) (i), find the speed of the train. [speed of sound in air 340ms^{-1}]. (04marks)

(d) Describe a simple experiment to show interference of longitudinal waves. (04marks)

Uneb 2008

(a) (i) what is **a wave?**. (01mark)

(ii) Explain why an open tube is preferred to a closed tube as a musical instrument. (03marks)

(b) (i) State **two** factors that affect the speed of sound in air. (01mark)

(ii) Explain the term **reverberation**. (02marks)

(iii) What are the implications ooof reverberation in a concert hall? (02marks)

(c) Describe an experiment to determine the velocity of sound in air using a resonane method. (06marks)

(d) (i) What is **a harmonic** in sound (01marks)

(ii) A string of length 0.50m and mass $5.0g$ is stretched between two fixed points. If the tension in the string is 100N. Calculate the frequency of the second harmonic. (04marks)

Uneb 2007

(a) State **three** differences between sound and light waves. (03marks)

(b) Distinquish between **free oscillations** and **damped oscillations.** (02marks)

(c) (i) What is meant by **resonance**? (01mark)

(ii) Describe with aid of a diagram, an experiment to investigate the variation of frequency of a stretched string with length. (06marks)

(d) (i) Calculate the frequency of beats heard by a stationary observer when a source of sound of frequency 80Hz is receeding with a speed of 5.0ms^{-1} towards a vertical wall. [speed of sound in air =340ms^{-1}]. (05marks)

(ii) state **two** uses of beats (02marks)

Uneb 2006

(a) (i) What is meant by **amplitude** and **wavelenght** of a wave (02marks)

(ii) State the differences between a progressive and a stationary wave. (03marks)

(b) The displacement, y of a wave travelling in the x- direction is given at time, t by

$$y = asin2\pi\left(\frac{t}{0.5} - \frac{x}{2.0}\right) \text{ meters}$$

Find the peed of the wave. (04marks)

(c) (i) What is meant bythe terms **overtones** and **beats.** (03marks)

(ii) State **two** uses of beats. (02marks)

(d) A tube 1m long closed at one end has its lowest resonance freqeucny at 86.2Hz. With a tube of identical dimensions but open at both ends, the first resonance occurs at 171Hz. Calculate the speed of sound in air and end corrections. (04marks)

Uneb 2005

(a) Distguish between **progressive** and **stationary** waves. (04marks)

(b) Briefly describe an experiment to show that a wire under tension can vibrate with more than one frequency. (05marks)

(c) A uniform wire of length 1.00m and mass $2.0x10^{-2}kg$ is stretched between two fixed points. The tension of 200N. The wire is plucked in the middle and released. Calculate the ;

(i) Speed of the transverse waves (03marks)

(ii) frequency of the fundamental note. (03marks)

(d) (i) Explain how beats are formed (02marks)

(ii) Derive an expression for beat frequency. (03marks)

Uneb 2004

(a) (i) Distguish between **transverse** and **longitudinal** waves. (02marks)

(ii) Define the wavelenght of a wave. (01marks)

(b) Describe with the aid of a diagram, an experiment to show that how fundamental frequency varies with tension in a given wire. (06marks)

(c) A sound wave propagating in the x- direction is given by an equation

$$y = 2x10^{-7}sin2\pi(8000t - 25x) \text{ meters}$$

Find;

(i) The amplitude. (01marks)

(ii) The speed of the wave (05marks)

(d) Explain why the amplitude of a wave goes on decreasing as the distance from the source increases. (05marks)

WAVE THEORY OF LIGHT

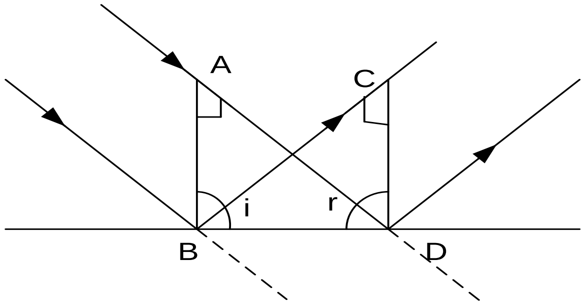
Huygens' principle

It states that every point on a wave front may be regarded as a source of secondary spherical wavelets which spread out with the wave velocity. The new wavefront is the envelope of these secondary wavelets.

Aplications of Huygens principle

(i) Reflection at palne surfaces

Consider a parallel beam of monochromatic light incident on a plane surface



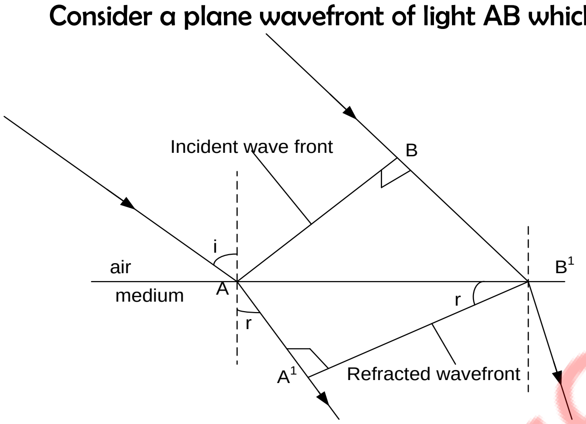
If particles A and B on the same wave fornt in time t, B travels to C while A travels to D

$BC = AD$ and $< BAD =< BCD = 90°$

Since BD is common then $< i =< r$

(ii) Refraction at plane boundary

Consider a plane wavefront of light AB which is about to cross form one medium into another



Let C and V be the velocities of light in air and the medium respectivelt.

If the wave particle at B takes time t to move to B^1, then the distance B$B^1 = Ct$.

In the same time interval wave particle at A moves to A^1, distance A$A^1 = Vt$

From triangle ABB^1 and AA^1B^1

$$\frac{\sin i}{\sin r} = \frac{\left(\frac{BB^1}{AB^1}\right)}{\left(\frac{AA^1}{AB^1}\right)} = \frac{BB^1}{AA^1} = \frac{Ct}{Vt} = \frac{C}{V}$$

But $\frac{C}{V} = n$, refractive index of the material

$$V = \frac{C}{n}$$

Note

When light moves from one medium to another, the frequency of light remains the same

If f_a and f be the frequencies of light in the vacuum (air) and in the medium then

$$f_a = f$$

$$n = \frac{C}{V} = \frac{f_a\lambda_a}{f\lambda}$$

$$n = \frac{C}{V} = \frac{\lambda_a}{\lambda}$$

Example

If the wavelenght of light in air is 620nm, find its wavelenght in a material of refractive index 1.6

Solution

$$n = \frac{C}{V} = \frac{f_a\lambda_a}{f\lambda}$$

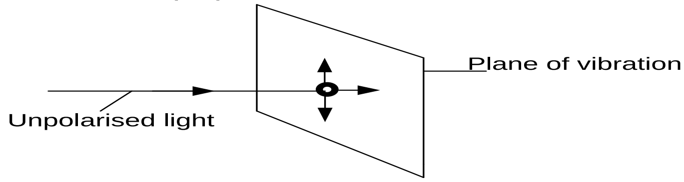
$$n = \frac{\lambda_a}{\lambda}$$

$$1.6 = \frac{620}{\lambda}$$

$$\lambda = 387.5nm$$

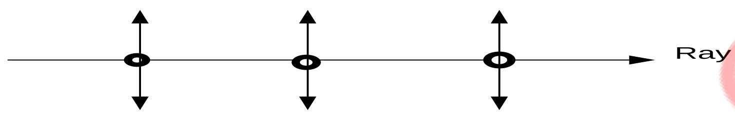
POLARISATION

Light is a transverse wave so its vibrations of electric vector occur in all directions perpendicular to the diection of propagation of light



Unpolarised light

This is light whose vibrations of the electric vectors occur in all direcions perpendicular to the direction of propagation of the wave.
Unpolarised light can be represented as below.



Where; ⚬ — Electric vibrations perpendicular to the plane

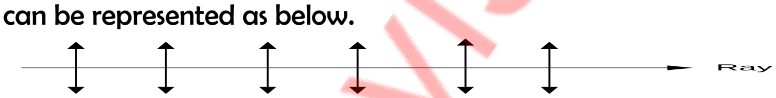
↕ — Represent vibrations parallel to the palne

Why sound waves can not be polarised

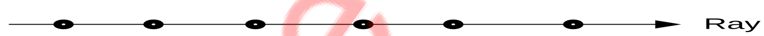
Sound waves are longitudinal waves, so its vibrations are parallel to the direction of propagation

Plane Polarised light

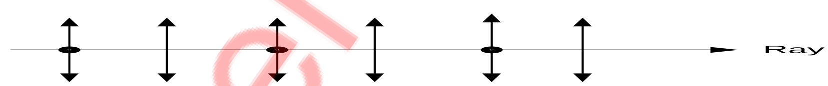
This is light whose vibrations of the electric vectors are confined to one plane perpendicular to the direction of propagation of the wave.
Polarised light can be represented as below.



Or

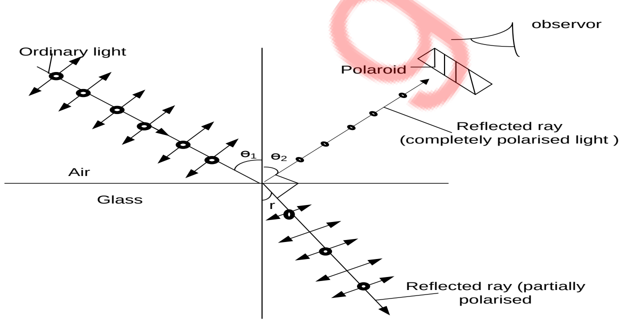


However light can under go partial polarization as shown below



PRODUCTION OF POLARISED LIGHT

(a) Reflection



- When light is incident on a boundary between air and glass, part of the light is partially reflected and the other partially transmitted into the denser medium.
- At one angle of incidence (polarizing angle), the reflected ray is completely plane-polarised while the refracted ray is partially plane polarized and the two rays are perpendicular to each other and vibrations in the reflected ray are parallel to the reflecting surface.

From sneils law: $n \sin i = constant$

$$n_a sin\theta_1 = n_g sinr \text{.........................(1)}$$
$$r + 90 + \theta_2 = 180$$
$$r = 90 - \theta_2$$

By law of reflection $\theta_1 = \theta_2 = \theta$

$$r = 90 - \theta$$

From (1) $n_a sin\theta_1 = n_g sin(90 - \theta_1)$

$$1xsin\theta = n_g cos\theta$$

$\boxed{n_g = tan\theta}$ Brewster's law

Where $\theta - polarising\ or\ Brewster\ angle$

Examples

1. The polarizing angle of ight incident in air on a glass plate is $56.5°$. What is the refractive index of glass

Solution

$n_g = tan\theta$ | $n_g = tan(56.5)$ | $n_g = 1.51$

2. A parallel beam of unpolarized light incident on a transparent medium of refractive index 1.62 is reflected as plane polarized light. Calculate the angle of incidence in air and the angle of refraction in the medium.

Solution

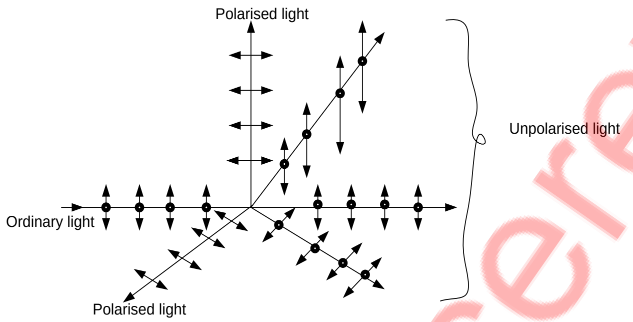
$$n_g = tan\theta$$
$$\theta = \tan^{-1}(1.62)$$
$$\theta = 58.3°$$

$$r = 90 - \theta$$
$$r = 90 - 58.3$$
$$r = 31.7°$$

Exercise

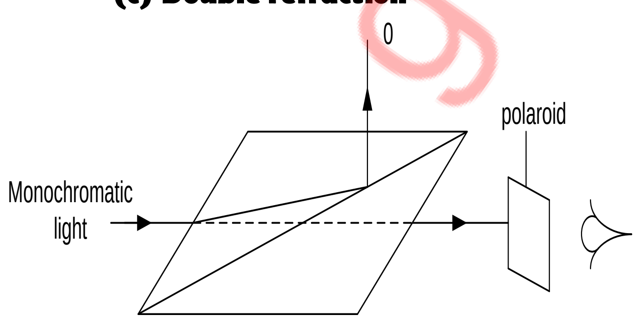
The polarising angle for light in air incident on a glass plate is 57.5°, what is the refractive index of the glass? **An(1.57)**

(b) By scattering



- When plane unpolarised light is incident on air molecules part of it is scattered.
- The light that passes through the air molecules is unpolarised and the light that is sattered in the direction perpendicular to the incident ray is polarized totally.

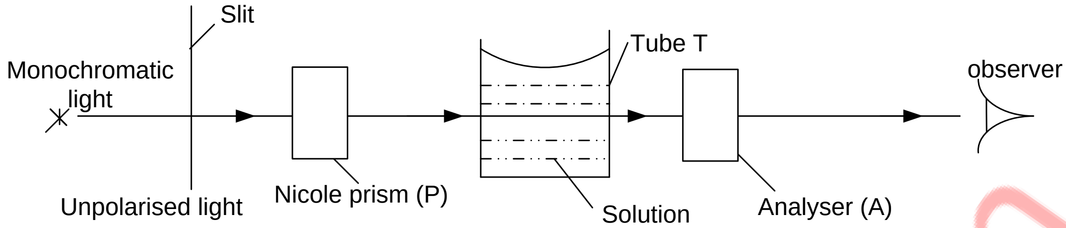
(c) Double refraction



- A narrow beam of ordinary light is made incident on a nicol prism and viewed through the analyser.
- The angle of incidence is gradually increased.
- For each angle of incidence, the emergent light is viewed through the analyser while rotating it about an axis perpendicular to the palne of the analyser.
- At a certain angle of incidence light gets cut off completely. At this point the emergent light is completely plane polarized.

Applications of polarization

(a) **Measurement of concentration of sugar in solution**



- Apparatus is arranged as above.
- With out tube T in place, the analyser A is rotated until the emergent light from T is completely cut off. The position of A is noted.
- Tube T is filled with a solution to be tested and on looking through A, light can now be seen
- Viewing through the analyzer A, it is rotated until when light is cut off and note this point.
- Measure the angle of rotation θ of the analyser.
- The concentration of the solution is proportional to the angle of rotation therefore the concentration can be determined

(b) **Reducing glare in sun glasses**

Polaroids are used in sun glasses to reduce intensity of incident sunlight and eliminate the reflected light.
When a polarized quoting is applied on sun glasses, the reflected light is partially or completely polarized and thus glare is reduced

Other applications include

- Holography
- In phot elasticity for stress analysis
- Used in L.C.D's

INTERFERNCE

Interference of waves is the superposition of waves from different two coherent soruces resulting into alternate regions of maximum and minimium intensity.
Where the path difference is an odd multiple of half a wavelength, cancellation occurs resulting itno minimum intensity. Where the path difference is an integral multiple of a full wavelength, reineforcement occurs resulting into maximum intensity.

Coherent sources

These are sources whose waves have the same frequency but nearly the same amplitude and a constant phase difference.

Conditions for observable interference to take place

- Wave trains must have nearly equal amplitudes
- There must be a constant phase relationship between the two wave trains (Wave sources must be coherent).
- The coherent sources must be close to each other.
- The screen should be as far as possible from the source

Types of interference

- Constructive interference
- Destructive interference

(a) Constructive interference

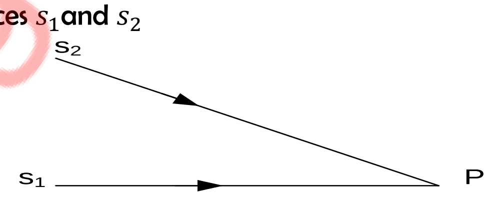
This is the re-enforcement of the intensities of two coherent sources to give maximum intensity when two wave distrubances from two sources are superimposed.
It takes place when a crest of one wave meets a crest of another wave and a troug meets a trough resulting into a large resultant amplitude.

(b) Destructive interference

This is the cancellation of two intensities of two coherent sources to give minimum intensity when two wave distrubances from two sources are superimposed.
It takes place when a crest of one wave meets a trough of another wave resulting into a small resultant amplitude.

Path difference

This is the difference in the length of the path taken by two waves from the source to a point of over lap.
Where they meet, the two waves superpose leading to reinforcement or cancellation. Where the path difference is an integral multiple of a full wavelength constructive interfence takes place. Where the path difference is an odd multiple of half a wavelength, destructive interference takes place.
Consider two coherent sources $s_1$and s_2



Wave forms from $s_1$and s_2 meet at P after taveling difffernt distances.
Waves from $s_1$travel a distance s_1P while waves from s_2 travel a distance s_2P
But $s_2P \; > s_1P$
$s_2P - \; s_1P = path\ difference$

If the path difference is zero or a whole number of wavelength. Then the brght band (constructive interference) will be formed

$\boxed{s_2P - s_1P = n\lambda}$ $n = 0,1,2,3$ — — — — — — — — — — — —

If the path difference is an odd number of half wavelength. Then the darkband (destructive interference) will be formed

$\boxed{s_2P - s_1P = (n - 1/2)\lambda}$ $n = 1,2,3$ — — — — — — — — — — — —

Optical path

It is the length in a medium that contains the same number of waves as a given length in a vacuum. OR

This is the product of the geometrical path length in air and refractive index of the medium

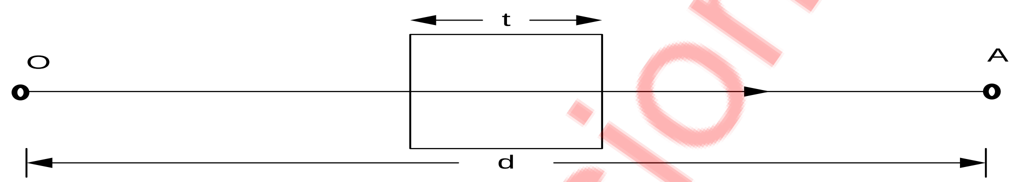
Consider a light travellign from O to A a distance d in air.

Optical path $= n_a d$

But $n_a = 1$

Optical path $= d$

If a thin transparent slab of thickness t and refractive index n is placed between O and A



Optica path between O and A is

Optical path $= n_a(d - t) + nt$

$n_a = 1$

Optical path $= d + (n - 1)t$

Phase diffference

This is the difference in the phase angles of two wave at a given time

Consider light travelling a distance x in the medium of refractive index n. if the wavelength of the medium is λ then, the phase difference, Ø is given by

$$\boxed{\emptyset = \frac{2\pi x}{\lambda}}$$

$\Rightarrow$ Phase difference $= \frac{2\pi}{\lambda}(optical\ path\ difference)$

When crests of two waves meet, then waves are said to be in phase

$\Rightarrow$ Phase difference $= 0$

Hence constructive interference occurs

When crest and trough of two waves meet, then waves are said to be out of phase

$\Rightarrow$ Phase difference $= \pi$

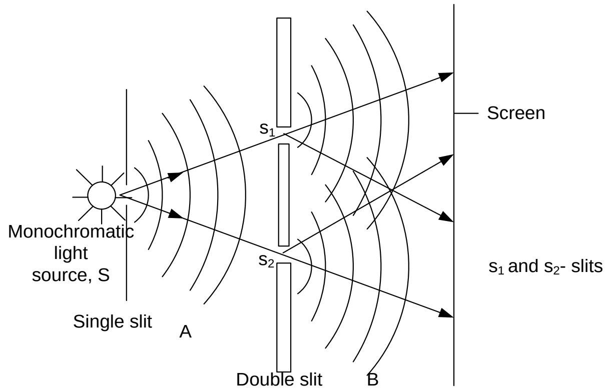
Hence destructive interference occurs

Production of coherent sources from a single source of light

(a) By division of a wave front

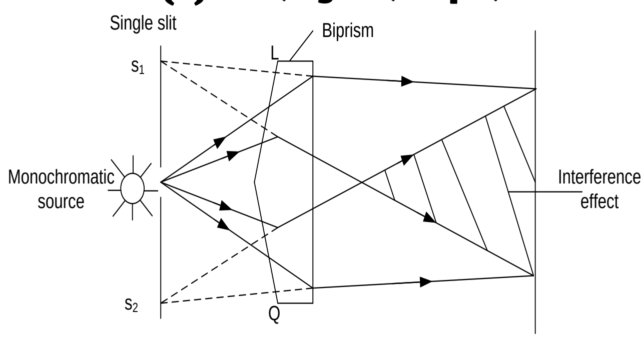
This is the process of obtaining <u>two coherent wave sources</u> from <u>a single wavefront</u>

(i) Using a double slit



- S, s_1 and s_2 are narrow slits which are parallel to each other.
- Waves from source, s diffreact into region and travel towards s_1 and s_2
- Diffraction also takes place at s_1 and s_2 and interference occurs in the region where the light from s_1 overlaps that from s_2
- Since s is narrow, the light which emerges from s_1 and s_2 comes form the same wave front as that which emerges from s. thus s_1 and s_2 are coherent

(ii) Using Fresnel prism



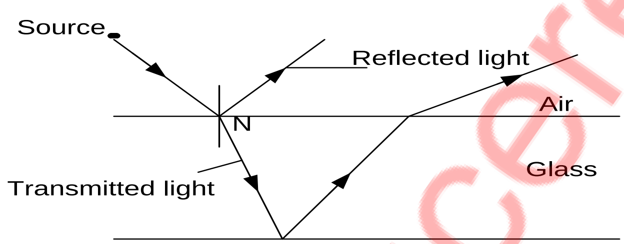
- A biprism of very large angle is placed with its refracting edge facing a narrow source of monochromatic light, s
- Light incident on face L is refracted and appear to come from a point S_1 and that incident on Q appears to come from S_2 due to refraction.
- The two sources are thus coherent since the light which emergent originates from the same wave front

Note : Biprism method is always preferred because it produces brighter fringes since the biprism converges most of the light on to the screen

(b) By division of amplitude

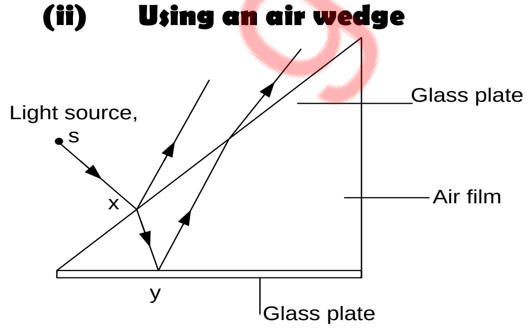
This is the process of dividing the amplitude into two parts by successive reflections

(i) When light is incident on a boundary of two media



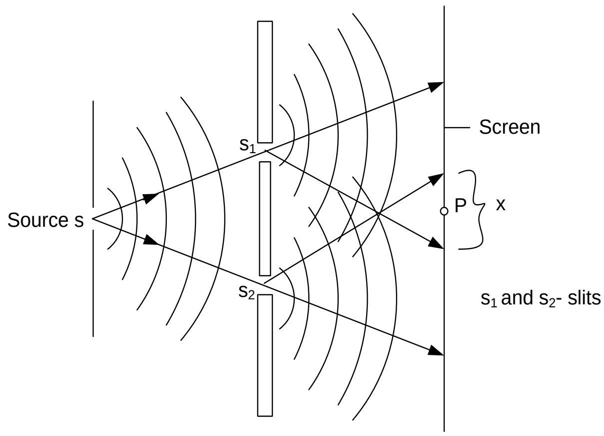
- When light is incident on a boundary between air and glass, part of the light is partially reflected and the other partially transmitted into the denser medium.
- At N, there is divison of intensity and since intensity is proportional to the square root of amplitude, then division of amplitude at N takes place.

(ii) Using an air wedge



- Monochromatic light is mad incident almost normally onto the upper glass slide.
- It is partly reflected at X and partly transmitted in the air film and reflected at Y.
- The light reflected at X and Y are coherent. When they overlap above the upper slide, they interfere
- Where the path difference is an odd mu;tiple of half a wavelenght, bright fringe is formed and where the path difference is an integral multiple of a full wavelength, a dark fringe is formed .

Young's double slit interference



When a wave front from the source, s is incident on a double slit s_1 and s_2, division of wave front take place and therefore s_1 and s_2 act as coherent sources. Waves from s_1 and s_2 superimpose in region x and interference takes place.

When a crest from s_1 meet a crest from s_2 and a trough from s_1 meets a trough from s_2 then maximum interference is achieved and a brght fringe is formed (constructive interference).

When a crest from s_1 meet a trough from s_2 and a trough from s_1 meets a cest from s_2 then minimum interference is achieved and a dark fringe is formed (destructive interference).

This results into a series of alternating dark and bright bands which areequally spaced and are parallel to the slits.

At the central point P waves from s_1 and s_2 travel equal distances and they arrive at the same time (they are in phase). This implies constructive interference hence a bright fringe is formed at P

Note;

(i) When one of the double slits is covered, no interference takes place

(ii) When the source of monochromatic light is moved close to the slits, the intensity increases and bands become brighter

(iii) When the distance between the double slits and single slit is reduced, fringe separation remains the same but bands become bright since the intensity increases

(iv) When the doube slit separation is reduced, the fringe seapraion increases and when the slit separation is increased, the fringe separation decreases unitl a stage is reached when no fringes are observed

Effect of using white light other than monochromatic light

Sets of coloured fringes are seen on the screen. The central fringe is white, with coloured fringes on either side. For each set, blue fringes is nearest to the central one while red is furthest.

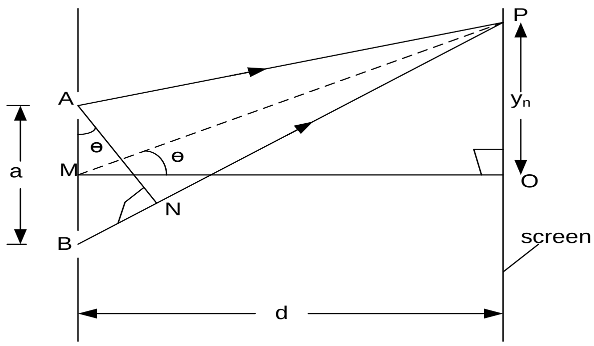
Effect of widening the single slit,s

The fringes gradually disappear. The slit s is now equivalent to a large number of slits each producing its own fringe system on the screen. The fringe systems overlap producing uniform illumination

Effect of narrowing the double slit separation

- When the slit separation is large $(a \gg \lambda)$, bright band of approximately the same width as the slit is observed.
- As the slit width is reduced so that $a \approx \boldsymbol{\lambda}$, a diffraction pattern is observed. A central white band having dark bands on either sides is obtained. The dark bands have coloured fringes running from blue to red, the blue fringes being nearest to the direction position
- As the slit width is reduced further $(a < \lambda)$, the central bright band widens and extends well into the geometrical shadow of the slit.
- When the slit finally closes, no light passes through.

Derivation of fringe separation



Suppose waves from A and B superpose at P to form <u>a bright fringe</u>

$path\ difference, BN = BP - AP = a sin\theta \ldots(\mathbf{1})$

For $d \gg a$, θ is very small in radians and

$sin\theta \approx tan\theta$

$BN = a tan\theta = \frac{ay_n}{d}$......................(2)

For n^th <u>bright fringe</u> at P

$path\ difference, BN = n\boldsymbol{\lambda} \ldots(\mathbf{3})$

where $\lambda - wavelenght$

$\Rightarrow \frac{ay_n}{d} = n\boldsymbol{\lambda}$

$\boxed{y_n = \frac{n\boldsymbol{\lambda}}{a}d}$..........................(4)

For $(n+1)^{th}$ bright fringe

$y_{n+1} = \frac{(n+1)\boldsymbol{\lambda}}{a}d$..........................(5)

Fringe separation

$\boldsymbol{y} = y_{n+1} - y_n$

$\boldsymbol{y} = \frac{(n+1)\boldsymbol{\lambda}}{a}d - \frac{n\boldsymbol{\lambda}}{a}d$

$\boxed{\boldsymbol{y} = \frac{\boldsymbol{\lambda d}}{a}}$

For dark fringes

Suppose waves from A and B superpose at P to form <u>a dark fringe</u>

$path\ difference, BN = BP - AP = a sin\theta \ldots(\mathbf{1})$

For $d \gg a$, θ is very small in radians and

$sin\theta \approx tan\theta$

$BN = a tan\theta = \frac{ay_n}{d}$......................(2)

For n^th <u>dark fringe</u> at P

$path\ difference, BN = \left(n + \frac{1}{2}\right)\boldsymbol{\lambda} \ldots(\mathbf{3})$

where $\lambda - wavelenght$

$\Rightarrow \frac{ay_n}{d} = \left(n + \frac{1}{2}\right)\boldsymbol{\lambda}$

$\boxed{y_n = \left(n + \frac{1}{2}\right)\frac{\lambda\, d}{a}}$..........................(4)

For $(n+1)^{th}$ dark fringe

$y_{n+1} = \left(n + 1 + \frac{1}{2}\right)\frac{\lambda\, d}{a}$..........................(5)

Fringe separation

$\boldsymbol{y} = y_{n+1} - y_n$

$\boldsymbol{y} = \left(n + 1 + \frac{1}{2}\right)\frac{\boldsymbol{\lambda}\, d}{a} - \left(n + \frac{1}{2}\right)\frac{\boldsymbol{\lambda}\, d}{a}$

$\boxed{\boldsymbol{y} = \frac{\boldsymbol{\lambda d}}{a}}$

Examples

1. In Youngs double slit experiment, 21 bright fringes occupying a distance of 3.6mm were visible on the screen. The distance of the screen form the double slit was 29cm and the wavelength of light used in the experiment was $5.5x10^{-7}m$. Calculate the separation of the slits.

Solution

$y = \frac{3.6x10^{-3}}{21}$

y= $0.171x10^{-3}m$

$\frac{\boldsymbol{y}}{\boldsymbol{d}} = \frac{\boldsymbol{\lambda}}{a}$

$\boldsymbol{a} = \frac{5.5x10^{-7}x29x10^{-2}}{0.171x10^{-3}}$

$\boldsymbol{a} = 9.327x10^{-4}m$

2. In Youngs double slit experiment, the slits are separated by 0.28mm and the screen is 4m away. The distance betwee the 4^th bright fringe and the central fringe is 1.2cm. Find the wavelength of light used in the experiment.

Solution

Fringe separation;

$y_n = \frac{n\boldsymbol{\lambda}}{a}d$

$y = y_4 - y_0$

$y = \frac{4\boldsymbol{\lambda}}{a}d - \frac{0\boldsymbol{\lambda}}{a}d$

$\Delta y = \frac{4\boldsymbol{\lambda}}{a}d$

$1.2x10^{-2} = \frac{4x4x\boldsymbol{\lambda}}{0.28x10^{-3}}$

$\lambda = 2.1x10^{-7}m$

3. In Youngs double slit experiment, the 6^th bright fringe is formed 4mm away from the center of the fringe system when the wave length of the light used is $6.0x10^{-7}m$. Calculate the separation of the two slits if the distance from the slits to the screen is 60cm.

Solution

$$y_n = \frac{n\lambda}{a}d$$
$$y = y_6 - y_0$$
$$y = \frac{6\lambda}{a}d - \frac{0\lambda}{a}d$$
$$y = \frac{6\lambda}{a}d$$
$$4x10^{-3} = \frac{6x6.0x10^{-7}x0.6}{a}$$
$$a = 5.4x10^{-4}m$$

4. In Youngs double slit experiment, the 8th bright fringe is formed 5mm away from the center of the fringe system when the wave length of the light used is $6.2x10^{-7}m$. Calculate the separation of the two slits if the distance from the slits to the screen is 80cm.

Solution

$$y_n = \frac{n\lambda}{a}d$$
$$y = y_8 - y_0$$
$$y = \frac{8\lambda}{a}d - \frac{0\lambda}{a}d$$
$$y = \frac{8\lambda}{a}d$$
$$5x10^{-3} = \frac{8x6.2x10^{-7}x0.8}{a}$$
$$a = 7.94x10^{-4}m$$

5. In Youngs double slit experiment, the slits 0.2mm apart and are placed a distance of 1m from the screen. The slits are illuminated with light of wavelength 550nm. Calculate the distance between the 4th and 2nd bright fringes of interference patterns.

Solution

Bright fringe position;
$$y_n = \frac{n\lambda}{a}d$$
$$y = y_4 - y_2$$
$$y = \frac{4\lambda}{a}d - \frac{2\lambda}{a}d$$
$$y = \frac{2\lambda}{a}d$$
$$y = \frac{2x550x10^{-9}x1}{0.2x10^{-3}}$$
$$y = 5.5x10^{-3}m$$

6. In Youngs double slit experiment, the distance between adjacent bright fringes is $10^{-3}m$. If the distance between the slits and the screen is doubled, the slit separation halved and light of wavelength 650nm changed to light of wavelenght 400nm. Find the new separation of the fringes.

Solution

$$y = \frac{\lambda}{a}d$$

Case 1:
$$10^{-3} = \frac{650x10^{-9}d}{a} \quad ....(1)$$

Case 2:
$$y = \frac{400x10^{-9}(2d)}{\left(\frac{1}{2}a\right)} \quad ....(2)$$

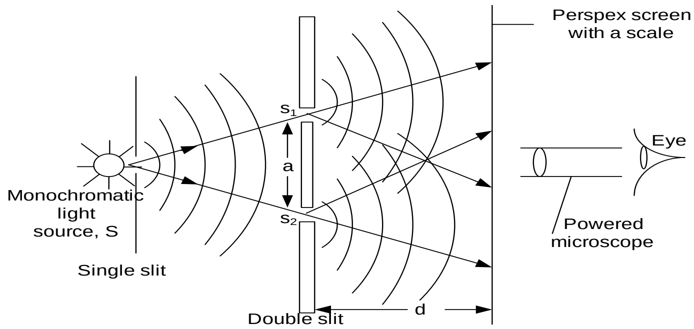
$equation\ 2 \div equation\ 1$

$$\frac{y}{10^{-3}} = \frac{\left[\frac{400x10^{-9}(2d)}{\left(\frac{1}{2}a\right)}\right]}{\left[\frac{650x10^{-9}d}{a}\right]}$$
$$y = 492m$$

Exercise

1. In Young's double-slit experiment, the 5th bright fringe is formed 7 mm away from the centre of the fringe system when the wavelength of light used is $4.6\ x\ 10^{-7}$ m. Calculate the separation of the two slits if the distance from the slits to the screen is 90 cm. **An($2.96\ x\ 10^{-4}m$)**
2. Two slits 0.5mm apart are placed at a distance of 1.1m from the screen. The slits are illuminated with light of wavelength 580nm. Calculate the distance between the sixth and second bright fringes of the interference pattern. **An($5.1\ x\ 10^{-3}m$)**
3. In Young's experiment, an interference pattern in which the tenth bright fringe was 3.4 cm from the centre of the pattern was obtained. The distance between the slits and the screen was 2.0m while the screen separation was 0.34mm. Find the wavelength of the light source **An($5.78\ x\ 10^{-7}m$)**

Experiment to measure wavelength of light using Youngs double slit interference



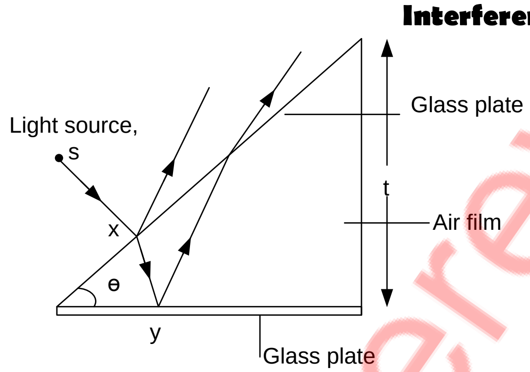
- Apparatus is arranged as above.
- A monochromatic light is used to illuminate double slits s_1 and s_2.
- The microscope is placed at such a distance d that fringes are observed in its field of view
- The number of bright fringes in a fixed length on the screen is counted and the fringe separation y is determined
- Measure the distance d using a meter rule.
- Measure the slit separation a using a travelling microscope.
- Wavelenght of light can be calculate from

$$\lambda = \frac{ya}{d}$$

Comparing wavelenght of red light and blue light

- Apparatus is arranged as above.
- A source of white light is used and a red filter filter is palced infornt of the slit, s
- The number of bright fringes in a fixed length on the screen is counted and the fringe separation y_r is determined
- The filter is now replaced by a blue one and the experiment repeated, and the fringe separation y_b determined.
- It is found that $y_r > y_b$ and since $\lambda = \frac{ya}{d}$ then $\lambda \propto y$
- Wavelenght of red light is greater than that of
- blue light

Interference in thin films



- Monochromatic light is mad incident almost normally onto the upper glass slide.
- It is partly reflected at the bottom part of X and partly transmitted into the air film and reflected at the top surface of Y.
- The light reflected at X and Y are coherent. When they overlap above the upper slide, they interfere
- Where the path difference is an odd multiple of half a wavelenght, bright fringe is formed and where the path difference is an integral multiple of a full wavelength, a dark fringe is formed.

Note : when white light is used coloured fringes are observed

Derivation of fringe separation

Consider two slides inclined at an angle θ

For n dark fringes

Path difference, $2t_n = n\lambda \ldots\ldots\ldots\ldots (1)$

where n=1,2,3.....

For $(n+1)^{th}$ dark fringes

$2t_{n+1} = (n+1)\lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$

where n =0,1,2.....

Eqn 2- Eqn 1

$$2t_{n+1} - 2t_n = (n+1)\lambda - n\lambda$$

$$t_{n+1} - t_n = \frac{\lambda}{2}$$

$$\tan\theta = \frac{t_{n+1} - t_n}{y_{n+1} - y_n}$$

$$y_{n+1} - y_n = y$$

$$\tan\theta = \frac{\lambda}{2y}$$

Since θ is very small in radians, $\tan\theta \approx \theta$

$$\boxed{y = \frac{\lambda}{2\theta}}$$

For n bright fringes

Path difference, $2t_n = \left(n - \frac{1}{2}\right)\lambda \ldots\ldots\ldots\ldots(1)$

For $(n+1)^{th}$ dark fringes

$$2t_{n+1} = \left(n + 1 - \frac{1}{2}\right)\lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Eqn 2- Eqn 1

$$2t_{n+1} - 2t_n = \left(n + 1 - \frac{1}{2}\right)\lambda - \left(n - \frac{1}{2}\right)\lambda$$

$$t_{n+1} - t_n = \frac{\lambda}{2}$$

$$\tan\theta = \frac{t_{n+1} - t_n}{y_{n+1} - y_n}$$

$$y_{n+1} - y_n = y$$

$$\tan\theta = \frac{\lambda}{2y}$$

Since θ is very small in radians, $\tan\theta \approx \theta$

$$\boxed{y = \frac{\lambda}{2\theta}}$$

Examples

1. Two glass slides in contact at one end are separated by a wire to form an air wedge. When the wedge is illuminated normally by light of wavelenght $5.6x10^{-7}m$ a total of 20 firnges occupying a distance of 15mm are obtained. Calculate the angle of the wedge.

Solution

$$y = \frac{15x10^{-3}}{20} = 0.75x10^{-3}$$

$$\tan\theta = \frac{\lambda}{2y}$$

$$\theta = \tan^{-1}\left(\frac{5.6x10^{-7}}{2x0.75x10^{-3}}\right)$$

$$\theta = 0.021°$$

2. Two glass slides in contact at one end are separated by a wire of diameter 0.04mm at the other end to form awave fringes observed when light of wavelenght $5x10^{-7}m$ is incident normally onto the slides. Find the number of dark fringes tha can be observed

Solution

For dark fringes

$$2t_n = n\lambda$$

$$n = \frac{2x0.04x10^{-3}}{5x10^{-7}}$$

$$n = 160\ dark\ fringes$$

3. Two glass slides in contact at one end are separated by a sheet of paper 16cm from the the line of contact, to form an air wedge. When the wedge is illuminated normally by light of wavelenght $5.8x10^{-7}m$ interference firnges of separation 2.0mm are obtained in reflection. Calculate the thickness of the paper.

Solution

$$\tan\theta = \frac{\lambda}{2y} = \frac{t}{16x10^{-2}}$$

$$\frac{\lambda}{2y} = \frac{t}{16x10^{-2}}$$

$$y = \frac{5.8x10^{-7}x16x10^{-2}}{2x2.0x10^{-3}}$$

$$y = 2.32x10^{-5}m$$

4. Two glass slides in contact at one end are separated by a metal foil 12.5cm from the the line of contact, to form an air wedge. When the wedge is illuminated normally by light of wavelenght $5.4x10^{-7}m$ interference firnges of separation 15mm are obtained. Calculate the thickness of the metal foil.

Solution

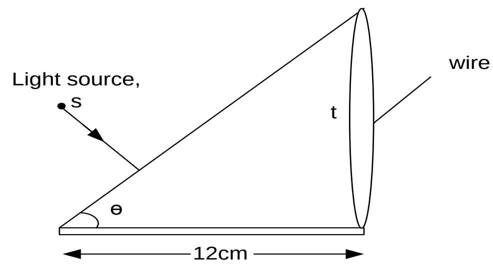
$$\tan\theta = \frac{\lambda}{2y} = \frac{t}{12.5x10^{-2}}$$

$$\frac{\lambda}{2y} = \frac{t}{12.5x10^{-2}}$$

$$y = \frac{5.4x10^{-7}x12.5x10^{-2}}{2x1.5x10^{-3}}$$

$$y = 2.25x10^{-5}m$$

5. Two glass slides 12cm long are in contact at one end and separated by a metal wire of diameter $2.5x10^{-3}cm$ at the other end. When the slides are illuminated normally as shown below with the light of wavelenhgt 500nm, a fringe system is observed



Calculate;

(i) Fringe separation
(ii) Number of dark fringes formed
(iii) Number of bright finges formed

Solution

(i) $\tan\theta = \frac{\lambda}{2y} = \frac{t}{12x10^{-2}}$

$$\frac{\lambda}{2y} = \frac{t}{12x10^{-2}}$$

$$y = \frac{500x10^{-9}x12x10^{-2}}{2x2.5x10^{-5}}$$

$$y = 1.2x10^{-3}m$$

(ii) For dark fringes

$$2t_n = n\lambda$$

$$n = \frac{2x2.5x10^{-5}}{500x10^{-9}}$$

$$n = 100\ dark\ fringes$$

(iii) For bright fringes

$$2t_n = \left(n+\frac{1}{2}\right)\lambda$$

$$n = \frac{2t_n}{\lambda} - \frac{1}{2}$$

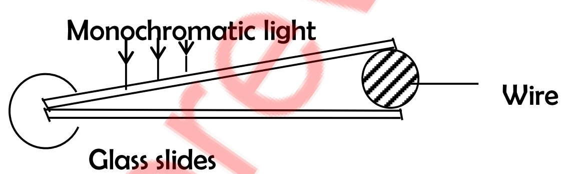
$$n = \frac{2x2.5x10^{-5}}{500x10^{-9}} - \frac{1}{2}$$

$$n = 99\ bright\ fringes$$

Exercise

1. Two glass slides in contact at one end are separated by a metal foil 12.50 cm from the line of contact, to form an air-wedge. When the air-wedge is illuminated normally by light of wavelength 5.4 x 10 $^{-7}$ m interference fringes of separation 1.5 mm are found in reflection. Find the thickness of the metal foil. **An** $(\mathbf{2.25x10^{-5}m})$

2. An air wedge is formed by placing two glass slides of length 5.0 cm in contact at one end and a wire at the other end as shown in figure 2



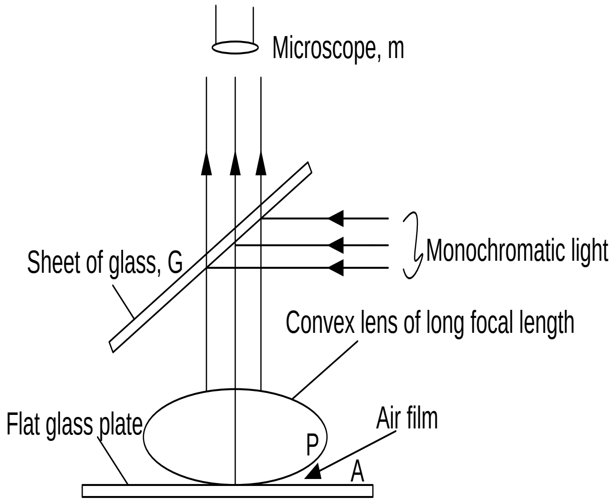
Viewing from vertically above, 10 dark fringes are observed to occupy a distance of 2.5 mm when the slides are illuminated with light of wavelength 500 nm.

(i) Explain briefly how the fringes are formed
(ii) Determine the diameter of the wire

Briefly explain why interference effect are not observed in thick films (air wedges)

- Bright fringes occur when the path difference for the wavelength is equal to $\left(n-\frac{1}{2}\right)\lambda$ where $n = 1,2,3...$
- When the film is thick, each colour attains this path difference forming bright band. The different colours thus overlap leading to uniform white illumination (blurring of the fringes).

NEWTON'S RINGS



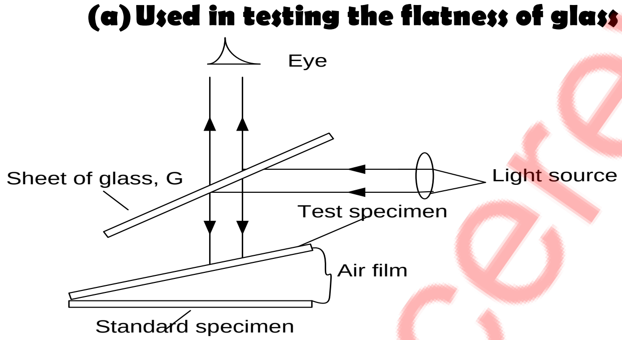
❖ When a convex lens of long focal length is made to rest on an optical flat glass plates, a layer of air between the lens and the plate acts as an air wedge.
❖ Monochromatic light is reflected by glass plate G such that it falls normally on an air film formed between the convex lens and the flat glass plate.
❖ Light reflected upwards and transmitted through G is observed through a travelling microscope M. A series of dark and brght rings is observed. Light rays from P and A interfere constructively if the path difference is
$2t_n = \left(n - \frac{1}{2}\right)\boldsymbol{\lambda}, \boldsymbol{n} = \mathbf{1}, \mathbf{2}, \mathbf{3} \ldots \ldots$
❖ If rays interfere destructively if the path difference is $2t_n = n\boldsymbol{\lambda}, \boldsymbol{n} = \mathbf{1}, \mathbf{2}, \mathbf{3} \ldots \ldots$
❖ Thus interference patterns observed consist of a series of dark and bright rings with a central sport being dark

Appearance of colours on an air film

❖ Colours on an oil film on a water surface appear due to interference of light
❖ When light from the sky meets the oil film, it is partially reflected and partially refracted. The refracted light is totally internally reflected at the oil- water boundary.
❖ When the colours reach the eye, they interfere. The interference colours for which the waves are in phase are seen while those for which they are out of phase are not seen. The particular colour seen depends in the positon of the eye

Applications of interference

(a) Used in testing the flatness of glass surface



➢ The surface under test is made to for an air wedge with a plane galss surface of standard smoothness
➢ When a parallel beam of monochromatic light from source s is reflected from the glass G, it falls almost normally to the air wedge
➢ Interference fringes caused by the air wedge between the plate are observed
➢ Irregularities in the surface of the test specimen will show up when unparalllel, equal spaced fringes are formed.

(b) Blooming of lenses

➢ When light is incident on a lense, some percentage of the light is reflected from each surface. This reults into reduction in intesniyt of light due to losss of light being transmitted. This reduces clarity of the finaal image produced.
➢ This defect therefore can be reduced by evaporating a thin coating of magnesium fluoride unot the lense surface. This process is called blooming

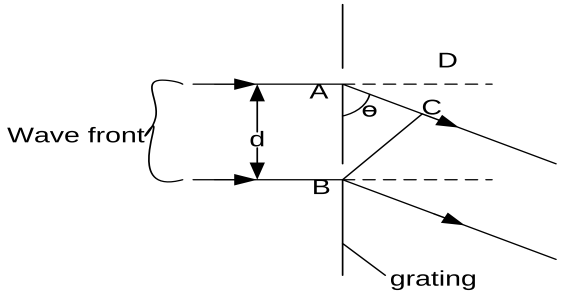
DIFFRACTION OF LIGHT

This is the spreading of waves beyond geometrical boundaries leading to interference

Diffraction grating

This is a transparent plate with many equidistant small parallel lines drawn on it using a diamond pencil

Explanation of formation of fringes by transmission grating



- Consider a transmission grating of narrow slits AB whose width is compared to the wavelength of light illuminated normally by monochromatic light
- Light is diffracted through spaces of the grating into region D where they superpose.
- Where the resultant path difference of wave through a pair f consecautive slits is an integral multiple of full wavelength, constructive interference occurs and a bright band is formed. Where the resultant path difference is an odd multiple of half a wavelenght, destructive inteference occurs and a dark band is formed
- This spreading of light along the obstacle beyond the geometrical shadow leading to interference pattern is called diffraction.

Effect of increasing the number of narrow slits in the diffraction grating on intensity

- When number of slits are increased, the intensity of the principal maxima increases and the subsidiary decreases.
- The interference at the principal maxima are always constructive hence intensity increases. Interference at the subsidiary maxima are destructive hence intensity decreases

Note;

(i) For diffraction grating
- Lines are ruled on glass
- The spaces transmit light

(ii) For reflection grating
- Lines are ruled on a polished metal
- The spaces reflect light

Condition for diffraction maxima

Consider a transmission diffraction grating of spacing d illuminated normally with light of wavelength λ.
Path difference between waves form A and B(distance BC)$= dsin\theta$
For diffraction maxima, path difference $= n\lambda$
$dsin\theta = n\boldsymbol{\lambda}$ **where** $\boldsymbol{n = 0, 1, 2}$

Examples

1. Sodium light of wavelenght 589nm falls normally on a diffraction grating which has 600 lines per mm. calculate the angle between the directions in which the first order maxima, on the same side of the straight through positions are observed.

Solution

$dsin\theta = n\lambda$

$$\boldsymbol{\theta = sin^{-1}\left(n\lambda x\frac{1}{d}\right)}$$

But $\frac{1}{d} = \frac{600}{10^{-3}}$ *lines per meter* and for first ordr maxima $n = 1$

$$\theta = sin^{-1}(1x589x10^{-9}x600x10^{3})$$

$$\theta = 20.70°$$

2. When monochromatic light of wavelenght 600nm is incident normally on a transmission grating, the second order diffraction image is observed at an angle of 30°. Determine the number of lines per centimeter on the grating

Solution

$dsin\theta = n\lambda$

$\frac{1}{d} = \frac{sin\theta}{n\lambda}$

$\frac{1}{d} = \frac{sin30}{2x600x10^{-9}}$

$\frac{1}{d} = 4.17x10^5$ *lines per meter*

$\frac{1}{d} = 4.17x10^3$ *lines per cm*

3. A diffraction grating of 600 lines per mm is illuminated normally by monochromatic, the first order maxima is observed at an angle of 20°. Find the;

(i) The wavelenght of the light

(ii) number of diffracton maxima possible

Solution

(i) $dsin\theta = n\lambda$

$\lambda = \frac{dsin\theta}{n}$

$\frac{1}{d} = \frac{600}{10^{-3}}$ *lines per meter* and for first order $n = 1$

$\lambda = \frac{\left(\frac{10^{-3}}{600}\right)sin20}{1}$

$\lambda = 5.7x10^{-7}m$

(ii) $dsin\theta_{max} = n_{max}\lambda$

But $sin\theta_{max} = 1$

$n_{max} = \frac{d}{\lambda}$

$n_{max} = \frac{\left(\frac{10^{-3}}{600}\right)}{5.7x10^{-7}} = 2.92$

Maxima value $n = 2$

4. A diffraction grating of 500 lines per mm is illuminated normally by light of wavelenght 526nm. Find the total number of images seen

Solution

$dsin\theta_{max} = n_{max}\lambda$

But $sin\theta_{max} = 1$

$n_{max} = \frac{d}{\lambda}$

$\frac{1}{d} = \frac{500}{10^{-3}}$ *lines per meter*

$n_{max} = \frac{\left(\frac{10^{-3}}{500}\right)}{526x10^{-9}} = 3.8$

Total number of images seen is **7**

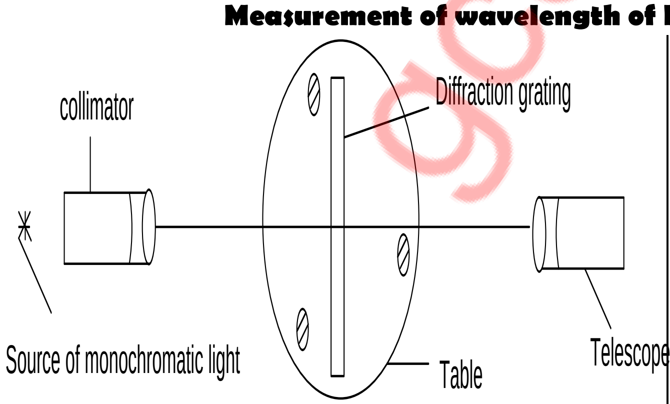
Uses of diffraction

(i) Measurement of the wavelenght of light usng a diffraction graitn

(ii) Used in spectrographic studies

(iii) Used in holograms(3-D photographs)

Measurement of wavelength of light using diffraction grating



- The telescope is adjusted to focus parallel light. The collimator is adjusted to produce parallel light and the table is leveled.
- The grating is placed on the table so that its plane is perpendicular to the incident light
- Zero order image is now received at the telescope. This position on T_1 on the scale is noted. The telescope is now turned in one direction until the first order image is obtained. The angle θ_1 of rotation form position T_1 is recorded.
- The telescope is restored to position T_2 and rotated in the opposite direction until the first

order image is again obtained. The angle θ_2 of rotation from T_1 noted.

❖ The wavelength λ is calculated from

$\lambda = d\sin\left(\frac{\theta_2+\theta_1}{2}\right)$ where d is the spacing of the grating

Condition for observable diffraction

The dimension of the obstacle must be of the same orde as the wavelenght of the light

Differences between spectra produced by a prism and tat by a diffraction grating

Prism	Diffraction grating
— Produce single spectrum at a time	Produce many spectra at a time
— Shorter wavelengths are deviated most	Longer wavelengths are deviated most
— Produce less pure spectrum	Produce more pure spectrum

Exercise

1. When monochromatic light of wavelength $5.8\ x\ 10^{-2}\ \ m$ is incident normally on a transmission grating, the second order diffraction line is observed at an angle of $27°$. How many lines per centimetre does the grating have? **An**$(\mathbf{3.91\ x\ 10^{3}\mathit{lines\ cm}^{-1}})$
2. A diffraction grating has 550 lines per mm. When it is illuminated normally by monochromatic light, the angle between the central maximum and first maximum is $19.1°$. Find the
   (i) wavelength of the light
   (ii) number of diffraction maxima obtainable **An**$(\mathbf{5.95\ x\ 10^{-7}\mathit{m}, 3})$

Uneb 2016

(a) What is menat by **diffraction**. (01mark)

(b) Explain using Huygens's principle, the diffractin pattern produced by a single slit. (06marks)

(c) Light of wavelength $5x10^{-7}m$ falls on a grating with 600 lines per mm. Determine the highest order of diffraction that can be observed. (04marks)

(d) (i) Explain what is meant by **plane of polarization of light**. (02marks)

(ii) A liquid of refractive index 1.3 is used to produce polarized light by reflection. Calculate the angle of incidence of light on the liquid surface. (02marks)

(e) (i) Describe how the polarized light can be produced by reflection. (03marks)

(ii) State two uses of polarized light . (02marks)

Uneb 2015

(a) What is meant by the following terms as applied to waves

(i) Phase difference. (01mark)

(ii) Optical path difference. (01mark)

(b) (i) Explain how interference fringes are formed in an air wedge. (04marks)

(ii) Two glass slides are separated by a thin wire to form an air wedge. When the wedge is illuminated normally by light of wavelength $5.6x10^{-7}m$, a total of 20 fringes occupying a distance of 15mm are obtained. Calculate the angle of the wedge. (03marks)

(c) In Young's double slit experiment, 21 bright fringes occupying a distance of 3.6mm were visible on the screen. The distance of the screen from the double slit was 29cm and the wavelength of the light used was $5.5x10^{-7}m$. Calculate the separation of the slits. (03marks)

(d) (i) Describe how plane polarized light can be produced by double refraction.

(e) (02marks)

(ii) Describe **one** practical use of polarized light. (05marks)

Uneb 2013

(a) Explain the formation of fringes by transmission gratings. (05marks)

(b) Describe how the wavelength of monochromatic light can be measured using a diffraction grating and a spectrometer. (07marks)

(c) Explain why an oil layer on the water surface appears coloured on a rainy day. (03marks)

(d) Explain

(i) What is meant by plane polarized light. (02marks)

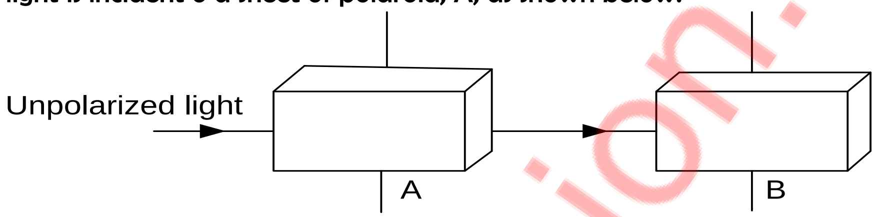
(ii) One application of polarized light. (03marks)

Uneb 2012

(a) (i) What is plane polarized light?. (01mark)

(ii) Why is it not possible to polarize sound waves? (01mark)

(b) (i) Unpolarized light is incident o a sheet of polaroid, A, as shown below.



Explain what would be observed if a second polaroid sheet B is rotated about an axis perpendicular to the directionof incidence. (03marks)

(ii) Sunlight is reflected off a glass window of refractive index 1.55. What should the elevation of the sun be if the reflected light is to be completely polarized. (03marks)

(c) Given the diffraction grating and a spectrometer, describe how you would use them to measure the wavelength of light from a given source. (07marks)

(d) A parallel beam of monochromatic light of wavelength 650nm is directed normally to a diffraction grating which has 600 lines per mm. Determine;

(i) The number of diffraction images. (03marks)

(ii) The angle of diffraction of the highest order diffractionimage. (02marks)

Uneb 2011

(a) What is meant by the following terms as applied to waves

(i) unpolarized light. (01mark)

(ii) plane polarized light. (01mark)

(b) (i) Describe briefly how plane polarized light is produced by double refraction. (03marks)

(ii) Explain briefly **one** application of polarized light. (02marks)

(c) Explain;

(i) How two coherent sources are obtained using a biprism. (03marks)

(ii) Why interference effects are not observed in thick films. (03marks)

(d) In Young's double slit experiment, the slits are separated by 0.28mm and the screen is 4m away. The distance between the fourth bright fringes and the central fringe is 1.2cm. determine the wavelength of the light used in the experiment. (04marks)

(e) Explain the effect of increasing the number f narrow slits diffraction grating on the intensity of diffraction fringes. (03marks)

Uneb 2010

(a) (i) Define the term **diffraction** (01mark)

(ii) What is meant by **plane polarized light** (01mark)

(b) (i) Describe briefly how plane polarized light is produced by double refraction. (05marks)

(ii) State **two** uses of plane polarized light. (02marks)

(iii) A parallel beam of unpolarized light incident on a transparent medium of refractive index 1.62 is reflected as plane polarized light. Calculate the angle of incidence in air and angle of refraction in the medium. (03marks)

(c) (i) what is a **diffraction grating**?. (01mark)

(ii) Sodium light of wavelenght $5.890x10^{-7}m$ and $5.896x10^{-7}m$ falls normally on a diffraction grating. If in the first order beam, the two sodium lines are separated by 2 minutes, find the spacing for the grating. (04marks)

(d) State **three** differences between the spectra produced by a prism and that by a diffraction grating. (03marks)

Uneb 2008

(a) Distinguish between **constructive** and **destructive** interference. (03mark)

(b) (i) Explain how interference fringes are formed ina an air wedge film between two glass slides when monochromatic light is used. (06marks)

(ii) Describe the appearance of the fringes when white light is used. (02marks)

(c) Two glass slides in contact at one end are separated by a sheet of paper 15cm from the the line of contact, to form an air wedge. When the wedge is illuminated normally by light of wavelenght $6.0x10^{-7}m$ interference firnges of separation 1.8mm are obtained. Calculate the thickness of the paper. (04marks)

(d) (i) Describe with an aid of a labeled diagram, one method of producing plane polarized. (04marks)

(ii) State **two** uses of plane polarized light. (02marks)

Uneb 2007

(a) State **Huygen's principle** (01mark)

(b) Monochromatic light propagating in air is incidentl obliquely onto a plane boundary with a dielectric of refreactive index n.

(i) Use Huygen's principle to show that the speed, V, of the light in the dielectric is given by $V = \frac{C}{n}$ where C is speed of light (06marks)

(ii) If the wavelength of the light is 600nm in air, what will it be in a dielectric of refractive index 1.50. **An(400nm)** (03marks)

(c) (i) What is meant by **interference of waves.** (01mark)

(ii) State the conditions necessary for interference fringes to be observed (02marks)

(iii) Explain the term **path difference** with reference to interference of two wave motions. (03marks)

(d) Two glass slides in contact at one end are separated by a wire of diameter 0.04mm at the other end to form wedge. When the wedge is illuminated normally by light of wavelenght $5.0x10^{-7}m$ interference fringes are observed. Find the number of fringes which can be observed. **An(160)** (04marks)

Uneb 2006

(a) (i) State **the superposition principle** as applied to wave motion (01mark)

(ii) What is meant by **optical path** (01mark)

(b) (i) State the conditions which must be satisfied inorder to observe an interference pattern due to two waves (02marks)

(ii) Explain why an oil film on a water surface appears to be coloure. (04marks)

(c)

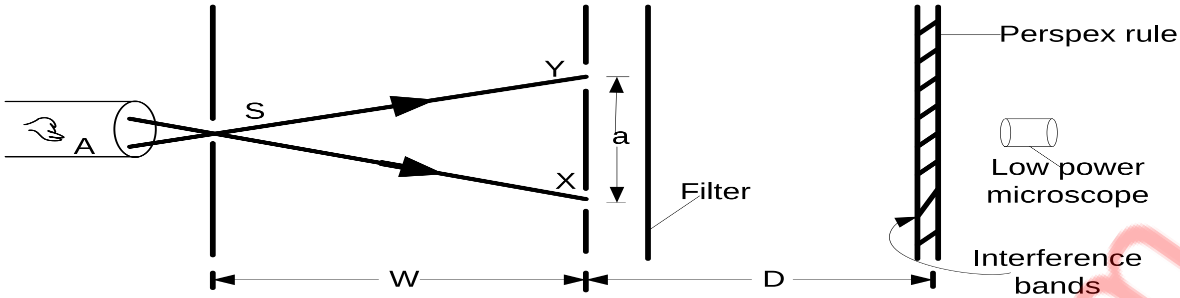


Figure above shows an experiment set up to demonstrate Young's interference fringes. Explain what is observed when the

(i) Slit X is covered (02marks)
(ii) Slit S is widened (02marks)
(iii) Separation, a, of the slits X and Y is reduced keeping W constant (02marks)
(iv) Distance W is reduced (02marks)

(d) A parallel beam of monochromatic light of wavelength 600nm is directed normally to a diffraction grating which has 500 lines per mm. Determine;

(i) The number of diffraction maxima observed. **An(3)** (03marks)
(ii) The angular position of the first diffraction maxxima **An(0.3)** (2marks)

Uneb 2004

(a) **(i)** What is meant by **plane polarized light** (01mark)
(ii) Describe briefly how plane polarized light is produced. (02mark)
(iii) Sketch the time variation of electric and magnetic vectors in a plane polarized light wave. (02marks)

(b) Two coherent sources a distance, s, apart produce light of wavelength, λ, which overlaps at a point on a screen a distance, D, from the sources to form an interference pattern.

(i) What is meant by coherent sources (02marks)
(ii) Show that the fringe width, w, is given by $w = \frac{\lambda D}{s}$ (04marks)
(iii) If λ=546nm, $S = 0.05mm$ and $D = 0.3m$. Find the angular position of the first diffraction maxima . (04marks)

(c) (i) what is meant by **diffraction of light** (02mark)
(ii) A parallel beam of monochromatic light of wavelength 600nm is directed normally to a diffraction grating which has 500 lines per mm. Determine the diffraction angle of the first order image. (03marks)

SECTION C: ELECTROMAGNETISM AND A.C CIRCUITS

A magnet is a substance which is able to attract a magnetic substance and it always points south-north direction when freely suspended.

Ferro magnetic / magnetic substances

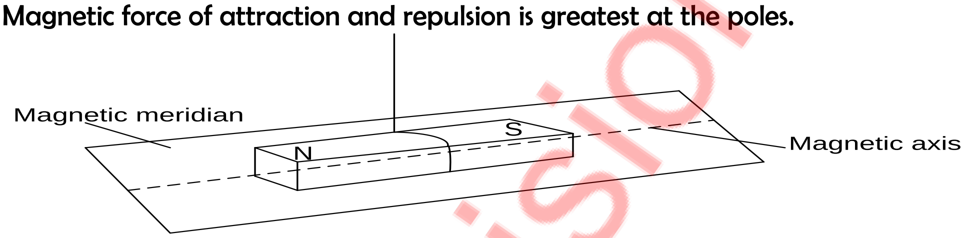
These are substances which can be attracted by magnets.
$e.g$ Iron, steel, cobalt, tin, nickel $e.t.c$

Non-magnetic substances

These are substances which cannot be attracted by a magnet.
$e.g$ Rubber, glass, copper tin, bronze, plastic $e.t.c$

Properties of a magnet

- Like poles repel and unlike poles attract
- When freely suspended it rest in the north- south direction with the north facing geographic north.
- Magnetic field lines run from north to south
- Magnetic force of attraction and repulsion is greatest at the poles.



When a freely suspended magnet rest in an approximate North- South direction.

The pole which points the north is called the North Pole (N) and the other is called South Pole (S).

(i) Magnetic axis

It is the central line joining the two poles
Or it is a line through the magnet about which the magnets magnetism is symmetrical.

(ii) Magnetic meridian

It is a vertical plane in which a freely suspended magnet rests.

(iii) Pole

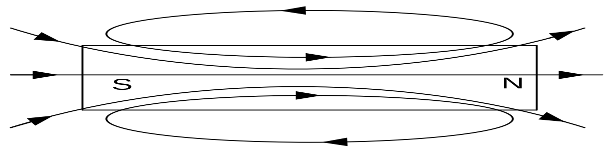
This is a point on a magnet where the resultant attractive force appears to be concentrated.

The magnetic field:

A magnetic Field is a region or space in which:

a) a magnetic dipole (magnet) experiences a force.
b) a current carrying conductor experiences a force or a moving charge experiences a force
c) an emf is induced in a moving conductor

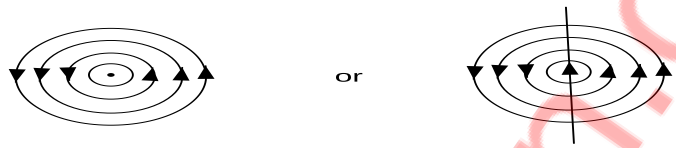
Field lines are used to represent the direction and magnitude of the magnetic field. The strength of the magnetic field is proportional to the density of the field lines.
The direction o the magnetic field is represented by the magnetic field lines. The magnetic field lines are taken to pass through the magnet, emerging from the North Pole and returning via the South Pole. The lines are continuous and do not cross each other.



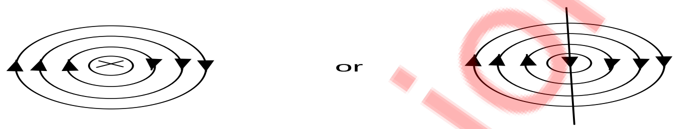
Magnetic fields due to a straight wire carrying current

Maxwell's right hand rule:
This is used to find the direction of the field.
If one grasps the current carrying straight wire in the right hand with the thumb pointing in the direction of current, then the fingers curl pointing in the direction of the magnetic field.

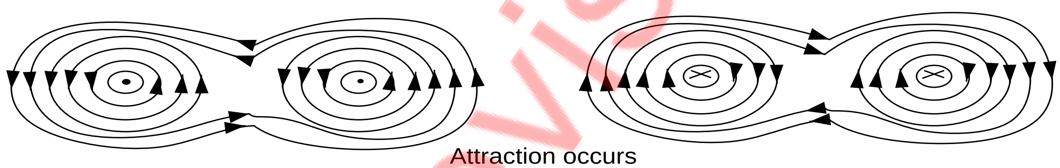
1. **Field due to a straight wire carrying current**
   i) **Upwards or out**



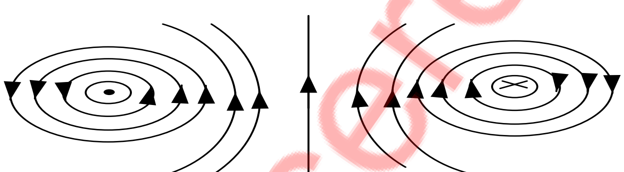
   ii) **Down or into**



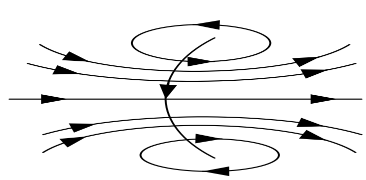
2. **Two wires carrying current in the same direction**



3. **Two wires carrying current in opposite direction**



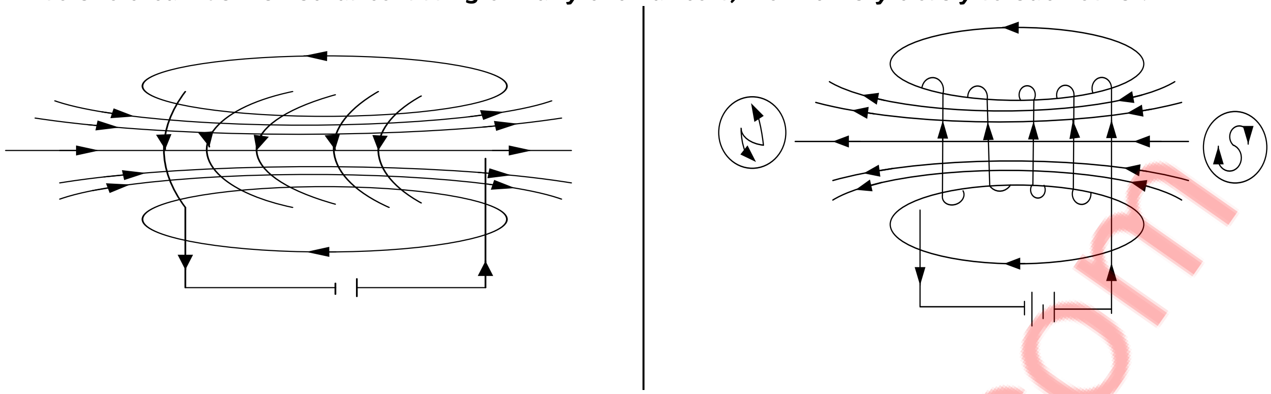
Magnetic field due to a current carrying circular coil



Magnetic fields near the center of the circular coil are uniform hence the magnetic field lines are nearly straight and parallel.

Magnetic fields due to a solenoid

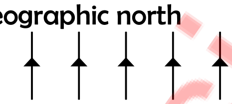
A solenoid can be viewed as consisting o many circular coils, wound very closely to each other.



Use right hand grip to determine direction of field. (fingers show direction of current; thumb direction)
In the middle of the solenoid, the magnetic field lines are parallel to the axis of the solenoid. At the ends of the solenoid, the lines diverge from the axis. The magnetic field due to a current carrying solenoid resembles that of a bar magnet. The polarities of the field are identified by looking at the ends of the solenoid. If current flow is clockwise, the end of the solenoid is South pole, and if anticlockwise then it is North pole.

Earth's magnetic field

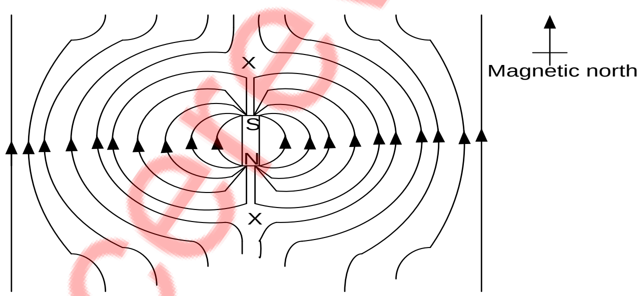
They run from geographic south to geographic north



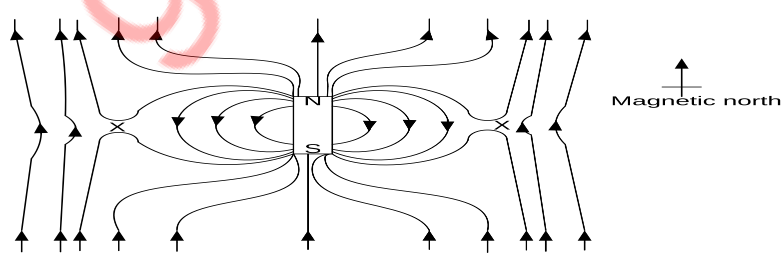
This shows that the earth behaves like a magnet with the North Pole in the southern hemisphere and South Pole in the north hemisphere.

A bar magnet in earth's magnetic field (Combined magnetic field)

a) A bar magnet with its north pole on the earth geographic south



b) A bar with its north pole in the earth geographic south



***X*- Is the neutral point:**

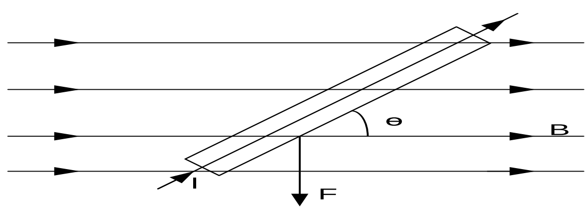
Neutral point is defined as a point at which the resultant magnetic flux density is zero

Magnetic force due to a current carrying conductor in the magnetic field

Electric currents cause magnetic fields around them, therefore when placed in a magnetic field the two magnetic fields interact and produce a force. The two forces can move wires and turn coils which carry electric current.

Factors affecting magnitude and direction of force

(i) Current $(F \propto I)$. Increase in current increases the force
(ii) Length of wire in the field $(F \propto L)$
(iii) Strength of magnet/magnetic flux density$(F \propto B)$
(iv) $sin\theta$ Where θ is the angle between the conductor and the magnetic field.



$F \propto BILsin\theta$

$F = kBILsin\theta$ Where k = constant of rotation.
k = 1

Therefore $\boxed{F = BILsin\theta}$

Magnetic flux density,B

This is the force acting on a 1m long conductor carying current of 1A in a direction perpendicular to the magnetic field.
When $\theta = 90°$

$\Rightarrow F = BILsin90$

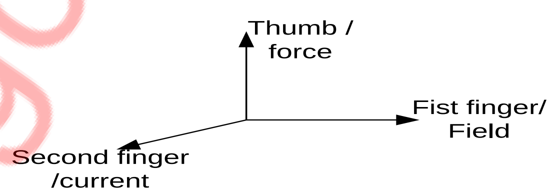
$F = BIL$

The unit of B is the *Tesla* (T)

Definition

The ***Tesla*** is the magnetic flux density in a magnetic field when a force of 1N acts on a conductor of length 1m, carrying a current of 1A, in a direction perpendicular to the magnetic field.

The direction of the **force** can be predicted by **Flemings left hand rule.**
It states, the thumb, the first finger and second finger are held at right angles with the **F**irst finger pointing in the direction of magnetic **F**ield and se**C**ond finger pointing in the direction of the **C**urrent, the **T**humb gives the direction of the **T**hrust /force



Examples

1. A conductor of length 10cm carrying a current of 5A is placed in magnetic field of flux density 0.2T. calculate the force on the conductor when placed;
   (i) At right angle to the field
   (ii) At 30° to the field
   (iii) Parallel to the field

Solution

$F = BILsin\theta$
$F = 0.2x5x0.1xsin90$
$\boldsymbol{F = 0.1N}$

$F = BILsin\theta$
$F = 0.2x5x0.1xsin30$
$F = 0.087N$

$F = BILsin\theta$
$F = 0.2x5x0.1xsin0$
$\boldsymbol{F = 0N}$

2. A horizontal wire carrying current of 4A lies in a vertical magnetic field of 0.03T. calculate the force on the wire per unit length

Solution

$F = BILsin\theta$

$\frac{F}{l} = 0.03x4xsin90$

$F = 0.12Nm^{-1}$

4. A straight horizontal rod of mass 140g and length 0.6m is placed in a uniform horizontal magnetic field 0.16T perpendicular to it. Calculate the current through the rod if the force acting on it just balances its weight.

Solution

$F = BILsin\theta$
$mg = BILsin\theta$

$I = \frac{140x10^{-3}x9.81}{0.16x0.6xsin90}$
$I = 14.31A$

5. A straight horizontal wire 5cm long weighing $1.2gm^{-1}$is placed perpendicularly to a uniform horizontal magnetic field of flux density $0.6T$. if the resistance of the wire is $3.8\Omega m^{-1}$. Calculate the p.d that has to be applied between the ends of the wire to make it just self supporting.

Solution

$F = BILsin\theta$
$mg = BILsin90$
$\left(\frac{m}{l}\right) g = BI$
$1.2x10^{-3}x9.81 = 0.6x\ I$
$I = 0.01962A$

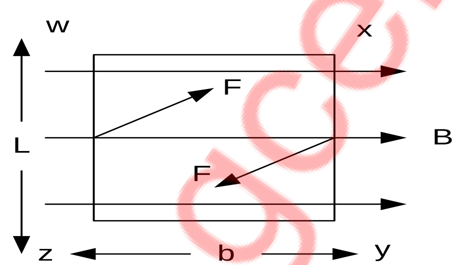
$V = IR$
$V = 0.01962x3.8x5x10^{-2}$
$V = 3.73x10^{-3}V$

Torque on a current carrying coil in a uniform magnetic field

Torques is sometimes called moment of a couple
Torque is defined as the product of the force and the perpendicular distance

(i) Plane of coil parallel to field

Consider a rectangular coil carrying a current *I*, in a uniform magnetic field of flux density B. Suppose the plane of the coil is parallel to the magnetic field.



- By Flemings left hand rule
  Force on WX= $BINb\ \ (upwards)$

Force on ZY= $BINb\ (\ down\ wards)$
Force on WZ= $BINL\ (inwards)$
Force on WZ= $BINL\ \ (outwards)$

- Forces on WX and ZY cancel since they are equal and opposite
- Forces on XY and WZ constitute a couple of moent of a torque given by
  $\tau = force\ x\ perpendicular\ distance$

$\tau = (BIL\ N)\ b$

$\boxed{torque = BINA}$ where $Lb = A$

Examples

1. A circular coil of 20 turns each of radius 10 cm is suspended with its plane along uniform magnetic field of flux density 0.5T. find the initial torque on the coil when a current of 1.5A is passed through it.

Solution

$torque = BINA$ | $\tau = 0.5x1.5x20x3.14x10x10^{-2}$ | $torque = 4.71Nm$

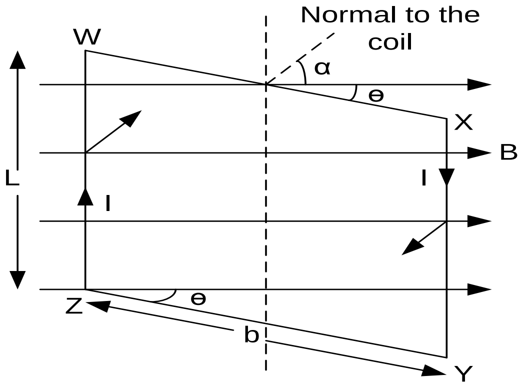
2. The coil in a certain galvanometer is rectangular with sides 4cm by 3cm and with 150 turns. Calculate the initial deflecting couple due to the current of 4mA if the magnetic flux density is 0.02T

Solution

$torque = BINA$ | $\tau = 0.02x4x10^{-3}x150x4x3x10^{-4}$ | $torque = 1.44x10^{-5}Nm$

(ii) Plane of coil at angle θ to the field

Consider a rectangular coil carrying a current *I*, in a uniform magnetic field of flux density B. Suppose the plane of the coil makes an angle θ with the magnetic field.



- By Flemings left hand rule
  - Force on WX= $BINb\ Sin\theta$ ($upwards$)
  - Force on ZY= $BINb\ Sin\theta$ ($down\ wards$)
  - Force on WZ= $BINL$ ($inwards$)
  - Force on WZ= $BINL$ ($outwards$)
- Forces on WX and ZY cancel since they are equal and opposite
- Forces on XY and WZ constitute a couple of moent of a torque given by

$$\tau = force\ x\ perpendicular\ distance$$

$$\tau = (BIL\ N)\ bCos\theta$$

$$\boxed{torque = BINACos\theta}$$ where $Lb = A$

Examples

1. A vertical square coil of side 5 cm has 100 turns and carries a current of 1 A. Calculate the torque on the coil when it is placed in a horizontal magnetic field of flux density 0.2 T with its plane making an angle of 30° to the field

Solution

$torque = BINAcos\theta$

$\tau = 0.2x1x100x5x5x10^{-4}cos30$ | $torque = 2.34Nm$

2. A vertical square coil of sides 15cm has 200 turns and carries a current of 2A, if the coil is placed in a horizontal magnetic field of flux density 0.3T with its plane making an angle of $30°$ to the field. find the initial torque on the coil.

Solution

$torque = BINAcos\theta$

$\tau = 0.3x2x200x15x15x10^{-4}cos30$ | $torque = 2.34Nm$

3. A vertical rectangular coil is suspended from the middle of is upper side with its plane parallel to the uniform horizontal magnetic field of 0.06T. the coil has 50 turns and the length of its vertical and horizontal sides are 4cm and 5cm respectively. Find the torque on the coil when the current of 4A is passed through it.

$torque = BINAcos\theta$ | $\tau = 0.06x4x50x4x5x10^{-4}cos0$ | $torque = 6x10^{-3}Nm$

Exercise

A circular coil of 10 turns each of radius 10 cm is suspended with its plane along a uniform magnetic field of flux density 0.1 T. Find the initial torque on the coil when a current of 1.0 A is passed through it.

An($3.14x10^{-2}Nm$)

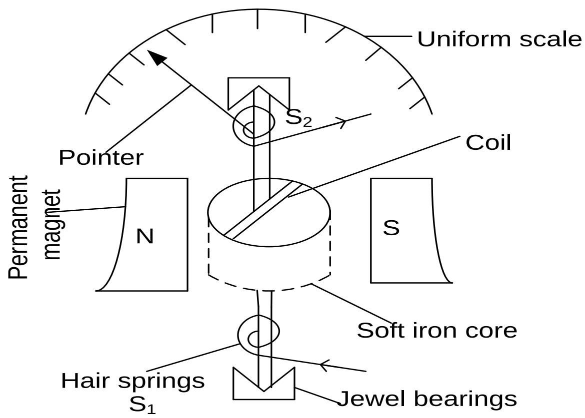
Electromagnetic moment

Electromagnetic moment of a current carrying coil is the product of the number of turns, the current in the coil and area of the coil

Electromagnetic moment is sometimes called magnetic moment of a current carrying coil or magnetic dipole moment of the coil

$$\boxed{\text{Magnetic moment } = INA}$$

Moving coil Galvanometer



- Current to be measured is allowed hrough hair spring S_1. The coil experiences a magnetic torque, $\tau = BINA$
- The coil turns together with the pointer until it is stopped by the restoring torque of the hair springs, $\tau = k\theta$
- At this point $\tau = BINA = k\theta$

$\Rightarrow I = \frac{K}{BNA}\theta$

But $\frac{K}{BNA}$ is constant , hence $I \propto \theta$ (or scale is linear)

Note

(i) The two hair springs are wound in opposite direction so as to provide a restoring couple and they allow current to be measured to enter and leave

(ii) The coil is put in soft iron cylinder which concentrates the magnetic flux radially in the annular space. For this reason, the magnetic flux density is constant and in the plane of the coil, hence force on the sides of the coil will be proportional to the current.

(iii) A radial magnetic field is used to provide a linear scale in which the plane of the coil in all position remains parallel to the direction of the magnetic field.

Example

A rectangular coil of 100 turns is suspended in uniform magnetic filed of flux density 0.02T with the plane of the coil parallel to the field. The coil is 3 cm high and 2 cm wide. If a current of 50A through the coil causes a deflection 30°, calculate the torsional constant of the suspension.

Solution

$k\theta = BINA$

$k = \frac{0.02x50x100x3x2x10^{-4}}{\left(\frac{30\,\pi}{180}\right)}$

$k = 0.115Nmrad^{-1}$

Sensitivity of the galvanometer

(a) Current sensitivity $\left(\frac{\theta}{I}\right)$

Current sensitivity is the deflection per unit current

$$BINA = k\theta$$

$$\boxed{\frac{\theta}{I} = \frac{BNA}{k}}$$

Current sensitivity can be increased by

i) Using a coil of large area

ii) Increase the number of turns of the coil
iii) Using a strong magnet to provide large magnetic flux density
iv) Using very fine hair springs (Small tortional constant)

(b) voltage sensitivity $\left(\frac{\theta}{V}\right)$

Current sensitivity is the deflection per unit current

$$BINA = k\theta$$

$$B\frac{V}{R}NA = k\theta$$

$$\boxed{\frac{\theta}{V} = \frac{BNA}{kR}}$$

Conversion of galvanometer to an ammeter

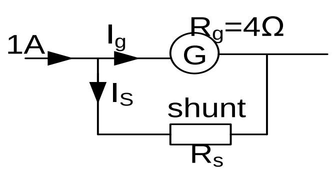
To convert a galvanometer to an ammeter, a low resistance called **a shunt** is connected in parallel with the galvanometer

Most of the current to be measured takes the path through the shunt and very small current through the galvanometer.

Examples

1. A galvanometer of resistance 4Ω and full scale deflection ($f.s.d$) $10mA$ is to be used for the purpose of measuring current to $1.0A$. Find the value of the shunt to be used.

Solution



Current through galvanometer $I_g = 10mA$

$$I_g = \frac{10}{1000} = 0.01A$$

Current through shunt $I_S = 1 - 0.01$

$I_S = 0.99A$

Since the shunt and galvanometer are in parallel, they have the same $p.d$

$V_g = V_S$

$I_gR_g = I_SR_S$

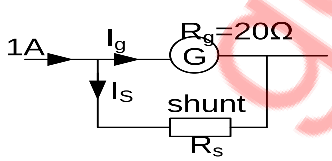
$0.01x4 = 0.99xR_S$

$$R_S = \frac{0.01x4}{0.99}$$

$R_S = 0.04\Omega$

2. A galvanometer has a resistance of 20Ω and gives a full scale deflection for a current of 2000μA. If the galvanometer is converted to an ammeter which can read up to $1.0A$. What is the size of the extra low resistance?

Solution



Current through galvanometer $I_g = 2000\mu A$

$$I_g = \frac{2000}{1000000} = 0.002A$$

Current through shunt $I_S = 1 - 0.002$

$I_S = 0.998A$

Since the shunt and galvanometer are in parallel, they have the same $p.d$

$V_g = V_S$

$I_gR_g = I_SR_S$

$0.002x20 = 0.998xR_S$

$$R_S = \frac{0.002x20}{0.998}$$

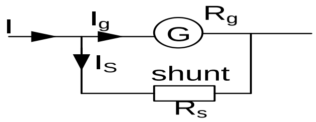
$R_S = 0.04\Omega$

3. A moving coil galvanometer of internal resistance 4 gives a maximum deflection when a current of 2mA flows through it. A shunt of resistance 0.06 is used to convert the galvanometer into an ammeter.

a) Find the current through the shunt

b) The maximum current that can be measured by the set up.

Solution



$I_gR_g = I_SR_S$

$0.002x4 = 0.06xI_S$

$I_S = \frac{0.002x4}{0.06}$

$I_S = 0.133A$

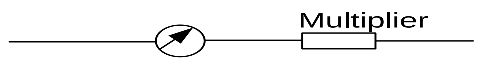
Maximum current $I = I_g + I_S$

$= 0.002 + 0.133$

$=0.135A$

Conversion of a galvanometer to a voltmeter

Large $p.d$ can be measured by placing a high resistance called **multiplier** in series with galvanometer.

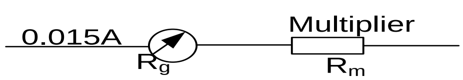


Same current passes through the galvanometer and multiplier

Example

How can you measure $a\ p.d$ of up to 30V using a galvanometer of resistance 10Ω and $f.s.d$ of $15mA$.

Solution



Maximum current through the galvanometer $= 0.015A$

$Total\ p.d = V_g + V_m$

$30 = I_gR_g + I_SR_m$

$30 = 0.015\ x10 + 0.015R_m$

$30 - 0.15 = 0.015R_m$

R_m=1990Ω

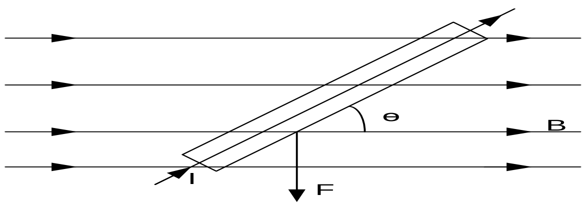
Galvanometer should be connected with a high resistance multiplier of 1990Ω

Exercise

1. Consider a full scale deflection when a current of 15mA flow through it. If the resistance of the galvanometer is 5Ω, find the magnitude of the resistance (multiplier) to be used for it to measure a maximum $p.d$ of 15V **[995Ω]**
2. A moving coil galvanometer has resistance of 0.5Ω and full scale deflection of 2mA. How can it be adopted to read current to voltage 10V **[4999Ω]**
3. A moving coil galvanometer has resistance of 0.5Ω and full scale deflection of 2mA. How can it be adopted to read current 6A An**[$1.67x10^{-4}$Ω]**
4. Consider a moving coil galvanometer which has resistance of 5Ω and full scale deflection when a current of 15mA. A suppose a maximum current of 3A is to be measured using this galvanometer. What is the value of the shunt required **[0.025Ω]**
5. A galvanometer of internal resistance of 20Ω and full scale deflection of 5mA. How can it be modified for use as;
   (i) 1.0A ammeter
   (ii) 100V voltmeter **[(i). 1.05Ω (ii). 1980Ω]**
6. A milliameter has a full scale reading of 0.01A and has resistance 20Ω. Show how a suitable resistor may be connected in order to use this instrument as a voltmeter reading up to 10V. **[980Ω]**

Force of a charge in a magnetic field.

Consider a wire o length l, carrying a current I, in a uniform magnetic field of flux density B.



The current $I = neVA$ where *n* is the number of electrons per unit volume, *A* is the cross-sectional area, *V* is the drift velocity of electrons, and *e* is the electron charge.

The magnetic force on the wire $F = BILsin\theta$

Hence $F = B(neVA)Lsin\theta$

but $nAl = N =$ total number of electrons.

$$F = BNeVsin\theta$$

but Ne = total charge = q

$\boxed{F = BqVsin\theta}$ force on any charged particle.

Force on one electron; $\boxed{F = BeVsin\theta}$

If the particles velocity is at right angles to *B*, $\vec{\theta} = 90^0$

Hence $\boxed{F = BqV}$

$$B = \frac{F}{qV}$$

Definition

magnetic flux density B in a magnetic field, is the force acting on a charge of 1C, moving with a velocity of 1ms^{-1}, at right angles to the magnetic field.

Example

1. An electron beam moving with velocity of 100m/s passes through a uniform magnetic field of flux density 0.04T which is perpendicular to the direction of the beam. Calculate the fore on each electron

Solution

$F = BeVsin\theta$ | $F = 0.04x1.6x10^{-19}x100sin90$ | $F = 6.4x10^{-19}N$

2. A metal rod of length $50cm$ is moves with a velocity of $5ms^{-1}$ in aplane perpendicular to a uniform magnetic field of flux density 0.05T. Find the
   (i) Magnetic force on the electron in the rod
   (ii) Electric field intensity in the rod
   (iii) Potential difference between the ends of the rod

Solution

i) $F = BeV$

$F = 0.05x1.6x10^{-19}x5$

$F = 4.0x10^{-20}N$

ii) At steady state:

magnetic force = electric force

$4.0x10^{-20} = Ee$

$$E = \frac{4.0x10^{-20}}{1.6x10^{-19}}$$

$E = 0.25V$

(iii) $V = El$

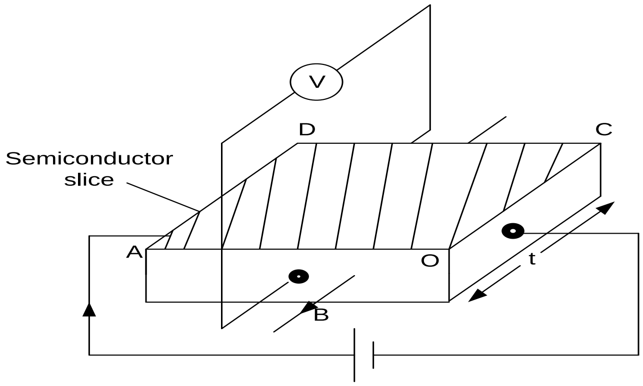
$V = 0.25x0.5$

$V = 0.125V$

HALL EFFECTS

This is the setting up of an $e.m.f$ transversely across a conductor when a perpendicular magnetic field is applied

A current carrying conductor in a magnetic field has a small potential difference across its sides at right angles to the field.



If the metal is placed in a magnetic field B, at right angles to the face ADCO of the slab and directed out of paper, a force BeV then acts on each electron in the direction from CD to AO. (Fleming's left hand rule)

Therefore electrons accumulate along then side AO of the metal making AO negatively charged and DC positively charged. Hence a $p.d$ or emf which opposes the electron flow is set up.

A equilibrium: $Ee = BeV$

$\frac{V_H}{d}e = BeV$ where $E = \frac{V_H}{d}$

$V_H = BVd$

But $I = neVA$ ∴

$Drift\ velocity, V = \frac{I}{neA}$

$$V_H = B\frac{I}{neA}d$$

But $A = txd.$

$$V_H = B\frac{I}{netd}d$$

$\boxed{V_H = \frac{BI}{net}}$ V_H —Hall voltage and t is thickness

Definition

Hall voltage is the maximum p.d set up across a conductor when a perpendicular magnetic field is applied

Examples

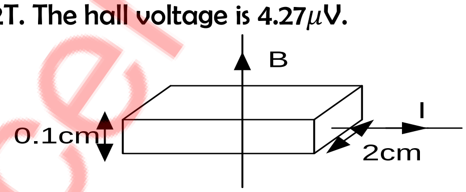
1. A metallic strip of width 2.5cm and thickness 0.5cm carries a current of 10A. When a magnetic is applied normally to the broad side of the strip, a hall voltage of 2mV develops. Find the magnetic flux density if the conduction electron density is $6.0x10^{28}m^{-3}$.

Solution

$$V_H = \frac{BI}{net}$$

$$B = \frac{2x10^{-6}x6.0x10^{28}x1.6x10^{-19}x0.5x10^{-2}}{10}$$

2. A metal strip 2cm wide and 0.1cm tick carries a current of 20A at right angles to a uniform magnetic field of flux density 2T. The hall voltage is 4.27μV.



Calculate

(i) the drift velocity of the electrons in the strip
(ii) the density of charge carriers in the strip.

Solution

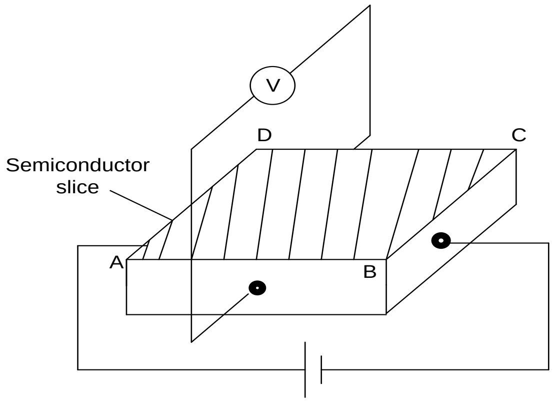
(i) $V_H = BVd$

$$v = \frac{V_H}{Bd} = \frac{4.27x10^{-6}}{2x2x10^{-2}} = 1.067x10^{-4}ms^{-1}$$

(ii) $V_H = \frac{BI}{net}$

$$n = \frac{BI}{V_Het} = \frac{2x20}{4.27x10^{-6}x1.6x10^{-19}x0.1x10^{-2}}$$

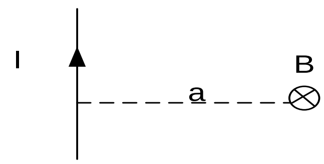
Measurement of magnetic flux density using hall probe



- A semiconductor slice is connected across a source of d.c voltage and a voltmeter aacross the slice as shown above
- The slice is now placed in the test magnetic field such that the face ABCD is perpendicular to the field.
- The voltmeter reading V is noted. The slic is now placed into a magnetic field whose flux density B_O is known
- The voltemeter reading V_o is noted
- The test flux density is now calculated from $B = \frac{B_o}{V_o}$

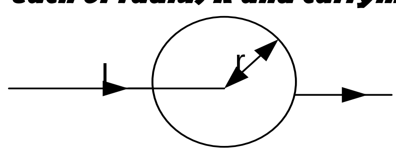
Magnetic flux density of current carrying conductors

(i) Magnetic flux density at a point a distance r from a long straight wire carrying current, I.



$B = \frac{\mu_0 I}{2\pi a}$ where μ_0 is the permeability in free space.($\mu_0 = 4\pi x 10^{-7} Hm^{-1}$)

(ii) Magnetic flux density at the center of a circular coil of N turns each of turns each of radius R and carrying a current I.



$$B = \frac{\mu_0 N I}{2r}$$

(iii) Magnetic flux density along the axis of a long solenoid of n turns per meter, each carrying a current I.

$B = \mu_0 nI$ or $B = \frac{\mu_0 NI}{l}$

Where N is the number of turns and l is the length of the solenoid.

Note:

At either end of the coil $B = \frac{\mu_0 nI}{2}$ or $B = \frac{\mu_0 NI}{2l}$

Examples

1. A solenoid of 2000 turns, 75cm long and carrying a current of 2.5A. Calculate the magnetic flux density at;
   (ii) Center of the solenoid
   (iii) End solenoid

Solution

$$B = \frac{\mu_0 NI}{l}$$
$$B = \frac{4\pi x 10^{-7} x 2000 x 2.5}{0.75}$$
$$\boldsymbol{B} = 8.38 x 10^{-3} T$$

$$B = \frac{\mu_0 NI}{2l}$$
$$B = \frac{4\pi x 10^{-7} x 2000 x 2.5}{2 x 0.75}$$
$$\boldsymbol{B} = 4.19 x 10^{-3} T$$

2. A small circular coil of 10 turns and mean radius 2.5 cm is mounted at the centre of a long solenoid of 750 turns per metre with its axis at right angles to the axis of the solenoid.. If the current in the solenoid is 2.0A, Calculate:-

(i) The magnetic flux density inside the solenoid.

(ii) The initial torque on the circular coil when a current of 1.0A is passed through it.

Solution

(i) $B = \mu_0 nI$

$B = 4\pi x10^{-7}x750x20$

$= 1.88x10^{-3}T$

(ii) $\tau = BINA$

$= 1.88x10^{-3}x\pi x(0.025)^2x10x1$

$= 3.7x10^{-5}Nm$

3. A solenoid of 2000 turns, 40cm long and resistance of 16Ω is connected to a 20V supply. Calculate the magnetic flux density at;

(i) The midpoint of the axis of solenoid

(ii) At the end of the solenoid

Solution

$B = \frac{\mu_0 NI}{l}$

$B = \frac{\mu_0 NV}{l\,R}$ where $I = \frac{V}{R}$

$B = \frac{4\pi x10^{-7}x2000x20}{0.4x16}$

$B = 7.9x10^{-3}T$

$B = \frac{\mu_0 NI}{2l}$

$B = \frac{\mu_0 NV}{2l\,R}$ where $I = \frac{V}{R}$

$B = \frac{4\pi x10^{-7}x2000x20}{2x\ 0.4x16}$

$B = 3.93x10^{-3}T$

4. A rectangular coil of 12 turns and dimensions 5 cm by 3 cm is suspended inside a long solenoid of 1200 turns per meter so that its plane lies along the axis of the solenoid. The coil is connected in series with the solenoid. If a current of 3.0A through the coil causes a deflection 40°, calculate the torsional constant of the suspension.

Solution

$B = \mu_0 nI$

$B = 4\pi x10^{-7}x1200x3$

$B = 4.52x10^{-3}T$

$k\theta = BINA$

$k = \frac{4.52x10^{-3}x3x12x5x3x10^{-4}}{\left(\frac{40\pi}{180}\right)}$

$k = 3.50x10^{-4}Nmrad^{-1}$

5. A fine wire of length 157.0m is wound into a solenoid of diameter 5.0cm and length 25cm. If a current of 2.0A passes through the coil, find the magnetic flux density at the end of the solenoid.

Solution

$circumferenc\ x\ number\ of\ turns = length$

$\pi dxN = 157$

$3.14x0.05xN = 157$

$N = 1000\ turns$

$B = \frac{\mu_0 NI}{2l}$

$B = \frac{4\pi x10^{-7}x1000x2.0}{2x0.25}$

$B = 5.03x10^{-3}\ T$

6. A current of 1.0A flows in a long solenoid o 1000 turns per meter. If the solenoid has a mean diameter of 80cm, find the magnetic flux linkage on one meter length of the solenoid.

$B = \frac{\mu_0 NI}{l}$

$B = \frac{4\pi x10^{-7}x`1000x1}{1}$

$B = 1.275x10^{-3}T$

$\Phi = NAB\ cosa\theta$

$\Phi = 1000x(40x10^{-2})^2x\pi x1.275x10^{-3}x1$

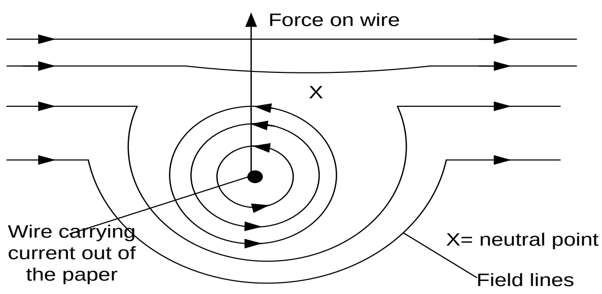
$\Phi = 0.6317Wb$

Exercise

1. A copper wire of length 7.85 m is wound into a circular coil of radius 5cm. A current of 2A is passed through the coil. Calculate the magnetic flux density at the centre of the coil

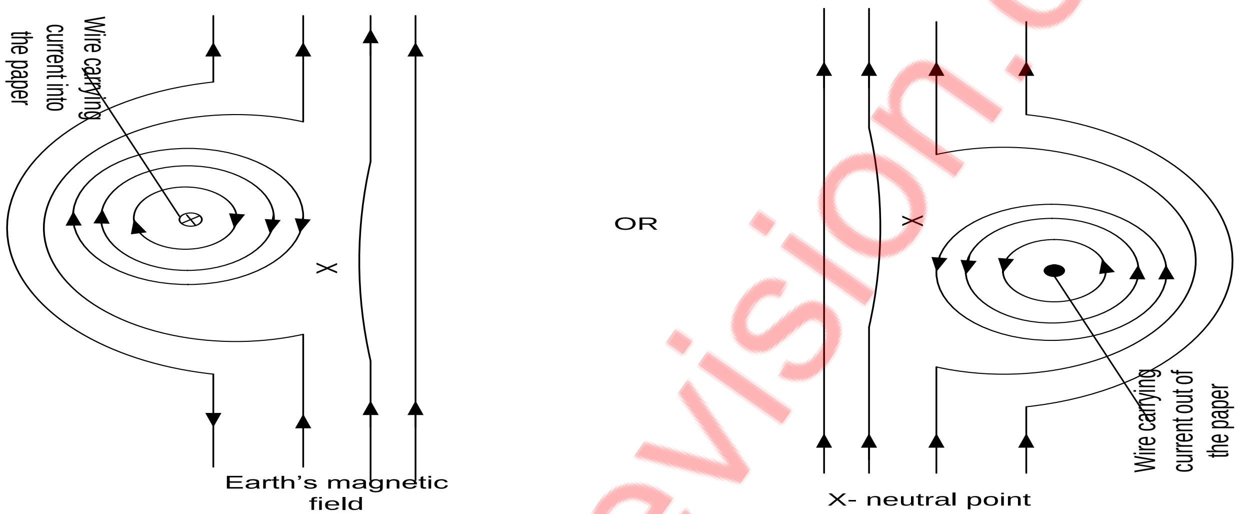
Magnetic force in a current carrying straight conductor

Exaplain why a current carrying conductor palced in a magnetic field experiences a force

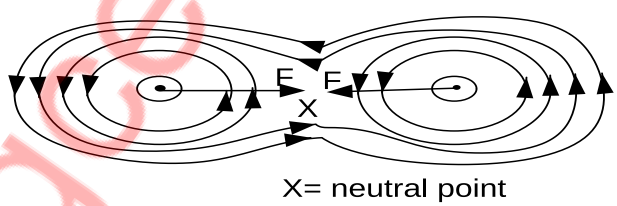


- Consider a single straight wire carrying current at right angles to a uniform magnetic field.
- Current in the wire produces a magnetic field around the wire.
- The external magnetic field interacts with the field due to the current.
- The resultant magnetic field is stronger below the wire than above .
- The forec due to the resultant magnetic field acts on the wire

Magnetic field patterns due to a straight wire carrying current in the earth's magnetic field

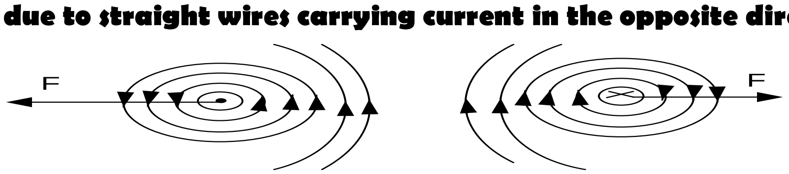


Magnetic field due to two straight wires carrying current in the same direction



A force on each wire acts from a region of strong field hence straight parallel wires carrying current to the same direction attract i.e. "like currents attract"

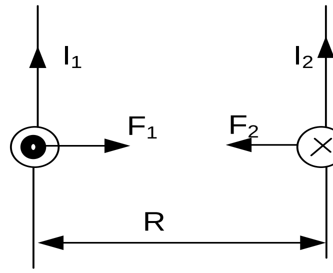
Magnetic field due to straight wires carrying current in the opposite direction



Straight parallel wires carrying current to the opposite direction repel.

Magnitude of the force between two wires carrying current.

Consider like currents.



The magnetic flux density B_2 due to current I_2 is given by $B_2 = \frac{\mu_0 I_2}{2\pi a}$

The force per unit length F_1 on wire 1 due to current I_1 is given by $F_1 = B_2 I_1 L = \frac{\mu_0 I_2 I_{1L}}{2\pi a}$

The magnetic flux density B_1 due to current I_1 is given by $B_1 = \frac{\mu_0 I_1}{2\pi a}$

The force per unit length F_2 on wire 2 due to current I_2 is given by $F_2 = B_1 I_2 L = \frac{\mu_0 I_2 I_{1L}}{2\pi a}$

Hence $\boxed{F_1 = F_2 = F = \frac{\mu_0 I_2 I_{1L}}{2\pi a}}$

If $I_1 = I_2 = 1$A, $d = 1m$,

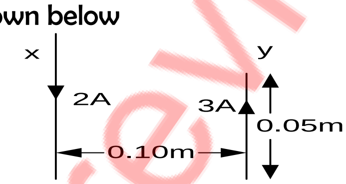
then $F = \frac{\mu_0}{2\pi} = \frac{4\pi x 10^{-7}}{2\pi} = 2x10^{-7} Nm^{-1}$

Definition

The **ampere** is the current which when flowing in each of the two inifinitely long parallel wires of negligible cross section area separated by 1m in a vacuum, produce a force of $2x10^{-7}\ Nm^{-1}$ on each other.

Examples

1. A long strainght conductor X carryng a current of 2A is placed parallel to a short conductor, y of length 0.05m carrying a current of 3A as shown below



The two conductors are 0.10m apat. Calculate the;

(i) The flux density due to x on y

(ii) The approximate force on y

(iii) At what point between the wire is the manetic flex density zero

Solution

(i) $B_x = \frac{\mu_0 I_x}{2\pi a}$

$B_x = \frac{4\pi x 10^{-7} x2}{2\pi x 0.1}$

$B_x = 4.0x10^{-6} T$

(ii) Force on y is due to field of x

$F_y = B_x I_y l_y$

$F_y = 4.0x10^{-6} x3x0.05$

$F_y = 6.0x10^{-7} N$

(iii) $B_x = B_y$

$\frac{\mu_0 x2}{2\pi x} = \frac{\mu_0 x3}{2\pi(0.1 - x)}$

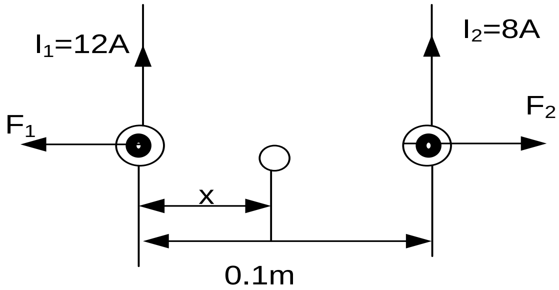
$x = 0.04m$

The flux is zero at a distance of 0.04m from conductor x

2. (i) Sketch the magnetic field due to two long parallel conductors carrying respective currents of 12A and 8A in the same direction.

(ii) If the wires are 10cm apart, find where a third parallel wire also carrying a current must be placed so that the force it experiences is zero.

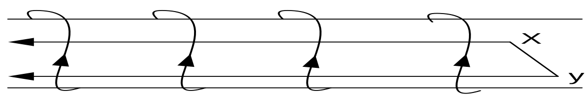
Solution



Let the third wire carry current **I** and be **x m** from wire carrying a current of 12A.
F_1 is force exerted on wire due to current 12A,
F_2 is force exerted on wire due to current of 8A.

$$B = \frac{\mu_0 I}{2\pi R}$$
$$B_1 = B_2$$
$$\frac{\mu_0 x12}{2\pi x} = \frac{\mu_0 x8}{2\pi(0.1 - x)}$$
$$x = 0.06m$$

3. A wire carrying a current of 2.4 A is palced inside a long solenoid of 2000 tunrs per meter as shown below



Part of the wire xy of length 1.1m experiences a force of 0.03N. Find the;

(i) Current flowing through the solenoid

(ii) Force on an electron which moves on the wire with the speed of $10.6ms^{-1}$

Solution

(i) Force on wire is due to field on the solenoid

$$F_W = B_s I_w l_w$$
$$F_W = \mu_0 n\, I_s I_w L_w$$
$$I_s = \frac{0.03}{4\pi x10^{-7}x2000x2.4x1.1}$$
$$I_s = 4.52A$$

(ii) $F = BeV$

$$F = \mu_0 n\, I_s eV$$
$$F = 4\pi x10^{-7}x2000x4.52x1.6x10^{-19}x10.6$$
$$F = 2x10^{-20}N$$

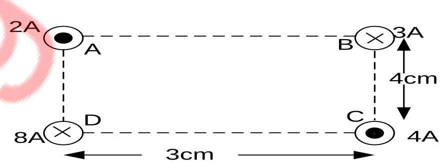
4. Two long thin parallel wires A and B carry currents of 5Aand 2A respectoively in opposite direction. If the wires are separated by a distance of 2.5cm in a vacuum. Calculate the force exerted by wire B on 1m of wire A

$$F = \frac{\mu_0 I_2\, I_1\, L}{2\pi a}$$
$$F = \frac{4\pi x10^{-7}x5x2x1}{2\pi x2.5x10^{-2}}$$
$$F = 8x10^{-5}N$$

5. Two parallel wires each of length 75cm are placed 1.0cm apart. When the same current is passed through the wires, a force of $5.0x\ 10^{-5}$N develops between the wires. Find the magnitude of the current.

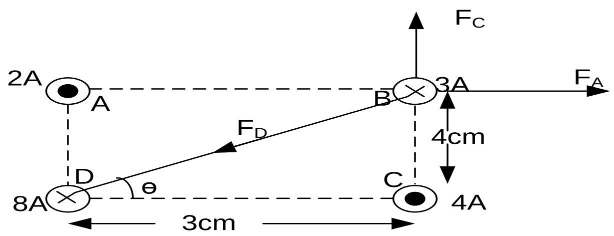
$$F = \frac{\mu_0 I_2\, I_1\, L}{2\pi a}$$
$$5x10^{-5} = \frac{4\pi x10^{-7}xI^2x0.75}{2\pi x0.1}$$
$$I = 5.77A$$

6.



Find the resultant force on B

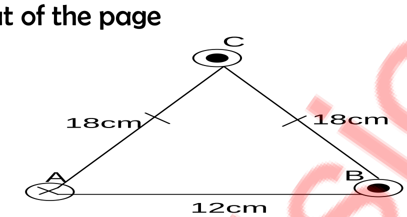
Solution



$$tan\theta = \frac{4}{3}$$
$$\theta = 53.13°$$
$$x = \sqrt{3^2 + 4^2} = 5cm$$
$$F = \frac{\mu_0 I_2\, I_1\, l}{2\pi a}$$
$$F_A = \frac{4\pi x10^{-7}x2\ x3}{2\pi x3x10^{-2}} = 4x10^{-5}Nm^{-1}(\rightarrow)$$

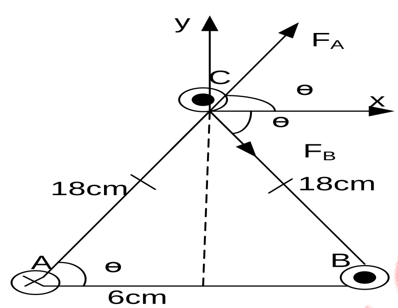
$$F_C = \frac{4\pi x10^{-7}x4\ x3}{2\pi x4x10^{-2}} = 6x10^{-5}Nm^{-1}(\uparrow)$$
$$F_D = \frac{4\pi x10^{-7}x8\ x3}{2\pi x5x10^{-2}} = 9.6x10^{-5}Nm^{-1}$$
$$F_R = \begin{pmatrix} 4x10^{-5} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 6x10^{-5} \end{pmatrix} + \begin{pmatrix} -9.6x10^{-5}cos53.13 \\ -9.6x10^{-5}sin53.13 \end{pmatrix}$$
$$F_R = \begin{pmatrix} -1.76x10^{-5} \\ -1.68x10^{-5} \end{pmatrix}$$
$$|F_R| = \sqrt{(-1.76x10^{-5})^2 + (-1.68x10^{-5})^2}$$
$$|F_R| = 2.433x10^{-5}\ Nm^{-1}$$
$$tan\alpha = \frac{1.68x10^{-5}}{1.76x10^{-5}}$$
$\alpha = 43.67°$ to the horizontal

7. A, B and C are long thin wires arranged so that their centres lie at the corners of an isosceles triangle. Wire A carries a current of 9.0A into the page, wire B carries a current of 9.0A out of the page and wire C carries a current of 3.0A out of the page



Find the net force on C

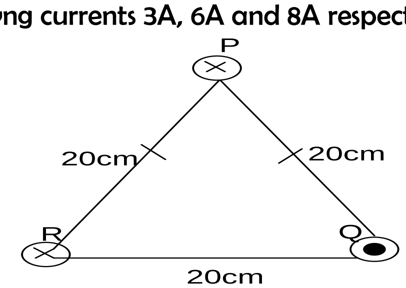
Solution



$$Cos\theta = \frac{6}{18}$$
$$\theta = 70.53°$$

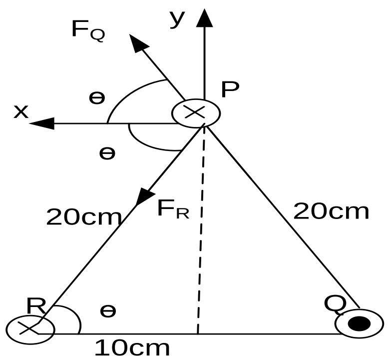
$$F = \frac{\mu_0 I_2\, I_1\, l}{2\pi a}$$
$$F_A = \frac{4\pi x10^{-7}x9\ x3}{2\pi x18x10^{-2}} = 3x10^{-5}Nm^{-1}$$
$$F_B = \frac{4\pi x10^{-7}x9\ x3}{2\pi x18x10^{-2}} = 3x10^{-5}Nm^{-1}$$
$$F_R = \begin{pmatrix} 3x10^{-5}Cos70.53 \\ 3x10^{-5}Sin70.53 \end{pmatrix} + \begin{pmatrix} 3x10^{-5}Cos70.53 \\ -3x10^{-5}Sin70.53 \end{pmatrix}$$
$$F_R = \begin{pmatrix} 2.0x10^{-5} \\ 0 \end{pmatrix}$$
$$|F_R| = \sqrt{(2.0x10^{-5})^2 + (0x10^{-5})^2}$$
$$|F_R| = 2.0x10^{-5}\ Nm^{-1}$$
$$tan\alpha = \frac{0}{2x10^{-5}}$$
$\alpha = 0°$ to the horizontal

8. Three conductors P, Q and R carryng currents 3A, 6A and 8A respectively are arranged as shown below



Find the net force on C

Solution



$$Cos\theta = \frac{10}{20}$$

$$\theta = 60°$$

$$F = \frac{\mu_0 I_2\, I_1\, l}{2\pi a}$$

$$F_R = \frac{4\pi x10^{-7}x8\,x3}{2\pi x20x10^{-2}} = 2.4x10^{-5}Nm^{-1}$$

$$F_Q = \frac{4\pi x10^{-7}x6\,x3}{2\pi x20x10^{-2}} = 1.8x10^{-5}Nm^{-1}$$

$$F_P = \begin{pmatrix} -2.4x10^{-5}Cos60 \\ -2.4x10^{-5}Sin60 \end{pmatrix} + \begin{pmatrix} -1.8x10^{-5}Cos60 \\ 1.8x10^{-5}Sin60 \end{pmatrix}$$

$$F_R = \begin{pmatrix} -2.1x10^{-5} \\ -0.52x10^{-5} \end{pmatrix}$$

$$|F_P| = \sqrt{(-2.1x10^{-5})^2 + (-0.52x10^{-5})^2}$$

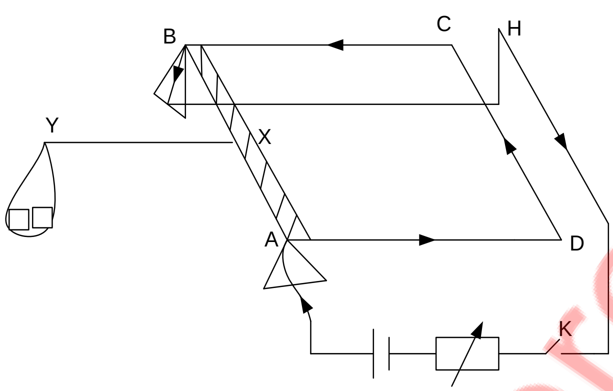
$$|F_P| = 2.163x10^{-5}\,Nm^{-1}$$

$$tan\alpha = \frac{0.52x10^{-5}0}{2.1x10^{-5}}$$

$\alpha = 13.91°$ to the horizontal

A, B and C are long thin wires arranged so that their centres lie at the corners of an isosceles triangle

Absolute measurement of current using the current balance



- ❖ Current is passed through a conducting frame ABCD which is pivoted about AB. The same current flows in opposite direction thrugh wire HG parallel and close to arm CD of the frame
- ❖ When the current content flows, CD is repelled downwards. Masses are placed in the scale pan at y to restore a horizontal balance.
- ❖ Since $\overline{XY}$ is equal to $\overline{BC}$ at balance, the force of repulsion is equal to the magnitude of weight placed at Y

Hence $mg = \frac{\mu_0 I^2 L}{2\pi R}$

- ❖ Curren flowing I can be obtained from

$I = \sqrt{\left(\frac{Rmg}{2x10^{-7}l}\right)}$ where l is the length of the paralle wire and R is the separation when the frame is horizontal

Advantage over ordinary pointer ammeter

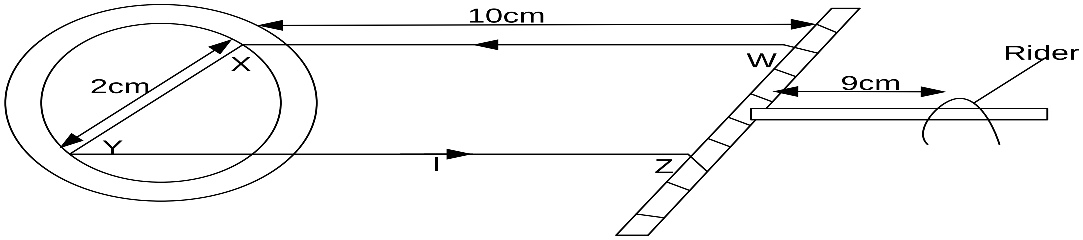
- it gives an accurate method because it involves measurement of fundamental quantities of length and mass.

Disadvantages

- It is not portable
- It doesn't give direct readings and requires a skilled person.

Examples

1. A rectangular wire WXYZ is balanced horizontally so that the length XY is at the center at a circular coil of 500 turns of means radius 10cm. The a current I is passed through XY and the circular coil, a rider of mass $5x10^{-4}$ kg has to be placed at a distance of 9.0cm from WZ to restore balance.



Find the value of I.

Solution

Taking moments about WZ.

$Fx10x10^{-2} = mgx0.09$

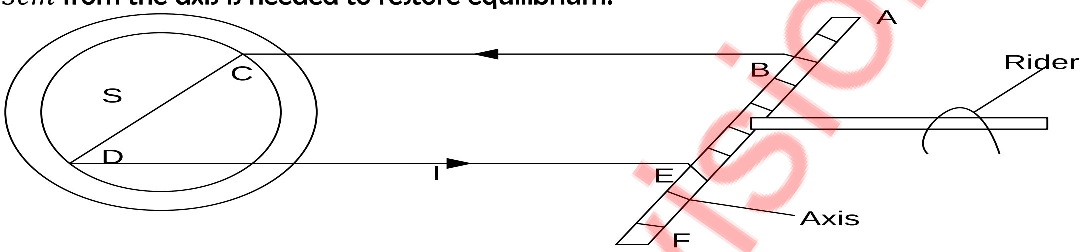
But $F = BIl$ and $B = \frac{\mu_0 NI}{2R}$

Hence $\frac{\mu_0 I^2 NL}{2R} x10x10^{-2} = 5x10^{-4}x9.81x0.09$

$$I = \sqrt{\left(\frac{5x10^{-4}x9.81x0.09x2x0.1}{4\pi x10^{-7}x500x2x10^{-2}x10x10^{-2}}\right)}$$

$$I = 8.38A$$

2. The figure below shows a simple form of a current balance. The long solenoid, s which has 2000 turns per meter is in series with a horizontal rengaular copper loop ABCDEF where $BC = 10cm$ and $CD = 3cm$. The loop which is freely pivoted on the axis AF goes well inside the solenoid and C is perpendicular to the axis of the solenoid. When a current I is switched on, a rider of mass $0.2g$ placed $5cm$ from the axis is needed to restore equilibrium.



Find the value of I.

Solution

Taking moments about WZ.

$Fx10x10^{-2} = mgx0.05$

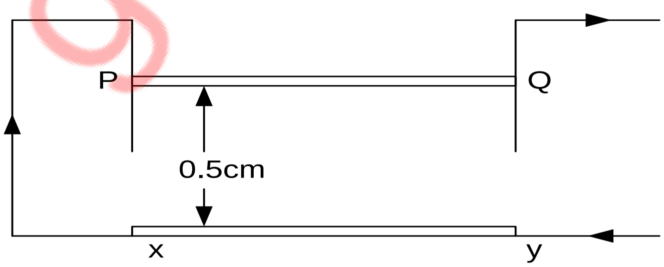
But $F = BIl$ and $B = \mu_0 nI$

$\mu_0 I^2 nLx10x10^{-2} = 0.2x10^{-3}x9.81x0.05$

$$I = \sqrt{\left(\frac{0.2x10^{-3}x9.81x0.05}{4\pi x10^{-7}x2000x3x10^{-2}x10x10^{-2}}\right)}$$

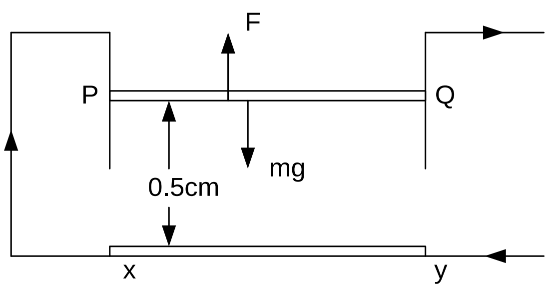
$$I = 3.65A$$

3. Two thin horizontal rods, xy and PQ each 1.0m in length carrying currents of equal magnitudes and are cconnected so that PQ is located $0.5cm$ above xy. The lower rod is fixed while the upper rod is kept in equilibrium by magnetic repulsion. The mass of each rod is $1.0x10^{-2}kg$



What is the magnitude of the current in each rod and state whether the curents.

Solution



At equilibrium $F = mg$

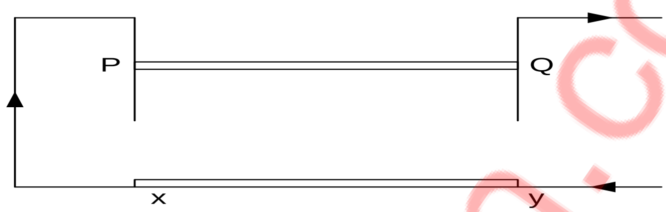
But $F = BIl$ and $B = \frac{\mu_0 I}{2\pi a}$

Hence $\frac{\mu_0 I^2 L}{2\pi a} = mg$

$$I = \sqrt{\left(\frac{2\pi x 0.5x10^{-2}x1.0x10^{-2}x9.81}{4\pi x10^{-7}x1.0}\right)}$$

$$I = 49.5A$$

4. A wire xy rests on a horizontal non conducting table and another wire PQ of length 12.0cm is free to move vertically in guides at the end P and Q above xy as shown below.

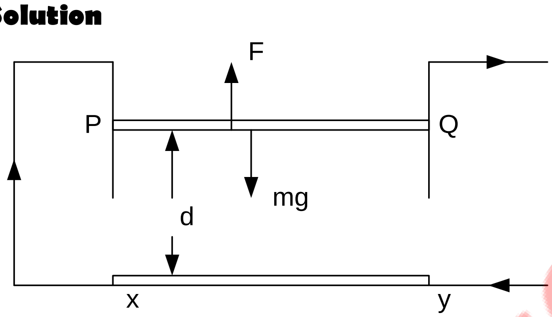


The mass per unit length of PQ $3mg\ cm^{-1}$. A current of 3.6A through the wires was enough to maintain the PQ at the distance dcm from xy.

(i) Calculate the distance of separation, d

(ii) Find the magnetic flux density due to PQ on xy

Solution



At equilibrium $F = mg$

But $F = BIl$ and $B = \frac{\mu_0 I}{2\pi d}$

Hence $\frac{\mu_0 I^2 L}{2\pi d} = mg$

$$d = \frac{\mu_0 I^2}{2\pi g}\left(\frac{L}{m}\right)$$

$$\frac{m}{l} = 3x10^{-6}kgcm^{-1} = \frac{3x10^{-6}}{10^{-2}}kgm^{-1}$$

$$\frac{m}{l} = 3x10^{-4}kgm^{-1}$$

$$d = \frac{4\pi x10^{-7}x(3.6)^2}{2\pi x9.8}\left(\frac{1}{3x10^{-4}}\right)$$

$$d = 8.82x10^{-4}m$$

$$B = \frac{\mu_0 I}{2\pi d} = \frac{4\pi x10^{-7}x3.6}{2\pi x8.82x10^{-4}}$$

$$B = 8.17x10^{-4}T$$

Exercise

1. The coil of a galvanometer is 0.02m by 0.08m. It consists of 200 turns of wire and is in a magnetic field of $0.2T$. The restoring torque constant of the suspension is $1x\mathbf{0}^{-5}$Nm per degree. Assuming the magnetic field is radial.
   (i) What is the maximum current that can be measured by the galvanometer if the scale accommodates a $45°$ deflection?
   (ii) What is the smallest current that can be detected if the minimum observed deflection of $0.1°$
   An ($7.03x10^{-3}A$, $1.56x10^{-5}A$)
2. A moving coil galvanometer has the following characteristics.
   - Number of turns = 80
   - Area o the coil = $50mm^2$
   - Flux density of the radial field = $0.2T$.
   - Torsional constant of the suspension wire = $5x10^{-9}Nm\ rad^{-1}$

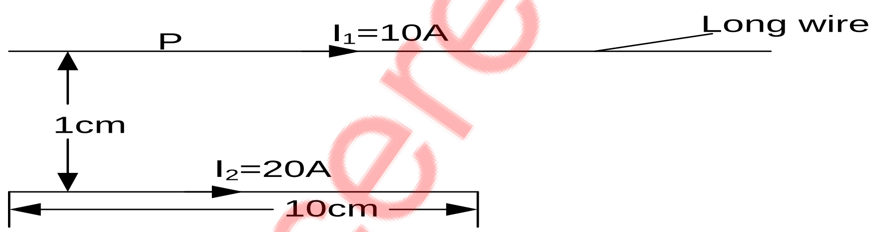
- Resistance of the coil = 20Ω

Calculate the angular deflection provided by

(i) a current of 0.01mA. **An** (1.6rad)

(ii) a p.d of 0.01mV. **An** (0.08rad)

3. A flat circular coil of 50 turns of mean diameter 40cm is in a fixed vertical plane and has a current of 5A flowing through it. A small coil 1cm square and having 120 turns is suspended at the center of the circular coil in a vertical plane at an angle of $30°$ to that of the larger coil. Calculate the torque that would act on the small coil when it carries a current o 2mA. **An** $(9.42x10^{-9})$
4. A square coil of side 1.2cm and with 20 turns of fine wire is mounted centrally inside and with its plane parallel to the axis of the long solenoid which has 50 turns per cm. The current in the coil is 70mA and the current in the solenoid is 6.2A.
   Find
   (i) the magnetic flux density inside the coil
   (ii) the torque on the square coil
5. Two parallel wires P and Q are placed $1.0cm$ apart and carrying current 10A and 20A respectively in the same direction. If wire Q is 10cm long, find the magnitude of the magnetic force acting on it. **An**$(4.0x10^{-4}N)$
6. Two parallel wires carrying currents of 5A and 3A respectively are 10cm apart. If the wire carrying current of 5A is 50cm long, find the force exerted on it. **An**$(1.5x10^{-5}N)$
7. Two thin, long parallel wires A and B carry currents of 5A and 2A respectively in opposite directions. If the wires are separated by a distance of 2.5 cm in a vacuum, calculate the force exerted by wire B on 1 m of wire A. **An**$(8.0x10^{-5}N)$
8. Two parallel wires each of length 75 cm are placed 1.0 cm aparat. When the same current is passed through the wires, a force of $50\ x\ 10^{-5}N$ develops between the wires. Find the magnitude of the current
9. Two long parallel wires X and Y are separately by 8cm in a vacuum. The wires carry currents of 10A and 5A respectively in the same direction. At what points between the wires is the magnetic flux density zero?
10. The diagram below shows two parallel wires P and Q placed 1cm apart and carrying currents of 10A and 20A respectively in the same direction.



If wire Q is 10 cm long, find the magnetic force acting on it. **An**$(4.0x10^{-4}N)$

ELECTROMAGNETIC INDUCTION

When a conductor wire moves across a magnetic field such that it cuts the magnetic field lines, an emf / current is induced in the wire. This process is called electromagnetic induction.

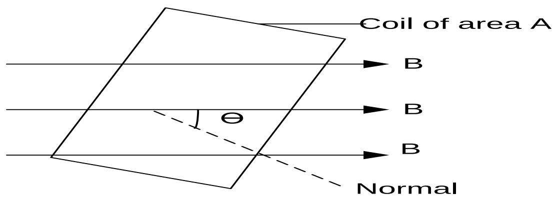
Definition

Electromagnetic induction is a process of generating an emf/ current is induced in a conductor when flux linking it changes.

MAGNETIC FLUX (Φ)

Magnetic flux is the product of the magnetic flux density B and projection of the area normal to the magnetic field.

Consider an area A, the normal of which makes an angle θ with the uniform magnetic field of flux density B.



The component of B along the normal to the area is $Bcos\theta$.

The magnetic flux through the area, $\Phi = ABcos\theta$

For N turns of the coil, total flux/(magnetic flux linkage) $\boxed{\Phi = NABcos\theta}$

If the magnetic field is perpendicular to the area A, then $\theta = 0$ magnetic flux $\Phi = ABcos\theta$,

Thus $\Phi = AB$

The unit of magnetic flux is the *Weber (Wb).*

Definition

Weber is the flux linking a circuit when the induced e.m.f is 1V and the flux is uniformly reduced to zero in one second

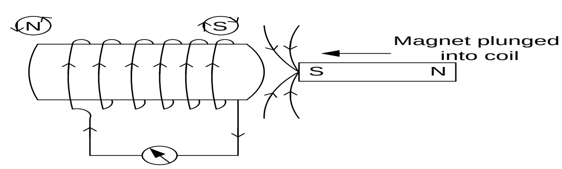
Laws of electromagnetic induction

Lenz's law:

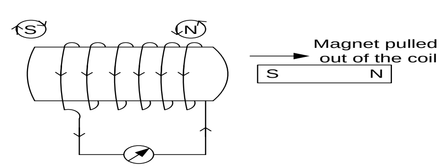
The induced emf acts in such a direction as to oppose the change in the flux causing it.

Illustration of Lenz's law

- When a magnet is suddenly pushed with its south pole towards a coil connected on a galvanometer, the galvanometer shows a deflection showing that current has been induced in the coil.



- On removal of the magnet from the coil, the galvanometer again deflects but in opposite direction.

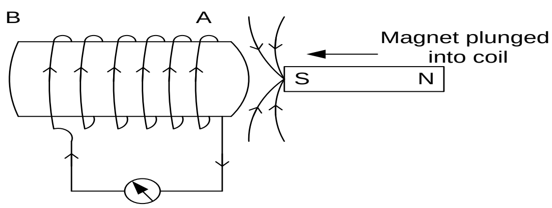


Note

If both the magnet and coil are stationary, the galvanometer gives no deflection because there is no change in the magnetic flux.

Likewise no deflection when the magnet and coil move with the same speed and same direction.

Lenz's law and conservation of energy



- When a magnet is suddenly pushed into a coil, current is induced in the coil and flows in such a direction that makes A a south pole.
- So work done by the external agent in moving the magnet and cause current to flow is the one converted to electrical energy, heat in the circuit and mechanical energy which deflects the pointer hence energy is conserved.

Faraday's law: It states that the magnitude of induced emf in a conductor is directly proportional to the rate of change of magnetic flux linking it.

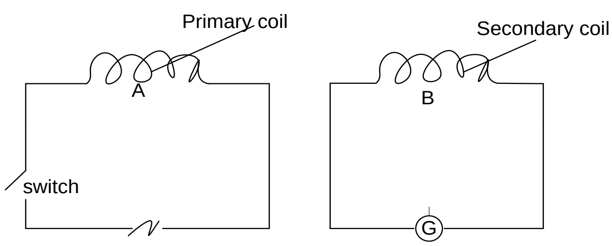
$$E \propto \frac{d\Phi}{dt}$$

$$E = -\frac{d\Phi}{dt}$$

For N turns of the coil

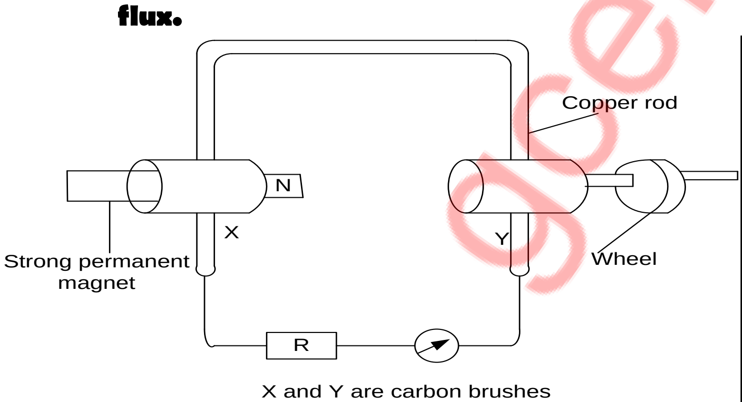
$$E = -N\frac{d\Phi}{dt}$$

Demonstration of Faraday's experiments



- As the switch K is closed, G shows a sudden temporary deflection showing that current is induced in a secondary coil. This is because the current in the primary coil increases from zero to a certain steady value, increasing the magnetic field and hence the number of field lines through the secondary.
- When K remains closed, G indicates no deflection, no emf is induced when the magnetic field lines through the secondary remain constant. As K is opened, G shows a sudden temporary deflection in the opposite direction. This is because the decrease in the primary current causes the field lines though the secondary to decrease. Therefore G only deflects when the current in the primary is changing and hence the magnetic flux through the secondary is changing.

Experiment to show that emf is directly proportional to the rate of change of magnetic flux.



- A copper rod which can be rotated round the north pole of a permanent magnet is connected as shown above
- The wheel is turned steadily until the deflection of the galvanometer is constant.
- The time, t for N revolutions is measured and the number number of revolutions per second is determined from $n = \frac{N}{t}$
- Note the deflection θ of the galvanometer.
- Repeat the experiment for different speeds of rtation of the wheel and the corresponding values of θ and n are recorded
- A graph of θ against n is plotted and it is a straight line through the origin.
- It implies $\theta \propto n$ and since $\theta \propto e.m.f$ induced and $n \propto$ speed of rotation of the rod
- Then e.m.f induced $\propto$ rate of change of magnetic flux

Factors which affect magnitude of the induced emf/current

a) Number of turns of the coil. Many turns give a large current
b) Strength of the magnetic field. Using a stronger magnet increases the induced emf
c) Speed at which the magnet moves. At a high speed the deflection is high.

Examples

1. A narrow coil of 60 turns and area $8cm^2$ is placed in a uniform magnetic field of flux density B of 0.4T so that the flux links the turns normally. If the flux is reduced to zero in 0.25s, find;
    a. Initial flux through the coil
    b. Initial flux linkage
    c. Induced e.m.f

Solution

(i) $\Phi = AB$

$\Phi = 0.4x8x10^{-4}$

$\Phi = 3.2x10^{-4}Wb$

(ii) $\Phi = NAB$

$\Phi = 60x0.4x8x10^{-4}$

$\Phi = 1.92x10^{-2}Wb$

(iii) $E = -N\frac{d\Phi}{dt}$

$E = -60\left(\frac{0 - 3.2x10^{-4}}{0.25}\right)$

$E = 7.68x10^{-2}V$

2. An air cored long solenoid of 500 circular turns per metre and radius 8.0 cm has a secondary coil of 20 turns tightly wound round its middle. The current in the solenoid is 2.0 A. Find the e. m.. f induced in the coil when the current in the solenoid is reduced to zero in 10^{-2} s.

Solution

$\Phi = AB$

$B = \mu_0 nI \quad and \quad \pi x r^2$

$\Phi = A\mu_0 nI$

$\Phi = 4\pi x10^{-7}x500x2x\pi x(8x10^{-2})^2$

$\Phi = 2.53x10^{-5}Wb$

$E = -N\frac{d\Phi}{dt}$

$E = -20\left(\frac{0 - 2.53x10^{-5}}{10^{-2}}\right)$

$E = 5.1x10^{-2}V$

3. A circular coil of 60 turns and area $6cm^2$ is placed in a uniform magnetic field of flux density B of 0.02T so that the flux links the turns normally.
    (i) Calculate the average induced e.m.f in the coil, if it is removed completely from the field in 0.7s.
    (ii) If the same coil is rotated about an axis through its middle so that it turns through $30°$ in 0.4s in the field B, find the average induced e.m.f

Solution

(i) $E = -N\frac{d\Phi}{dt} = -N\left(\frac{0-AB}{t}\right)$

$E = -60\left(\frac{0 - 0.02x6x10^{-4}}{0.7}\right)$

$E = 1.03x10^{-3}V$

(ii) $E = -N\frac{d\Phi}{dt} = -N\left(\frac{ABcos\theta - AB}{t}\right)$

$E = -60\left(\frac{0.02x6x10^{-4}cos30 - 0.02x6x10^{-4}}{0.4}\right)$

$E = 2.41x10^{-4}V$

4. A flat coil of 100 turns and area $0.2m^2$ is placed in a uniform magnetic field of flux density B of 0.01T so that the flux links the turns normally. If the coil is rotated about an axis so that it turns through $60°$ in 2s, find;

(i) Initial flux linkage through the coil
(ii) the average induced e.m.f

Solution

(i) $\Phi = NAB$

$\Phi = 100x0.2x0.01$

$\Phi = 0.2Wb$

(ii) $E = -N\frac{d\Phi}{dt} = -N\left(\frac{ABcos\theta - AB}{t}\right)$

$E = -100\left(\frac{0.2x0.01cos60 - 0.2x0.01}{2}\right)$

$E = 0.05V$

5. 500 circular turns per metre and radius 8.0 cm has a secondary coil of 20 turns tightly wound round its middle. The current in the solenoid is 2.0 A. Find the e. m.. f induced in the coil when the current in the solenoid is reduced to zero in 10^{-2} s.

Solution

$\Phi = NAB$

$B = \mu_0 nI \quad and \quad \pi x r^2$

$\Phi = NA\mu_0 nI$

$\Phi = 4\pi x10^{-7} x1000x1x\pi x(4x10^{-2})^2$

$\Phi = 6.31x10^{-6} Wb$

Exercise

A narrow circular coil of 10 turns and area $4x10^{-2}m^2$ is placed in a uniform magnetic field of flux density B of $10^{-2}T$ so that the flux links the turns normally. find;

(i) the average induced e.m.f. If the coil is completely removed from the field in $0.5s$.

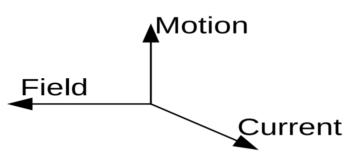
(ii) the average induced e.m.f. If the coil is rotated about an axis through its middle so that it turns through $60°$ in $0.2s$ in field B. **An($8.0x10^{-3}V$, $1x10^{-2}V$)**

Induced current

This a current obtained without use of a battery and the e.m.f which produces that current is called induced e.m.f

Direction of induced current in a straight wire

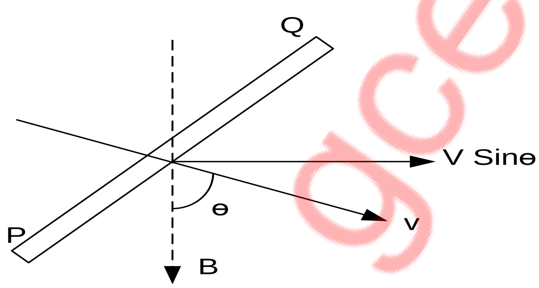
To predict the direction of induced emf (current), we use Fleming's right hand rule or dynamo rule.



Thumb-------motion
First finger----Field
Second finger---induced current

Emf induced in a moving conductor (rod)

Consider a conducting rod PQ pulled with uniform velocity, v of a rectangular frame with a force F.



Component of the velocity perpendicular to B is $vsin\theta$

Rea swept by the conductor in one second is $l\ vsin\theta$

Change in magnetic flux in one second is $B\ lvsin\theta$

But induced e.m.f $E = -\frac{d\Phi}{dt}$

$$|E| = Blvsin\theta$$

Note :

If the velocity is perpendicular to the field, then $\theta = 90$ to the field then $E = Blv$

Examples

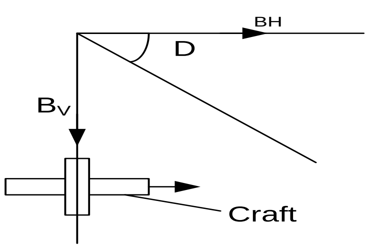
1. A metal rod of length $8.0cm$ is moved in the plane of the paper at $3.0ms^{-1}$ through a magnetic field of flux density 0.04T which is directed into the paper. Find the magnitude of the induced e.m.f

Solution

$E = Blv$ | $E = 0.04x0.08x3$ | $E = 0.0096V$

2. An air craft is flying horizontally at $800kmh^{-1}$ at a point where the earth's magnetic flux density is $2.31x10^{-5}$T and the angle of dip is 60°. If the distance between the wings tips is 50m. calculate the potential difference induced between its wings

Solution



$E = B_V lV$ *(wings cut the vertical component)*

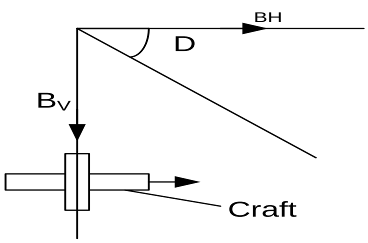
$$E = l\ V\ B\ Sin\ D$$

$$E = 2.31x10^{-5}x50x\left(\frac{800x1000}{3600}\right)xsin60$$

$$\boldsymbol{E = 0.22V}$$

3. A metallic air craft with the wing span of 40m flies with its ground velocity of $1000kmh^{-1}$ in the direction due east at a constant altitude in region of northern hemisphere whee the horizontal component of the earth's magnetic field is $1.6x10^{-5}T$ and the angle of dip is 71.6°. Find the p.d between the wing tips.

Solution



$E = B_V lV$ *(wings cut the vertical component)*

$$B_V = B_H \tan D$$

$$E = l\ V\ B_H\ tan\ D$$

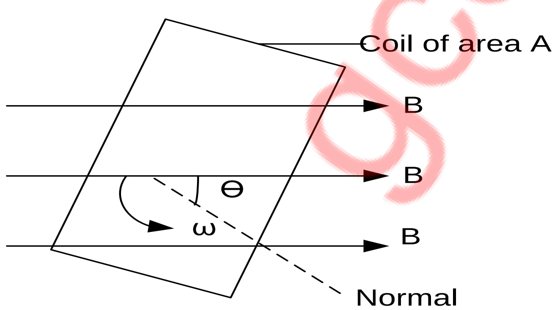
$$E = 1.6x10^{-5}x40x\left(\frac{1000x1000}{3600}\right)xtan71.6$$

$$\boldsymbol{E = 0.53V}$$

Exercise

1. An air craft is flying horizontally at $100m^{-1}$ at a point where the earth's magnetic flux density is $5.0x10^{-5}$T and the angle of dip is 71°. If the distance between the wings tips is 20m. calculate the e.m.f induced between its wings **An**$(\boldsymbol{9.46x10^{-2}V})$
2. An air craft is flying horizontally at $1000kmh^{-1}$ at a point where the earth's magnetic flux density is $2.4x10^{-5}$T and the angle of dip is 80°. If the distance between the wings tips is 60m. calculate the potential difference induced between its wings **An**$(\boldsymbol{0.39V})$

Emf induced in a rotating coil.



By Faraday's law

$$E = -\frac{d\Phi}{dt} = -\frac{d}{dt}(NABcos\theta) = -NAB\frac{d}{dt}cos\theta$$

Where $\theta = \omega t$

$$E = -NAB\frac{d}{dt}cos\omega t$$

$$E = -NAB(-\omega sin\omega t)$$

$$\boxed{E = NAB\omega sin\omega t}$$

OR

$$\boxed{E = NAB2\underbrace{\pi}_{3.14}fsin2\underbrace{\pi}_{180°}ft}$$

Maximum $\boldsymbol{e.m.f}$

$E.m.f$ is maximum when $\theta = 90°$

$$\boxed{E_{max} = NAB\omega}$$

Examples

1. A rectangular coil of 100 turns and area $400cm^{2}$ is rotated about a horizontal axis at a constant rate of 50 revolutions per second in a horizontal magnetic field of $0.21T$ to the axis. Calculate

(i) Maximum $e.m.f$ induced in the coil
(ii) Induced $e.m.f$ when the coil is 30° to the horizontal

Solution

(i) $E_{max} = NAB\omega$

$E_{max} = NAB2\pi f$

$E_{max} = 100x400x10^{-4}x0.21x2x3.14x50$

$E_{max} = 264V$

(ii) $E = E_{max}sin\theta$

$E = 264sin(90 - 30)$

$E = 228.6V$

2. A flat circular coil with 200 turns each of radius 40cm is rotated at a uniform rate of 480 revolutions per minute about its diameter at right angles to a uniform magnetic field of $5x10^{-4}T$. Calculate;
   (i) the peak value of emf induced in the coil.
   (ii) the instantaneous value of induced emf when the plane of the coil makes an angle of 20° with the field direction.

Solution

(i) $E_{max} = NAB\omega$

$E_{max} = NAB2\pi f$

$$E_{max} = 2000x\frac{22}{7}x(40x10^{-2})^2x5x10^{-4}x2x\frac{22}{7}x\frac{480}{60}$$

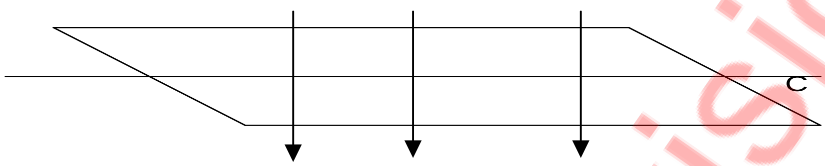
$E_{max} = 25.3V$

(iii) $E = E_{max}sin\theta$

$E = 25.3xsin(90 - 20)$

$E = 23.7V$

3. A coil of 100 turns and area $2\ x\ 10^{-2}\ m^2$ lies in a magnetic field of flux density $3x10^{-3}T$ and rotates uniformly at 100 revolutions per second about an axis perpendicular to the magnetic field as shown.



Calculate
(i) the emf induced when the plane of the coil makes 60° with B
(ii) the amplitude of the induced emf.

Solution

(i) $E = NAB\omega sin\omega t$

$$E = 100x2x10^{-2}x3x10^{-3}x2x\frac{22}{7}sin(90 - 60)$$

$\boldsymbol{E = 1.88V}$

(ii) $E_{max} = NAB\omega$

$$E_{max} = 100x2x10^{-2}x3x10^{-3}x2x\frac{22}{7}$$

$E_{max} = 3.77V$

4. A flat circular coil with 500 turns each of radius 10 cm is rotated at a frequency of 200 revolutions per minute about its diameter at right angles to a uniform magnetic field of flux density 0.18 T. Calculate the :
   (i) maximum magnetic flux linking the coil
   (ii) e.m.f induced in the coil when the plane of the coil makes an angle of 30° with the magnetic field.
   (iii) Root mean square value of the e.m.f induced in the coil

Solution

(i) $\Phi_{max} = NAB$

$\Phi_{max} = 500x\pi x(10^{-2})^2x0.18$

$\Phi_{max} = 2.83Wb$

(ii) $E = NAB\omega sin\omega t$

$$E = 2.83x2x\frac{22}{7}x\frac{200}{60}sin(30)$$

$\boldsymbol{E = 51.3V}$

(iii) $E_{rms} = \frac{E_{max}}{\sqrt{2}}$

$$E_{rms} = \frac{NAB\omega}{\sqrt{2}}$$

$$E_{rms} = 2x\frac{22}{7}x\frac{200}{60}x\frac{2.83}{\sqrt{2}}$$

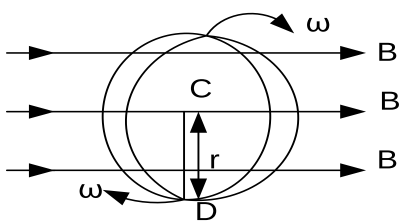
$E_{rms} = 41.9V$

Questions:

1. A metal disc of diameter 20cm rotates at constant speed of 60*rev min*$^{-1}$ about an axis through its center and perpendicular to a uniform magnetic field of density $5x10^{-3}$T established parallel to the axis of rotation. Calculate the emf developed between the axis and the rim of the disc. **An[1.57mV]**
2. A square coil of side 20cm and 100 turns is arranged to rotate at 240 revolutions per mnute about a vertical axis perpendicular to the coil placed in the horizontal uniform magnetic flux density of 1.5 T, the axis of rotation passes through the mid points of a pair of opposite sides of the coil. Calculate the e.m.f induced in the coil if the plane of the coil makes an angle of 30° with the field **An[130.6V]**
3. A search coil of 20 turns each of area 4cm^2 is connected in series with a ballistic galvanometer and the resistance which makes the total resistance of the circuit 2000Ω. When the search coil is suddenly withdrawn from between the poles of the magnet the galvanometer gives a throw of 20 scale divisions. When a capacitor of capacitance $1\mu F$ changed to a $p.d$ of 2V is discharged through the galvanometer, a throw of 10 divisions is obtained. What is the magnetic flux density between the magnetic poles? **An[1T]**
4. A horizontal straight wire 5cm long has a mass of 1.2gm^{-1} and is placed perpendicular to uniform horizontal magnetic field of flux of 0.6T. The resistance of the wire is 3.8Ω in the magnetic field. Find the potential difference that must be applied between the ends of the wire to make it self supporting.

Emf Induced in a disc rotating about its axis in a uniform field.

Consider a disc of radius *r*, rotating about its axis at a distance.



CD cuts the magnetic flux continuously;

Average velocity V of CD= $\frac{0+r\omega}{2}$

$$v = \frac{r\omega}{2}$$

Induced e.m.f in CD, $E = Blv$

$$E = Br\frac{r\omega}{2}$$

$E = \frac{Br^2\omega}{2}$ but $\omega = 2\pi f$

$$E = \frac{Br^2 x 2\pi f}{2}$$

$$\boxed{|E| = B\pi r^2 f}$$

or $\boxed{|E| = BAf}$

OR

Magnetic flux linking with the disc, $\Phi = AB$

Induced e.m.f, $E = -\frac{d\Phi}{dt} = -\frac{dAB}{dt} = -B\frac{dA}{dt}$

But $\frac{dA}{dt} = \frac{\pi r^2}{T} = \pi r^2 f$ since $\frac{1}{T} = f$

$$\boxed{|E| = B\pi r^2 f}$$

Examples

1. A circular metal disc of area $3x10^{-4}m^2$ is rotated about a horizontal axis at a constant rate of 50 revolutions per second in a horizontal magnetic field of $6.0x10^{-3}T$ to the axis. Calculate induced $e.m.f$ in the disc.

Solution

$E = BAf$ | $E = 6.0x10^{-3}x3.0x10^{-4}x50$ | $E = 9x10^{-5}V$

2. A circular aluminum disc of radius 40cm is mounted inside a long solenoid of 2000 turns per meter carrying current of 20A such that its axis coincides with that of the solenoid, if the disc is rotated about its axis at 30 revolutions per minute. What will be the $e.m.f$ induced in the disc

$E = B\pi r^2 f$ $\quad B = \mu_0 nI$

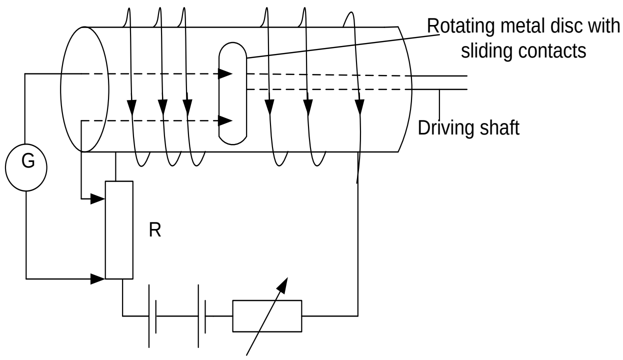
$$E = 4\pi x10^{-7}x2000x20x\pi x(40x10^{-2})^2 x\frac{30}{60}$$

$E = 0.013V$

Exercise

A circular aluminium disc, of radius 30 cm, is mounted inside a long solenoid of $2\ x\ 10^3$ turns per metre carrying a current of 20.0A such that its axis coincides with that of the solenoid. If the disc is rotated about its axis at 40 revolutions per minute, what will be the e.m.f induced? **An**($9.5x10^{-3}V$)

Absolute determination of resistance



- The circuit is connected as above. The metal disc of known radius r is placed at the centre of a solenod carrying current with the plane of the disc perpendicular to the magnetic field.
- The disc is rotated using a driving shaft. The speed of rotation of the disc is adjusted until the galvanometer shows no deflection. The number of revolutions for a given interval of time is counted and the frequency f determined.
- The resistance is calculated from $R = \mu_0 n\, \pi r^2 f$ where n is the number of turns per meter of the solenoid.

Theory

$E = B\pi r^2 f$

Where r is the radius of the disc, f is the frequency of revolution of the disc.

Potential difference across $\boldsymbol{R}, V = IR.$

Hence when current is zero, $IR = B\pi r^2 f$

$$R = \frac{B\pi r^2 f}{I} \qquad B = \mu_0 nI$$

$$R = \frac{\mu_0 nI\, \pi r^2 f}{I}$$

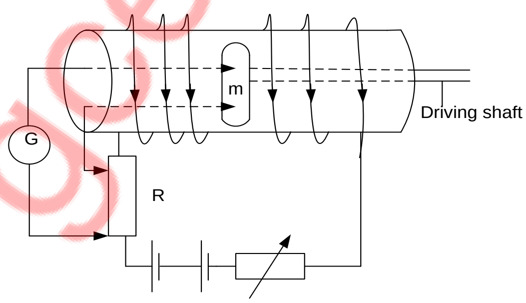
$$\boxed{R = \mu_0 n\, \pi r^2 f}$$

Limitations of the set up in measuring resistance

- The method is used for measuring only small resistance since e.m.f generated is very small.
- The earth's magnetic field affects the induced e.m.
- The effect of thermal e.m.f generated due to the friction at the contact.

Example

A circular metal disc , m of radius 10.0cm is mounted at the centre of along solenoid of 5000 turns per meter as shown below



A solenoid winding is connected in series with a resistor R which carries a current of 0.2 A. A metal disc is rotated about the axis of the solenoid at a rate of $100\pi\ rad\ s^{-1}$, the e.m.f induced between the centre and the rim of the disc balances the p.d across the resistor. Calculate the;

(i) Magnetic flux density along the axs of the solenoid

(ii) Resistance R

Solution

(i) $B = \mu_0 nI$ | $B = 4\pi x10^{-7}x5000x0.2$

$$B = 1.26x10^{-3}T$$

(ii) $IR = B\pi r^2 f$

but $f = \frac{\omega}{2\pi}$

$$R = \frac{B\pi r^2}{I}\frac{\omega}{2\pi}$$

$$R = \frac{1.26x10^{-3}\pi(0.1)^2}{0.2}\left(\frac{100\pi}{2\pi}\right)$$

$$R = 9.9x10^{-3}\Omega$$

BALLISTIC GALVANOMETER

The ballistic galvanometer consists of a rectangular coil of fine copper wire wound on a heavy insulating former and having a fine suspension.
The coil is made heavy and the suspension fine in order to obtain a large period of oscillation.
The former is made of insulating material to minimize damping which arises due to eddy currents.

Mode of operation

When a charge Q flows through the galvanometer, the mass of its coil makes it swing slowly so that the charge finishes circulating while the coil is just beginning to turn.

Theory shows that if dumping is negligible, the first deflection or "throw" of the galvanometer is directly proportional to the quantity of charge Q, which passes through its coil as it begins to move.

$$Q \propto \theta$$

Where θ is the first deflection of the galvanometer.

$$Q = k\theta$$

Where *c* is the charge sensitivity.
Unit of $k = C\ rad^{-1}\ or\ C\ div^{-1}$

Relationship between magnetic flux and induced charge.

Consider a coil of N turns with a changing magnetic flux.

Instantaneous induced e.m.f, $E = -N\frac{d\Phi}{dt}$

Instantaneous current $I = \frac{E}{R} = -\frac{N}{R}\frac{d\Phi}{dt}$

But $I = \frac{dQ}{dt}$

$$\frac{dQ}{dt} = -\frac{N}{R}\frac{d\Phi}{dt}$$

$$dQ = -\frac{N}{R}d\Phi$$

Suppose when time is zero flux linkage is Φ_0 and when time is t flux linkage is Φ_t. Then the total induced charge is

$$Q = -\frac{N}{R}\int_{\Phi_0}^{\Phi_t} d\Phi$$

$$Q = -\frac{N}{R}(\Phi_t - \Phi_0)$$

$$Q = \frac{N(\Phi_0 - \Phi_t)}{R}$$

Induced charge, $Q = \frac{chjange\ in\ flux}{total\ resistance}$

Note:

(i) Suppose the plane of the coil is perpendicular to B and is turned through $90°$ so that no magnetic flux threads it

$$Q = \frac{N(\Phi_0 - \Phi_t)}{R}$$

$$\Phi_0 = AB, \quad \Phi_t = 0$$

$$Q = \frac{NAB}{R}$$

But $Q = k\theta$

Where θ is the throw on B.G and k is the constant of the B.G

$$k\theta = \frac{NAB}{R}$$

$$B = \frac{k\theta R}{NA}$$

(ii) Suppose the plane of the coil is perpendicular to B and is turned through $180°$

$$Q = \frac{N(\Phi_0 - \Phi_t)}{R}$$

$$\Phi_0 = AB, \quad \Phi_t = -AB$$

$$Q = \frac{2NAB}{R}$$

But $Q = k\theta$

Where θ is the throw on B.G and k is the constant of the B.G

$$k\theta = \frac{2NAB}{R}$$

$$B = \frac{k\theta R}{2NA}$$

Examples

1. A circular coil of wire 24 turns with its end joined stands with its plane vertical and facing magnetic North and South. It's suddenly turned 180° about the vertical axis. If the radius of the coil is 40cm its resistance 0.5Ω and if the horizontal component of the earth field is $1.8x10^{-5}T$. Calculate the quantity of electricity set in motion.

Solution

$$Q = \frac{2\Phi}{R} = \frac{2NBA}{R}$$

$$Q = \frac{2x24x1.8x10^{-5}x(0.4)^2}{0.5}$$

$$Q = 8.686x10^{-4}C$$

2. A long solenoid carries a current which produces a flux density B at its center. A narrow coil M of 10 turns and are $4m^2$ is placed in the solenoid so that the flux links its turns normally and the ends of M are connected. If the charge of $1.6x10^{-3}C$. Circulates through M when current in the solenoid is reversed, given that the resistance of M is 0.2Ω. Calculate the magnetic flux density.

Solution

$$Q = \frac{2\Phi}{R} = \frac{2NBA}{R}$$

$$B = \frac{1.6x10^{-3}x0.2}{2x10x4}$$

$$B = 4x10^{-6}T$$

3. A ballistic galvanometer is connected to a flat coil having 40 turns of mean area $3.0cm^2$ to form a circuit of total resistance 80Ω. The coil is held between the poles of an electromagnet with its plane perpendicular to the field and is suddenly withdrawn form the field producing a throw of 30 scale divisions. Find the magnetic field density at the place where the coil was held assuming that the sensitivity of the galvanometer under the conditions of the experiment is $0.4div\ \mu C^{-1}$

Solution

$$\frac{1}{k} = 0.4div\ \mu C^{-1}$$

$$k = \frac{1}{0.4} = 2.5\mu C\ div^{-1}$$

$$B = \frac{k\theta R}{NA}$$

$$B = \frac{2.5x10^{-6}x30x80}{40x3.0x10^{-4}}$$

$$B = 0.5T$$

4. A ballistic galvanometer of sensitivity $2\ div\ \mu C^{-1}$ is connected across the coil having 10 turns wound tightly round the middle of a solenoid of 10^3 turns per meter and diameter 5.0cm. when the current in the solenoid is reversed, a ballistic galvanometer deflects through 8 divisions. If the total resistance of the coil is 20Ω. Find the current in the solenoid.

Solution

$$\frac{1}{k} = 2div\ \mu C^{-1}$$

$$k = \frac{1}{2} = 0.5\mu C\ div^{-1}$$

$B = \frac{k\theta R}{2NA}$ *and* $B = \mu_0 nI$

$$\mu_0 nI = \frac{k\theta R}{2NA}$$

$$I = \frac{0.5x10^{-6}x8x20}{2x4\pi x10^{-7}x10^3x\pi x(2.5x10^{-2})^2x10}$$

$$I = 1.62A$$

5. A circular coil of 50 turns and mean radius 0.5m is arranged so that its palne is perpendicular to the magnetic meridian. The coil is connected to a ballistic galvanometer of sensitivity $5.7x10^4\ rad\ C^{-1}$. The total resistance of the coil and galvanometer is 100Ω. When the coil is rotated through 180° about a vertical axis, the B.G deflects through 0.8 radians.
   (i) Find the horizontal component of the earth's manetic flux density
   (ii) Calculate the voltage along a solenoid of 2400 turns per meter and total resistance 8Ω required to produce a magnetic field along the axis of the solenoid and of the same strength as calculated in (i) above

Solution

(i) $\frac{1}{k} = 5.7x10^4\ rad\ C^{-1}$

$$k = \frac{1}{5.7x10^4} C\ rad^{-1}$$

$$B = \frac{k\theta R}{2NA}$$

$$B = \frac{0.8x100}{5.7x10^4\ 2x50x\pi x(0.5)^2}$$

$$B = 1.79x10^{-5}\ T$$

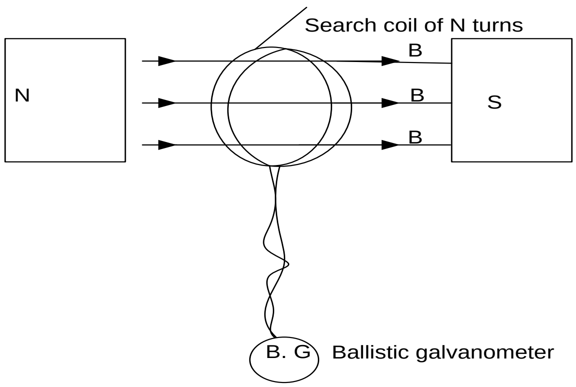
(ii) $B = \mu_0 nI$ and $V = IR$

$$V = \frac{BR}{\mu_0 n}$$

$$V = \frac{1.79x10^{-5}x8}{4\pi x10^{-7}x2400}$$

$$V = 4.75x10^{-2}V$$

Measurement of magnetic flux density of the poles of a strong magnet



- A search coil of known number of turns N, and area A, is connected in series with <u>a B.G</u> . the search coil is placed between pieces of the magnet with its palne <u>normal</u> to the <u>magnetic field.</u>
- After the B.G pointer has settled, the coil is <u>completely with drawn from the field</u> and the first deflection θ_1 of the B.G is noted.
  $c\theta_1 = \frac{NAB}{R} \ldots\ldots(i)$ where R is resistance of the coil
- A capacitor of known capacitance C is charged to a p.d V and then discharged through the B.G and the second deflection θ_2 is noted.
  $$c\theta_2 = CV \ldots\ldots\ldots(ii)$$
- The magnetic flux density, B is now calculated from $B = \frac{RCV}{NA}\frac{\theta_1}{\theta_2}$

Source of errors

- It may not be possible to egt the coil completely out of the field
- There is influence of the earth's magnetic field
- Improper positioning of the search coil initially
- Inaccurate reading of the B.G scale

Examples

1. A rectangular coil of area 20cm^2 and 10 turns is placed with its plane perpendicular to a magnetic field of flux density B. the ends of the coil are connected to ballistic galvanometer and total resistance in the circuit is 10Ω. When the coil is removed from the field, the charge of 60μC is measured by the galvanometer. Calculate the magnetic flux density

Solution

$Q = \frac{N(\Phi_0 - \Phi_t)}{R}$ $\quad \Phi_2 = 0$

$$Q = \frac{N(AB - 0)}{R}$$

$$60x10^{-6} = \frac{10x20x10^{-4}xB}{10}$$

$$B = 0.3T$$

2. A capacitor of apacitance 2000μF is fully charged to 10V. When the capacitor is discharged through a ballistic galvanometer gives a maximum throw of 20 divisions. A coil of 25 turns each of radius 10cm is palced with its plane perpendicular to a uniform magnetic field. The coil is connected in series with the ballistic galvanometer. When the coil is rotated through 180°, the galvanometer gives a maximum throw of 15 divisions. Calculate the magnetic flux density, if the total resistance in the circuit is 3Ω

Solution

$$B = \frac{RCV}{2\ NA}\frac{\theta_1}{\theta_2}$$

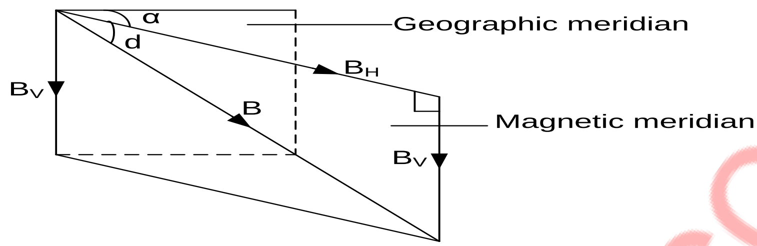
$$B = \frac{3x2000x10^{-6}x10x15}{2x\ 25x\pi(0.1)^2x20}$$

$$B = 2.86x10^{-2}\ T$$

Earth's magnetic field

The earth behaves as though it contains a short bar magnet inclined at a small angle to its own axis of rotation and its south pole is in the northern hemisphere

Specification of the earth's magnetic field



Geographic meridian:

This is a vertical plane containing the place and the earth's axis of rotation.

Magnetic meridian:

This is a vertical plane through the magnetic north and south pole of the earth's magnet

Angle of declination (α)**:**

This is the angle between the earth's meridian and geographic meridian

Angle of Dip (Angle of inclination) (d):

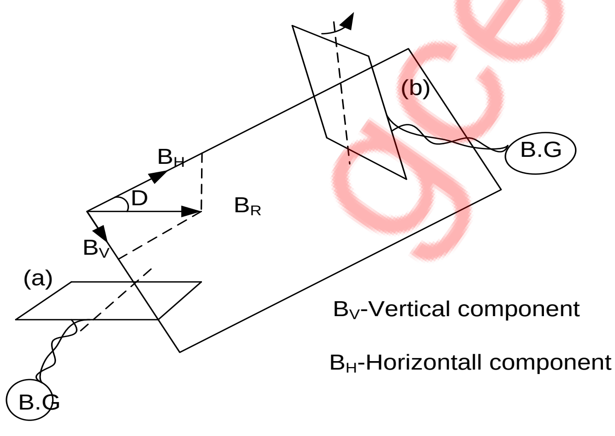
This is the angle between the axis of a freely suspended bar magnet and the horizontal.
OR angle between the earth's resultant magnetic flux density and the horizontal

$$\boxed{\tan d = \frac{B_V}{B_H}}$$

Earth inductor

This consists of a coil of a wire which can be rotated about an axis capable of beign set in any direction by turning the movable frame. This is the instrument used to measure the flux density of the earth.

Determining earth's magnetic flux density



- A search coil of known number of turns N, and area A, is connected to a calibrated ballistic galvanometer(B.G) so that the total resistance in the circuit is R.
- The coil is placed in a vertical plane perpendicular to the magnetic meridian of the earth. The coil is then rotated through $180°$ about the vertical axis and the maximum throw θ_1 of the B.G is noted
- The coil is then placed in a horizontal plane perpendicular to the magnetic meridian of the earth. The coil is again rotated through $180°$ about the horizontal axis and the maximum throw θ_2 of the B.G is noted

$$\Rightarrow \; B_H = \frac{k\theta_1 R}{2\,NA} \ldots \ldots (i)$$

$$\Rightarrow \; B_V = \frac{k\theta_2 R}{2NA} \ldots \ldots (ii)$$

- k is determined by charging a standard capacitor C to a known p.d V and discharging it through the B.G and the deflection α noted
  $\Rightarrow \ k = \frac{C\,V}{\alpha}$

- Earth magnetic flux density, B is got from:

$$B = \sqrt{{B_H}^2 + {B_V}^2}$$

Determining angle of Dip D

Its determined from $tanD = \frac{B_V}{B_H} = \frac{\theta_V}{\theta_H}$

Examples

A coil of an earth inductor is rotated through 90° from a position in which it's threaded by the horizontal component of the earth field to a position in which it's threaded. Throw of the galvanometer is 26 divisions. When rotated through 90° between positions in which I is threaded and not threaded by the vertical component the throw is 59.5 divisions. What is the angle of Dip.

Solution

$$tanD = \frac{B_V}{B_H} = \frac{\theta_V}{\theta_H}$$

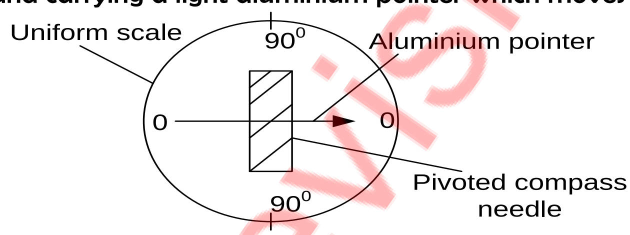
$$D = tan^{-1}\left(\frac{59.5}{26}\right)$$

D= 66.4°

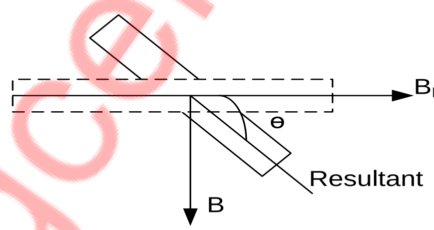
Defelection magnetometer(tangent galvanometer)

Structure and mode of operation

It is used to compare the strength of the magnetic fields. It consists of a small magnet (compass need) pivoted on a vertical axis and carrying a light aluminium pointer which moves over a uniform scale



- The horizontal component of the earht's magnetic field B_H and the other field B to be compared are arranged at right angles to each other
- The pivoted magnet then sets its self along the resultant of the two fields, at ange θ to its own direction when it is in the field B_H alone.



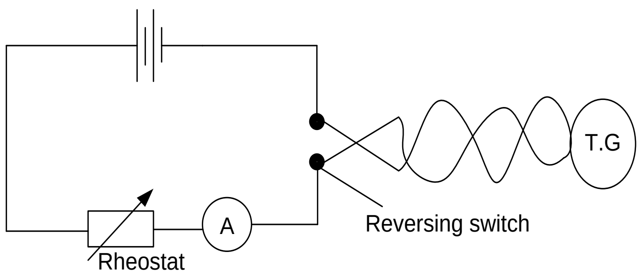
$$\boldsymbol{tan\theta} = \frac{\boldsymbol{B}}{B_H}$$

- It consists of a circular coil at the centre of which there is deflection magnetometer. If N is numbere of turns in the coil, R is the radius of the coil and I is the current flowing, then the flux density at the center of the coil is $B = \frac{\mu_0 NI}{2R}$

$$\boldsymbol{tan\theta} = \frac{\boldsymbol{B}}{B_H} = \frac{\frac{\mu_0 NI}{2R}}{B_H}$$

$$B_H = \frac{\mu_0 NI}{2R\ \boldsymbol{tan\theta}}$$

Measurement of horizontal component B_H of the earth's magnetic flux density by a tangent galvanometer



- A magnetometer is mounted on a vertical axis at the center of the coil such that the plane of the coil lies along the magnetic meridian
- Different currents I are passed thrpguh the coil. For each current, the deflections θ_1 and θ_2 of the magnetometer are measured
- The current is reversed and the deflections θ_3 and θ_4 of the magnetometer recorded. The average deflection $\theta = \frac{\theta_1+\theta_2+\theta_3+\theta_4}{2}$ is calculated.
- The results are tabulated including value of $\boldsymbol{tan\theta}$.
- A graph of $tan\theta$ against I is plotted and its slope, s determined
- The horizontal component of the earht's magnetic field is deteined from $B_H = \frac{\mu_0 N}{2R\,S}$ where , N is number of turns in the coil, R is the radius of the coil

Possible sources of errors

(i) Imperfect setting of the coil iin the magnetic meridian
(ii) The magnetometer not being well levelled
(iii) Magnetic material and stray magnetic fields within the vicinity of the T.G

Example

1. A circular coil of 50 turns and mean radius 7.0cm is set with its plane in the magnetic meridian. There is a small pivoted magnetic needle at its centre. When a current of 0.023A is passed through the coil, it is found that the coil must be rotated through $30°$ before the magnet is once again in the plane of the coil. Calculate the horizontal component of the earth's induction.

Solution

$$B_H = \frac{\mu_0 NI}{2R\,\boldsymbol{tan\theta}}$$
$$B_H = \frac{4\pi x10^{-7}x50x0.023}{2x7x10^{-2}x\,\boldsymbol{tan30}}$$
$$B_H = 1.79x10^{-5}T$$

2. A circular coil of 5 turns and mean diameter 10cm is mounted with its plane vertical and along the magnetic meridian. A small compasss needle is mounted on a vertical axis at the centre of the coil. When a current of 0.5A is passed through the coil, the compass needle deflects through $61°$. When the current is reversed, the compass needle deflects through $59°$. Calculate the horizontal component of the earth's magnetic field intensity.

Solution

$B_H = \frac{\mu_0 NI}{2R\,\boldsymbol{tan\theta}}$ where

$\theta = \frac{59+61}{2} = 60°$

$$B_H = \frac{4\pi x10^{-7}x5x0.5}{2x5x10^{-2}x\,\boldsymbol{tan60}}$$
$$B_H = 1.81x10^{-5}T$$

3. A circular coil of 50 turns and mean radius 4cm is placed with its plane in the magnetic meridian. A small compasss needle is placed at the centre of the coil. When a current of 0.1A is passed through the coil, the compass needle deflects through $40°$. When the current is reversed, the compass needle deflects through $43°$. Calculate the;
   (i) horizontal component of the earth's magnetic field intensity
   (ii) Magnetic flux density of the earth's field at the place given that the angle of dip at the place is $15°$.

Solution

(i) $B_H = \frac{\mu_0 NI}{2R\,\boldsymbol{tan\theta}}$ where

$\theta = \frac{40+43}{2} = 41.5°$

$$B_H = \frac{4\pi x10^{-7} x50x0.1}{2x4x10^{-2}x\ \boldsymbol{tan41.5}}$$

$$B_H = 8.88x10^{-5}T$$

(ii) $B_H = Bcos\ D$

$$B = \frac{8.88x10^{-5}}{Cos\ 15}$$

$$B = 9.19x10^{-5}\text{T}$$

Exercise

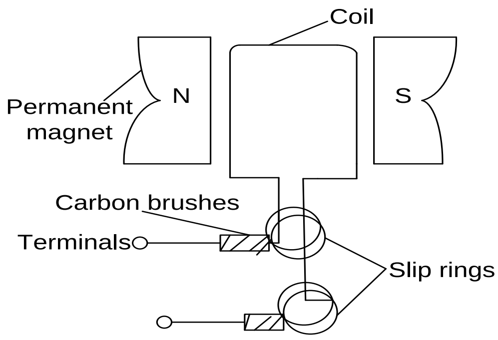
1. A ballistic galvanometer is connected to a flat coil having 50 turns of mean area $4.0cm^2$ to form a circuit of total resistance 100Ω. The coil is held between the poles of an electromagnet with its plane perpendicular to the field and is suddenly withdrawn form the field producing a throw of 60 scale divisions. Find the magnetic field density at the place where the coil was held assuming that the sensitivity of the galvanometer under the conditions of the experiment is $0.2div\ \mu C^{-1}$. **An(1.5T)**
2. A circular coil of 20 turns each of radius 10.0 cm lies flat on a table. The Earth's magnetic field intensity at the location of the coil is 43.8. Am^{-1} while the angle of dip is $67.0°$. Find the:
   (i) magnetic flux threading the coil
   (ii) torque on the coil if a current of 2.0 A is passed through it
   An $(3.18x10^{-5}$Wb, $2.70x10^{-5}$Nm)

3. A circular coil of 10 turns and diameter 12 cm carries current I. The coil is placed with its plane in the magnetic meridium. A small magnetic needle placed at the centre of the coil makes 30 oscillations per minute about a vertical axis. When the current is cut off, it makes 15 oscillations per minute. If the horizontal component of the earth's magnetic flux density is 2.0 x 10^5T , calculate the magnitude of I. (Assume that the square of frequency of oscillation is proportional to the magnetic flux density)

Generator / Dynamo

A dynamo changes mechanical energy to electrical energy

Alternating current $(A.C)$ generator / dynamo

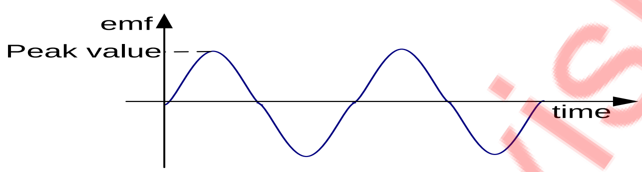


- The coil is rotated with a uniform angular velocity ω in the magnetic field.
- The magnetic flux threading the coil changes and an emf is induced across the terminals of the coil.
- $V = NAB\omega Sin\omega t$

Where B=magnetic flux density
A= area of the coil
N = number of turns of the coilNB:

During rotation emf increases to maximum when a coil is in horizontal position. Decreases and becomes zero when coil is in vertical position. It follows the same pattern but direction of emf is reversed.

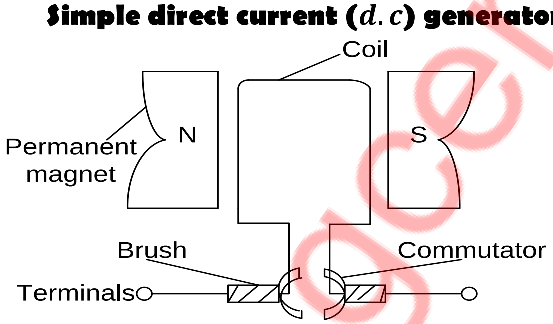
Variation of induced emf with time for A.C generator



Note;

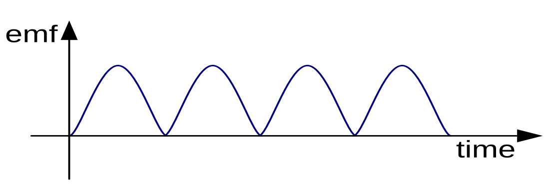
A.C generator can be changed to $d.c$ generator by replacing slip rings with split rings (commutators).

Simple direct current $(d.c)$ generator/ dynamo



- When the coil is rotated in a uniform magnetic field, it cuts the magnetic flux and an emf is induced in the coil.
- The emf is tapped off using carbon brushes pressed against slip rings
- However as the coil rotates, the commutators change contact from one brush to another although the current is received in the coil, the change over of brushes and commutators ensure that the direction of the current is maintained.

A graph of induced emf with time for $d.c$ generator



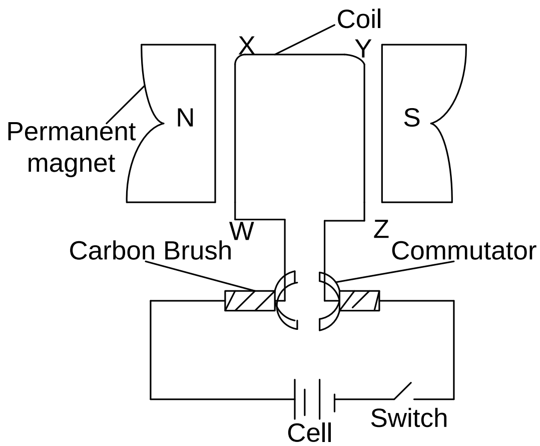
Note:
$d.C$ dynamo can be changed to $A.C$ by replacing commutators with slip rings

Factors which affect magnitude of the induced emf/current in conductor

1. Number of turns of the coil. Many turns give a large emf
2. Area of the conductors. Increasing the area of conductor increases the induced emf
3. Strength of the magnetic field. Using a stronger magnet increases the induced emf
4. Speed of rotation of the coil. At a high speed the emf is high.

Simple electric motor [uses L.H.R]

It is a device which changes electric energy to mechanical energy



- When the switched is closed, current flows in the coil in the direction WXYZ. The arm WX experiences downward force and the arm YZ experinces an upward force. The two forces thus form a couple, turning the coil in anticlockwise direction.
- When the coil is vertical, the commutators loose contact with the carbon brushes. The coil however is carried over by its momentum
- The two commutators change contact from one brush to another. The direction of the current in the coil thus reverses. WX now experiences an upward force and YZ experinces an a downward force.
- The directin of turning thus remains the same.

Note

Electric motor is improved by;
i) increasing current so as to increae the speed of rotation
ii) Increasing the number of turns in the coil
iii) Using a stronger magnet
iv) Winding the coil on a soft iron armature so as to intensify the magnetic flux through the coil.

Back emf in a motor

It is defined as the emf induced in a coil of a motor when the coil rotates in the magnetic field linking it.
Or it is the emf induced in the armature of the motor when the armature rotates in the magnetic field

Origin of back emf

When the armature coil in a motor rotates, it cuts the magnetic flux of the magnet and an emf, called back emf, $E_b$is induced in it which by Lenz's law opposes the applied p.d, V causing current I_a in the coil.

Significance of back emf

- Back e.mf provides the useful power of the motor
- It also limits the current flowing in the coil. Alarge current would burn the coil.

Relation of back emf and efficiency

If R_a is the coil/ armature resistance, then $\boxed{V = E_b + I_aR_a}$

Hence $VI_a = I_aE_b + I_a^2R_a$

Where VI_a is power supplied to the motor, $I_a^2R_a$ is power dissipated as heat in the armature coil, I_aE_b is mechanical power output power of the motor or the rate of working against the induced emf.

The efficiency of the motor $= \frac{mechnaicalpower\ developed}{power\ supplied} x100\%$

$efficiency\ = \frac{I_aE_b}{VI_a} x100\%$

$$\boxed{efficiency\ = \frac{E_b}{V} x100\%}$$

Example

1. A dc motor has an armature resistance of 1Ω and is connected to a 240 V supply. The armature current taken by the motor is 10A. Calculate
   (i) the back emf in the armature
   (ii) the mechanical power developed by the motor
   (iii) the efficiency of the motor

Solution

(i) $V = E_b + I_aR_a$

$240 = E_b + 1x10$

$E_b = 230V$

(ii) $\text{mechanical power } = E_bI_a$

$= 230x10 = 2300W$

(iii) $efficiency\ = \frac{E_b}{V} x100\%$

$efficiency\ = \frac{230}{240} x100\%$

$= 95.8\%$

2. A dc motor has an armature resistance of 0.5Ω and is connected to a 240V supply. The armature current taken by the coil is 30A. calculate:
   (i) the back emf generated by the motor
   (ii) the power supplied to the armature
   (iii) the mechanical power developed by the motor.
   (iv) the efficiency of the motor

Solution

(iv) $V = E_b + I_aR_a$

$240 = E_b + 30x0.5$

$E_b = 225V$

(v) $P = VI_a = 240x30 = 7200W$

(vi) $\text{mechanical power } = E_bI_a$

$= 225x30 = 6750W$

(vii) $efficiency\ = \frac{E_b}{V} x100\%$

$efficiency\ = \frac{225}{240} x100\%$

$= 93.8\%$

3. A d.c motor has armature resistance resistance of 0.5Ω and is connected to 220V supply. The armature current in the motor is 20A. calculate
   (i) the back emf generated by the motor
   (ii) the power supplied to the armature
   (iii) the mechanical power developed by the motor.
   (iv) the efficiency of the motor

Solution

(i) $V = E_b + I_aR_a$

$220 = E_b + 20x0.5$

$E_b = 210V$

(ii) $P = VI_a = 220x20 = 4400W$

(iii) mechanical power $= E_b I_a$
$= 210x20 = 4200W$

(iv) $efficiency = \frac{E_b}{V} x100\%$

$efficiency = \frac{210}{220} x100\%$
$= 95.45\%$

Relation of back emf and magnetic flux density

Consider a coil of N turns and area A of a motor rotating at ω radians per second in a radial magnetic field of flux density B.

Flux linking the coil, $\Phi = NABcos\omega t$

back $e.m.f$, $E_b = -\frac{d\Phi}{dt} = -\frac{d}{dt}(NABcos\omega t)$

$E_b = -(-NAB\omega sin\omega t)$

For a radial magnetic field $\omega t = 90°$

$E_b = NAB\omega sin90$

$\boxed{E_b = NAB\omega}$

Example

The coil of a dc motor is mounted in a radial magnetic field of flux density 1T. The coil has 20 turns each of area 40cm^2 and total resistance 2Ω. Calculate the maximum angular velocity of the motor when working on a 240V supply and drawing current of 1A.

Solution

$V = E_b + I_a R_a$
$240 = E_b + 1x2$
$E_b = 238V$

$E_b = NAB\omega$
$238 = 20x40x10^{-4}x1x\omega$
$\omega = 2975rads^{-1}$

Energy losses in a dc motor

- energy loss due to the resistance of the coil. This minimized using thick Copper wires.
- Energy losses due to eddy currents. Minimized by using laminated core.
- Hysteresis loss in the core, reduced by using soft iron core .

Explain the following observations

(ii) When a d.c motor is switched on, the initial current decreases to a steady value when the motor is running at a constant speed

(iii) If the motion of the d.c motor is slowed down, the current rises and then falls again when the motor is allowed to run freely.

Solution

(i) Initially back e.m.f is zero. When the motor is switched on , the back e.m.f increases due to increased rate of change of the magnetic field linking the motor coil and the current reduces. When the motor is running at a constant speed the back e.m.f is constant and current is steady

(ii) If the speed of the motor is reduced, back e.m.f reduces due to reduced rate of change of magnetic field linking the armature coil and current increase.
When the motor is running freely back e.m.f increases due to increased speed and hence increased rate of change of the magnetic field linking the armature coil and current reduces

Exercise

1. A dc motor has an armature resistance of 0.5Ω and is connected to a 200V supply. The armature current taken by the coil is 20A. calculate:
   (i) the back emf generated by the motor
   (ii) the power supplied to the armature

(iii) the mechanical power developed by the motor.
(iv) the efficiency of the motor
An(190V, 4000W, 3800W, 95%)

2. A motor which is 80% efficient is required to do mechanical work at a rate of 640W.
   (i) Find dthe current that the motor will take from a 200V d.c supply
   (ii) Calculate the back e.m.f in the armature of the motor in (i) above
   An(4A, 160V)

3 A motor whose armature resistance is 2Ω is operated on 240V mains supply. If the back e.m.f in the motor is 220V, calculate the armature current

4 A transformer is designed to work on a 240V, 60W supply. It has 3000 turns in the primary and 200 turns in the secondary and its efficiency is 80%. Calculate the current in the secondary coil

5 (a) A transformer connected to an ac supply of peak voltage 240V, is to supply a peak voltage of 9V to a mini lighting system of resistance 5Ω. Calculate
   (i) the ratio of the primary to the secondary turns
   (ii) the rms current supplied by the secondary
   (iii) the average power delivered to the lighting system

(b) (i) Explain why the voltage of the electricity generated at Owen Falls Dam has to be stepped up to about 132 kv for transmission places in the Western Region, and then stepped down for general use.
   (ii) Give any two power losses in a transformer and state how they are minimised.

6 An a.c transformer operates on 240 V mains,. It has 1200 turns in the primary and gives a voltage of 18 V across the secondary
   (i) Find the number of turns in the secondary
   (ii) If the efficiency of the transformer is 90% , calculate the current in the primary coil when a resistor of 50 Ω is connected across the secondary

Self-Induction

This s a process of generating an emf in a coil due to changing current in the same coil

The flux due to the current in the coil links that coil and if the current changes, the resulting flux change induces an emf in the same coil. This effect is called self-induction. The coil is said to have self-inductance, (L) and the coil is said to be an inductor. The induced emf tends to oppose the growth of current in the coil and its called back $e.m.f$

The induced e.m.f obeys the law of electromagnetic indution $E \propto \frac{dI}{dt}$

Hence induced emf, $E = -L\frac{dI}{dt}$

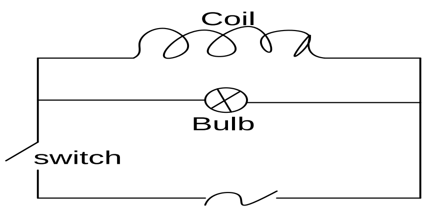
Where L is the inductance of the coil
hence $L = -\frac{E}{\frac{dI}{dt}}$
Self inductance, L, of a coil is the ratio of the induced emf to the rate at which the current in the coil changes,
Unit of self inductance is the Henry (H)

Definition

A henry (H) is the inductance of the a coil in which an e.m.f of 1V is induced when the current changes at a rate of $1\ As^{-1}$

Assignment: Prove that the energy stored in an inductor is given by $Energy = \frac{1}{1}LI^2$

Demonstration of self induction



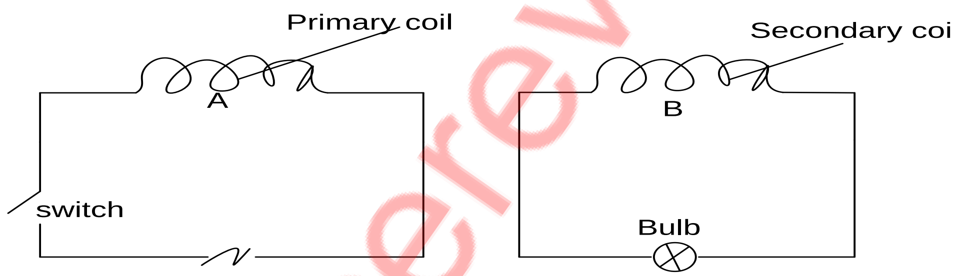
- A coil of many turns of wire is connected in parallel with an electric bulb an a.c supply as shown below

- When switch is closed, the bulb will light brightly and then it will go dim after some time.
- When the switch is opened, the bulb keeps the light for some time before going off. This is because as current decays from the coil. The back e.m.f is induced in it to oppose the decay. This e.m.f creates current which lights the bulb for some time

Mutual Inductance

This is the process of generating an emf in a coil due to changing current in the neighbouring coil.
If two coils i.e. the primary and the secondary coils, are near each other, and the current in the primary coil is changed, an emf is induced in the secondary coil. This process is called *mutual induction.*

Demonstration of mutual induction



- Two coils are placed close to each other. When switch is closed, the bulb lights momentarily.

  Hence e.m.f is induced in B. Since the circuit is closed current flows and the bulb lights

Mutual inductance, M between two coils is the ratio of the induced emf in the secondary coil to the rate of change of current in the primary coil.

$M = -\frac{E_S}{\frac{dI_P}{dt}}$, E_S is induced emf in the secondary coil, $\frac{dI_P}{dt}$ is rate of current change in primary coil.

Alternatively, $E_S = -\frac{d\Phi_S}{dt}$, where Φ_S is magnetic flux linkage in the secondary coil.

$E_S = -M\frac{dI_S}{dt}$

hence $-M\frac{dI_S}{dt} = -\frac{d\Phi_S}{dt}$

$$M = \frac{d\Phi_S}{dI_S} = \frac{\Phi_S}{I_S}$$

hence mutual inductance is the magnetic flux linkage in the secondary coil when the primary current is 1A.
Unit of mutual inductance is the Henry(H).

Calibration of a Ballistic galvanometer using a standard mutual inductance

A known mutual inductance can be used to calibrate a ballistic galvanometer so that its sensitivity C can be known.
The Ballistic galvanometer is connected to a secondary with a mutual inductance and its deflection θ is noted when a known current I_P as measured by the ammeter is reversed in a primary coil or switched off.

Case I [when current is switched off]

$\Phi_0 = MI_P$ *and* $\Phi_t = 0$

hence induced charge, $Q = \frac{\Phi_0 - \Phi_t}{R} = \frac{MI_P}{R}$

But from the theory of Ballistic galvanometer, $Q = c\theta$

Hence $\frac{MI_P}{R} = c\theta$

Hence $c = \frac{MI_P}{R\theta}$

Case II [when I_P is reversed]

$\Phi_0 = MI_P$ *and* $\Phi_t = -MI_P$

If the deflection of the B.G is θ_1, then induced charge $Q = \frac{\Phi_0 - \Phi_t}{R} = \frac{MI_P - -MI_P}{R} = \frac{2MI_P}{R}$

Hence $\frac{2MI_P}{R} = c\theta_1$

Hence $c = \frac{2MI_P}{R\theta_1}$

Examples

1. For calibration purposes, a B.G is connected to the secondary of mutual inductance $1.2x10^{-5}$H. The total resistance of the secondary circuit being 1000Ω. When a current of 2A is reversed in the primary, the B.G deflects through 30 divisions. What is the sensitivity in $Cdiv^{-1}$.

Solution

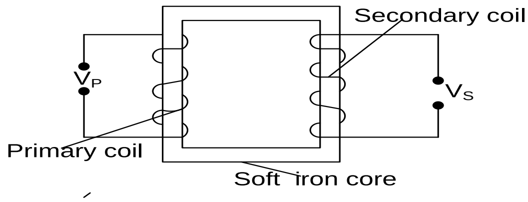
$c = \frac{2MI_P}{R\theta_1}$

$$c = \frac{2x1.2x10^{-5}x2}{1000x30}$$
$$c = 1.6x10^{-9}Cdiv^{-1}$$

2. A solenoid 50cm long and of mean diameter 3cm is uniformly wound with 2000 turns. A secondary coil of 800 turns is wound closely round the middle of the solenoid and is connected to a B.G. the total resistance of the secondary circuit is 400Ω. when a current of 4A in the solenoid is switched of, the deflection on the B.G is 12 divisions. Calculate the sensitivity of B.G in $Cdiv^{-1}$
3. A secondary coil of 500 turns is wrapped closely round the middle of a long solenoid which has 2000 turns per metre and area of cross section 9cm^2. the resistance of the secondary coil is 75Ω and is connected to a B.G of resistance 125Ω. When a current of $2.5A$ in the solenoid is reversed, a deflection corresponding to $28.3\mu C$ is registered. Calculate the permeability of air.

Transformer

This is a device that steps up and down voltage.
A transformer that steps up voltage is called step up transformer and the one that steps down voltage is called step down transformer.
A step up transformer has more turns on the secondary coil while step down transformer has more turns on the primary.



- Alternating voltage connected to the primary coil produces an alternating current in it. This sets up an alternating magnetic flux in the core which links up the secondary coils and thus induces an _ in the coils. The magnitude of the e.m.f induced in the secondary coil is $V_S = N_S A \frac{dB}{dt}$
- The changing magnetic flux also links the primary and induces a back e.m.f in the same coil whose $V_P = N_P A \frac{dB}{dt}$

Thus $\frac{V_S}{V_P} = \frac{N_S}{N_P}$. When $N_S > N_P$ and $V_S > V_P$ then it's a step up transformer,
When $N_S < N_P$ and $V_S < V_P$ then it's a step down transformer,

NOTE :
When the secondary coil is connected to the load, the current in the primary coil increases. This is because current flows in the secondary which induces a magnetic flux in the core in opposition to magnetic flux due to primary current, magnetic flux in the core thus reduces. This leads to reduction in back e.m.f in the primary and hence the primary current increases

Power losses/ energy losses in a transformer

1. Some of the energy is dissipated as heat due to the resistance of the coil (joule-ohmic energy loss) This loss is minimized by using thick copper wires of low resistance.
2. Eddy currents: eddy currents circulating in the core dissipate heat by I^2R mechanism This is minimized by using a laminated core made of thin strips or laminars separated from each other by a layer of insulating varnish.
3. Hysteresis loss: energy used to turn the magnetic domains of the core is dissipated as heat It can be minimized by using a soft magnetic material such as soft iron, mumental , perm alloy
4. some loss of energy occurs because a small amount of flux associated with the primary coil may not pass through the secondary coil.

This can be minimized by winding one coil on top of the other.

Example

1. An $a.c$ transformer is used to provide a voltage of 3000V for operating a T.V tube. If the transformer has 500 turns on primary and is connected to 240V mains supply. How many turns are in the secondary coil.

Solution

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$
$$N_S = \frac{V_S}{V_P} N_P$$

$$N_S = \frac{3000x\ 500}{240}$$
$$N_S = 6250\ turns$$

2. A transformer has 200 turns of the primary coil. Calculate the number of turns on the secondary coil if 240V is to be stepped up to 415V

Solution

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$
$$N_S = \frac{V_S}{V_P} N_P$$
$$N_S = \frac{415x\ 200}{240}$$
$$N_S = 345.8\ turns$$

Note:

Although there are a lot of energy losses in the transformer, the energy losses are so small such that the power, put into primary coil is equal to power got out of secondary coil for a transformer that is 100% efficient

$$power\ put\ into\ primary\ coil = power\ out\ of\ secondary\ coil$$
$$I_P\ \ V_P = I_S\ \ V_S$$
$$\boxed{\frac{V_S}{V_P} = \frac{I_P}{I_S}}$$

But

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$
$$\boxed{\frac{I_P}{I_S} = \frac{N_S}{N_P}}$$

1. A transfer steps up its $p.d$ from 12V to 48V. If the current is flowing in the primary coil is 2A. What is the current in the secondary circuit.

Solution

$$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$
$$I_S = \frac{V_P}{V_S} I_P$$
$$I_S = \frac{12x2}{48}$$
$$I_S = 0.5A$$

2. A transformer designed to operate a 12V lamp from 240V supply has 1200 turns on the primary coil. Calculate.
   i) Number of turns on the secondary coil.
   ii) Current passing through the primary coil when the 12V lamp has a current of 2A flowing through it.

Solution

$$N_p = 1200,\ N_S = ?$$
$$V_P = 240V, V_S = 12V$$
$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$
$$N_S = \frac{V_S}{V_P} N_P$$
$$N_S = \frac{12x\ 1200}{240}$$
$$N_S = 60\ turns$$

$$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$
$$I_P = \frac{V_S}{V_P} I_S$$
$$I_P = \frac{12x2}{240}$$
$$I_P = 0.1A$$

Efficiency of a transformer

$$Efficiency = \frac{Power\ out\ put}{Power\ input} x\ 100\%$$
$$power\ output = power\ on\ secondary\ coil = I_S\ \ V_S$$

$$power\ input = power\ on\ primary\ coil = I_P\ \ V_P$$

$$\eta = \frac{I_S\ \ V_S}{I_P\ \ V_P} x100\%$$

Examples

1. A transformer is used on the 240V supply to deliver 9A at 80C to a heating coil. If 10% of the energy taken from the supply s dissipated in the transformer it self. What is the current in the primary winding

SolutionSince 10% is dissipated,

$\eta = (100 - 10) = 90\%$

$$Efficiency = \frac{Power\ out\ put}{Power\ input} x\ 100\%$$

$$\eta = \frac{I_S\ \ V_S}{I_P\ \ V_P} x100\%$$

$$90\% = \frac{8\ x\ 9}{240\ x\ I_P} x100\%$$

$$I_P = \frac{8x9x100}{240x90}$$

$I_P = 3.33A$

2. A transformer is designed to operate at 240V main supply and deliver 9V. The current drawn from the main supply is 1A if the efficiency of the transformer is 90%. Calculate
   (i) maximum power output
   (ii) power lost

Solution

$\eta = 90\%, I_P = 1A,$

$V_P = 240V,\ V_S = 9V$

$$Efficiency = \frac{Power\ out\ put}{Power\ input} x\ 100\%$$

$$90\% = \frac{Power\ out\ put}{I_P\ \ V_P} x100\%$$

$$90\% = \frac{Power\ out\ put}{240x1} x100\%$$

$$Power\ out\ put = \frac{90x\ 240x1}{100}$$

$Power\ out\ put = 216W$

Power lost $= P_{In} - P_{out}$

$= I_P\ \ V_P - 216$

$= (240x1) - 216$

Power lost $= 24\ W$

3. An electric power generator produces 24kW at 240V, the voltage is stepped up to 400V for transmission to a factory where it is stepped down to 240V. The total resistance of the transmission wire is 0.5Ω.
   (i) What is the ratio of number of turns in primary to number of turns in secondary of the step down transformer.
   (ii) Find the power loss in transmission lines assuming both transformers are 100% efficient.
   (iii) What power would have been lost if same had been transmitted directly without transformers.

Solution

$V_S = 240\ V,\ V_P = 4000\ V$

i) $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$$\frac{240}{4000} = \frac{N_S}{N_P}$$

$$\frac{3}{50} = \frac{N_S}{N_P}$$

$N_P : N_S = 50 : 3$

(iii) power loss = I^2R

but $I = \frac{P}{V}$

$$I = \frac{24\ x\ 10^3}{4000}$$

$I = 6A$

power loss = I^2R

power loss = $6^2 x\ 0.5$

$= 1.8W$

(iv) power loss = I^2R

$$I = \frac{P}{V}$$

$$I = \frac{24\ x\ 10^3}{240}$$

$I = 100A$

power loss = I^2R

power loss = $100^2 x\ 0.5$

$= 5000W$

4. A setup transformer is designed to operate from a 240V supply with delivery energy at 250V. If the transformer is 90% efficient, determine the current into the primary winding when the output terminals are connected to 250V, 100W lamp.

Solution

$V_S = 250\ V,\ \ V_P = 20\ V,$

$\eta = 90\%,\ P_{out} = 100W$

$$Efficiency = \frac{Power\ out\ put}{Power\ input} x\ 100\%$$

$$90\% = \frac{100}{P_{In}} x 100\%$$

$P_{In} = 111.11W$

$P_{In} = I_P\ \ V_P$

$$I_P = \frac{111.11}{20}$$

$I_P = 5.56A$

5. A generator with a power out put of $20kW$ at 4kV distributes power to a workshop along cables having a total resistance of 16Ω. Calculate
   (i) the current in the cables
   (ii) the power loss in the cables
   (iii) the potential drop between the ends of the cables

Solution $P_{out} = 20kW, V_S = 4kW$

$P = I\ V$

$$I = \frac{20\ x\ 10^3}{4000}$$

$I = 5A$

ii) Power loss = $I^2 R$

Power loss = $5^2 x\ 10$

$= 400W$

iii) potential drop = $I\ R$

$= 5x16$

$= 80V$

6. A transformer steps up 200V, it has 10 windings in the primary and 100 windings in the secondary. If the current in the primary winding is 1.2A. What is the current in the secondary given that the efficiency is 80%

Solution

$V_S = 200\ V,\ \ V_P =?,$

$N_s = 100,\ N_P = 10$

$I_p = 1.2A, I_s =?$

$\eta = 80\%,$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$V_P = \frac{200x10}{100}$$

$V_P = 20V$

$$Efficiency = \frac{Power\ out\ put}{Power\ input} x\ 100\%$$

$$80\% = \frac{I_S V_S}{I_P V_P} x 100\%$$

$$I_S = \frac{80\ x\ I_P V_P}{100 x V_S}$$

$$I_S = \frac{80\ x\ 1.2x20}{200x100}$$

$I_S = 0.096A$

7. A transformer is designed to private an output of 220V when connected to a 25V supply. If the transformer is 80% efficient. Calculate the input current when the output is connected to a 220V,75W lamp.

Solution

$V_S = 220\ V,\ \ V_P = 25V,$

$\eta = 80\%, P_{out} = 75W$

$$Efficiency = \frac{Power\ out\ put}{Power\ input} x\ 100\%$$

$$80\% = \frac{P_{out}}{I_P V_P} x 100\%$$

$$I_P = \frac{75}{80x25} x 100$$

$I_P = 3.75A$

Exercise

1. An $a.c$ transformer operates on a 240V mains. The voltage across the secondary which has 960 turns s 20V.
   ii) find the number of turns in the primary
   iii) if the efficiency of the transformer is 80% calculate the in the primary coil when a resistor of 40Ω is connected across the secondary. **[11520turns, 0.0521A]**

2. A transformer whose secondary col has 60 turns and primary 1200 turns has its secondary connected to a 3Ω resistor if its primary is connected to a 240V $a.c$ supply. Calculate the current flowing in the primary assuming that the transformer is 80% efficient. **[0.25A]**
3. A transformer is designed to work on a 240V, 60W supply, it has 3000 turns in the primary and 200 turns in the secondary and its efficiency is 80%. Calculate the current in the secondary coil. **[3A]**
4. An a.c transformer operates on 240V mains. It has 1200 turns in the primary and gives 18V across the secondary.
   i) find the number of turns in the secondary
   ii) if the efficiency of the transformer is 90% calculate the current in the primary coil when a resistor of 50Ω is connected across the secondary
   . **[90turns, 0.03A]**

5. An a.c. transformer operates on a 240 V mains. The voltage across the secondary which has 960 turns is 20V.
   (i) find the number of turns in the primary
   (ii) If the efficiency of the transformer is 80%, calculate the current in the primary coil when a resistor of 40Ω is connected across the secondary.
6. An a.c transformer operates on 240 V mains,. It has 1200 turns in the primary and gives a voltage of 18 V across the secondary
   (iii) Find the number of turns in the secondary
   (iv) If the efficiency of the transformer is 90% , calculate the current in the primary coil when a resistor of 50 Ωis connected across the secondary

Transmission of electrical power

When power is transmitted, alternating current is used ($a.c$).
However power is lost due to heating of the cables during transmission as a result of resistance of the wires.
Power is transmitted at high voltage as this reduces the energy loss. It is transmitted using high voltage and low current because the heating effect depends on the square of current and the loss as heat is reduced by using a high voltage.
power carried by cable $= IV$
power lost in heating cables= I^2R

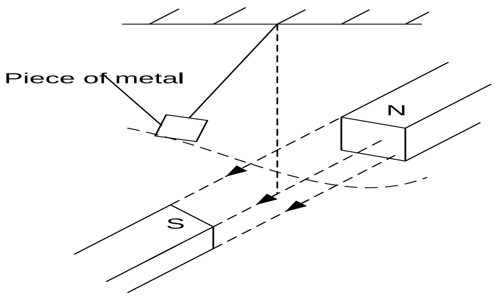
Note
Electric cables are always thin so that low current and high voltage is transmitted with low power loss and the cost of supporting the cable is also reduced.

Advantages of $a.c$ over $d.c$ power transmission
- Alternating current can be stepped up to high voltage and transmitted over long distances with minimum power loss
- It is very easy to generate.

EDDY CURRENTS

Eddy currents are currents which circulate in a piece of metal when the magnetic flux linking the metal changes.
Eddy currents can be minimized by laminating the coils
Uses of eddy currents
(i) Used in damping of moving coil galvanometers
(ii) They can be used to detect cracks in metals
(iii) They can act as electromagnetic brakes in electric trains
(iv) Used in sorting metallic objects from solid waste
Disadvantages
(i) Can lead to heat power loss in a transformer
(ii) Produce heat

Experiment to demonstrate eddy currents



- A metal plate is suspended in a magnetic field so that its plane is perpendicular to the the magnetic field.
- The plate is then set to oscillate, such that its plane cuts the magnetic field and the time taken for the oscillation to die down noted.
- The experiment is repeated with an identical metal plate which has vertical slots cut through.
- It is noted that the plate with slots osscillates for a long period of time.

Note

(i) If the weaker magnets are used the metal takes long to settle

(ii) Oscillation of the metal plate dies down because, when the beam moves between the magnetic poles, it cuts the magnetic field. Eddy currents are induced in the beam whose magnetic field opposes its movement and hence damping the motion

Uneb 2016

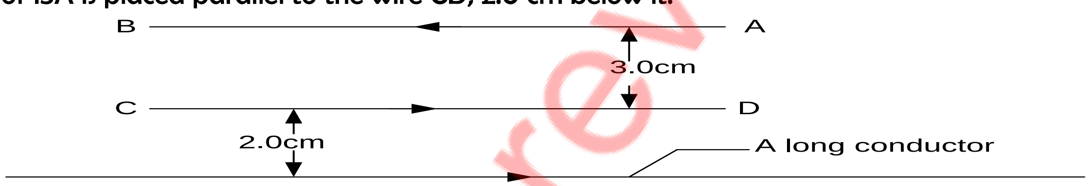
(a) (i) What is the difference between **a motor** and **a dynamo**. (01mark)

(ii) Describe with the aid of a labelled diagram the structure and mode of operation of a d.c generator. (06marks)

(iii) Describe briefly the factors that determine the peak value of the induced e.m.f. (03marks)

(iv) How can d.c generator be converted into an a.c generator. (01mark)

(b) Figure below shows two wires AB and CD of length 5.0cm each carrying a current of 10.0A in the direction shown. A long conductor carrying a current of 10.0A in the direction shown. Along conductor carrying a current of 15A is placed parallel to the wire CD, 2.0 cm below it.



(i) Calculate the net force on the long wire (06marks)

(ii) Sketch the magnetic field pattern between the long wire and the wire CD after removing wire AB. Use the field pattern to define a neutral point. (03marks)

Uneb 2016

(a) What is meant by the following as applie to the eathrs magnetic field?

(i) Magnetic meridian (01mark)

(ii) Magnetic variation (01mark)

(b) Describe the structure and mode of action of the deflection magnetometer. (06marks)

(c) A circular coil of 4 turns and diameter $11cm$ has its plane vertical and parallel to the magnetic meridian of the earth. Determine the resultant magnetic flux density at the centre of the coil when a current of $0.35A$ flows in it. (take the horizontal component of the earth's magnetic flux densitu to be $1.6x10^{-5}\ T$). (04marks)

(d) (i) Define **self-induction** and **mutual induction**. (02marks)

(ii) Give the causes of power loss in an **a.c** transformer and state how each can be minimized. (04marks)

(iii) Explain why the current in the primary coil of a transformer increases when the secondary is connected to a load. (02marks)

Uneb 2015

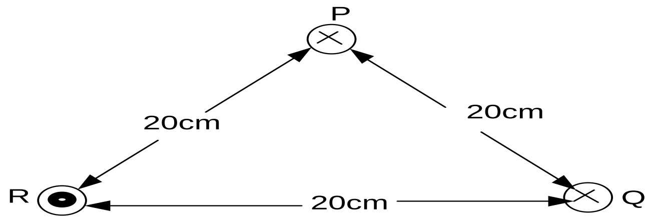
(a) What is meant by the following as applie to the eathrs magnetic field?

(i) Magnetic meridian (01mark)

(ii) Angle of dip (01mark)

(b) (i) Define the **ampere** (01mark)

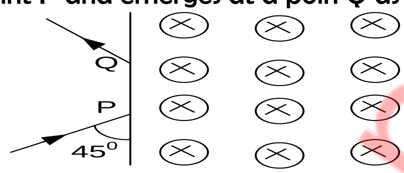
(ii) Three conductors P, Q and R carrying current 3A. 6A and 8A respectively are arranged as shown belwo



Calculate the force experienced by conductor P. (06marks)

(c) (i) Define **magnetic flux** and **magnetic flux density**. (02marks)

(ii) A charged particle of mass $1.4x10^{-27}\ kg$ and charge $1.6x10^{-19}\ C$ enters a region of uniform magnetic field of flux density 0.2T at a point P and emerges at a poin Q as shiwn in the figure below.



If the speed of the particle is $10^7 ms^{-1}$, calculate the distance PQ (04marks)

(d) Describe an experiment to measure the magnetic flux density between the pole pieces of a storng magnet (05marks)

Uneb 2015

(a) (i) State the **laws of electromagnetic induction**. (02marks)

(ii) Describe with the aid of a labelled diagram, an experiment to verify Faraday's law of electromagnetic induction. (05marks)

(b) Explain

(i) Why when a plate of copper is pushed into a storng magnetic field between the poles of a powerful electromagnet, considerable resistance to the motion is felt, but no scuh effect is felt with a sheet of glass. (04marks)

(ii) How damping is achieved in a moving coil galvanometer. (03marks)

(c) An aeroplane of wing-span 30m flies horizontally at a speed of $100kmh^{-1}$. What is the p.d across the tips of its wings, if the horizontal component of the earht's field is $1.46x10^{-4}T$. (Angle of dip at the place is 70°). (03marks)

(d) A coil of 500 turns and area $80cm^2$ is rotated 1200 revolutions per minute aabout an axis perpendicular to its plane and magnetic field of flux density $0.25T$. calculate the maximum e.m.f in the coil. (03marks)

Uneb 2013

(a) Define the following terms

(i) Magnetic flux density (01mark)

(ii) Magnetic flux linkage (01mark)

(b) (i) A rectangular coil of N turns, length l, and width, b, carrying a current , I is placed with its plane making an angle, θ, to a unifprm magnetic field of flux density, B. Derive the expression for the torque exerted on the coil. (05marks)

(ii) A current of 3.25A flows through a long solenoid of 400 turns and length 40.0cm. Determine the magnitude of thee force exerted on a particle of charge 15.0μC moving at $1.0x10^3 ms^{-1}$ through the centre of the solenoid at an angle of 11.5° relative to the axis of the solenoid. (04marks)

(c) Describe with the aid of a diagram, an absolute method of measuring current. (06marks)

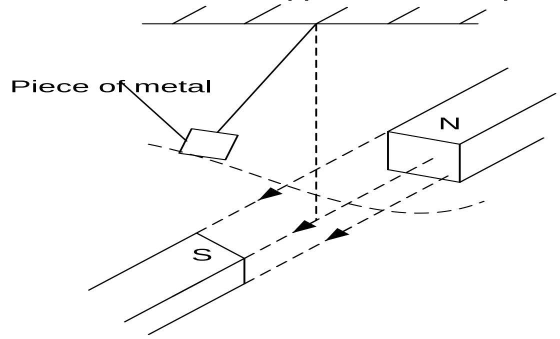
(d) Explain why a current carrying conductor placed in a magnetic filed experiences a force magnet (05marks)

Uneb 2013

(a) (i) State **Lenz's law** of **electromagnetic induction**. (01mark)

(ii) Describe with the aid of a labelled diagram, an experiment to verify Faraday's law of electromagnetic induction. (06marks)

(b) Figure below shows a piece of a metal swinging between opposite magnetic poles



Explain what will be observed after some period of time. (04marks)

(c) (i) Define **self-induction** and **mutual induction**. (02marks)

(ii) Describe an experiment which can be used to demonstrate self-induction. (03marks)

(d) A search coil has 40 turns of a wire and cross-sectional area 5cm^2. The coil is connected to a ballistic galvanometer and then placed with its plane perpendicular to a uniform magnetic field of flux density B. When the search coil is suddenly withdrawn from the field, the galvanometer gives a deflection of 240 divisions. When a capacitor of capacitance $4\mu F$ charged to a $p.d$ of 20V is discharged through the circuit, the galvanometer deflection is 180 divisions. What is the magnetic flux density between the magnetic poles. If the total resistance of the circuit 20Ω (04marks)

Uneb 2012

(a) Define the following terms

(i) Weber (01mark)

(ii) Ampere (01mark)

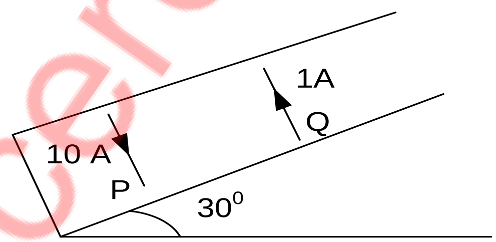
(b) A circular coil of N turns each of radius R carries a current , I

(i) Write an expression for the magnetic flux density at the centre of the coil. (05marks)

(ii) Sketch the magnetic fields pattern associated with the coil. (04marks)

(c) Describe how a deflection magnetometer can be used to investigate the variation f magnetic flux density at the centre of a circular coil with the current flowing through the coil. (06marks)

(d) Two paralle wire P and Q each of length 0.2m carrying currents of 10A and 1A respectively in opposite directions as shown below



The distance between the wire is 0.04m. If both wire remain statinaory and the angle of the planne with the horizontal is 30°, calculate the weight of Q (05marks)

(e) (i) State why the damping in the B.G should be as small as possible. (01mark)

(ii) Describe how damping can be reduced in practice. (06marks)

Uneb 2012

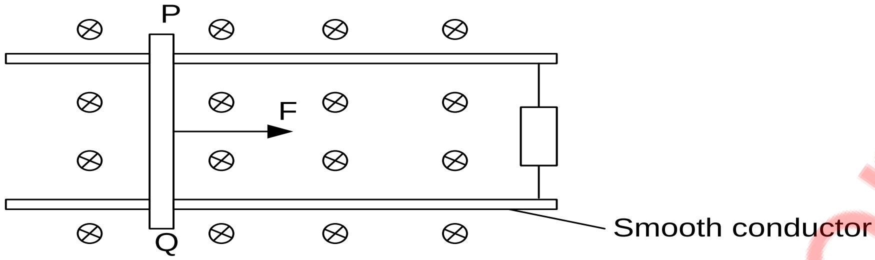
(a) (i) Define **self-induction** and **mutual induction**. (02marks)

(ii) State **Faraday's law** of **electromagnetic induction**. (01mark)

(b) (i) Describe the structure and action f an a.c transformer (06marks)

(ii) Explain why voltage at a generating power station must be stepped up to a very high value for long distance transmission (03marks)

(c) The figure below shows connecting rod PQ of length 20mm rests on a smooth conducting frame to form a complete circuit of resistance 4.0Ω. When a force, F, is applied, the rod moves at a constant velocity of $6.0ms^{-1}$ perpendicular to a uniform magnetic field of flux density 1.5T



(i) Explain why the rod PQ moves with a constant velocity (03marks)
(ii) Calculate the magnitude of the induced e.m.f (02marks)
(iii) Calculate the magnitude of the force F (03marks)

Uneb 2011

(a) (i) Draw a well labelled diagram to show a repulsive type moving iron ammeter. (02marks)
(ii) Explain how the ammeter in (a)(i) above is able to measure alternating current. (05mark)

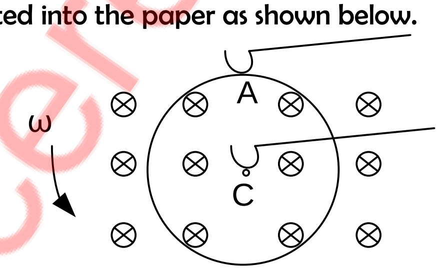
(b) (i) Write down an expression for the magnetic flux density at the centre of a flat circular coil of, N, turns each of radius, a, carrying a current, I. (01mark)
(ii) Describe how you would determine the value of thr earth's magnetic flux density at a place, using a search coil. (06marks)

(c) A circular coil of 50 turns and mean radius 4cm is placed with its plane in the magnetic meridian. A small compasss needle is placed at the centre of the coil. When a current of 0.1A is passed through the coil, the compass needle deflects through $40°$. When the current is reversed, the compass needle deflects through $43°$. Calculate the;
(i) horizontal component of the earth's magnetic field density (04marks)
(ii) Magnetic flux density of the earth's field at the place given that the angle of dip at the place is $15°$. (02marks)

Uneb 2011

(a) State the laws of **electromagnetic induction**. (02marks)

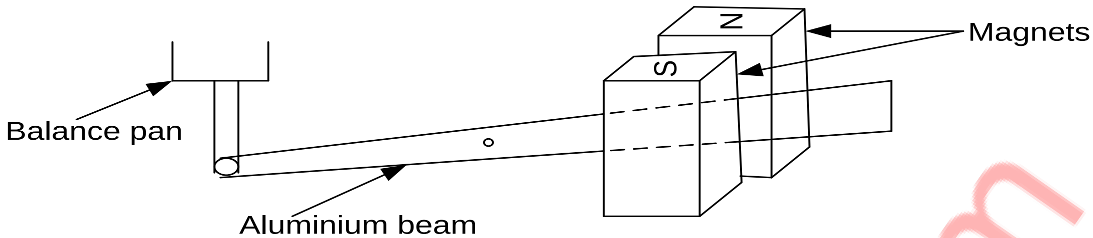
(b) (i) A circular metal disc of radius, R, rotates in anticlockwise direction at an agular velocity, ω, in a uniform magnetic field of flux density, B, directed into the paper as shown below.



Derive an expression for the e.m.f induced between A and C. (03marks)
(ii) A copper disc of radius 10cm is placed ina uniform magnetic field of flux density, 0.02T, with its plane perpendicular to the field. If the disc is rotated parallel to the field about an axis through its centre at 3000 revolutions per minute. Calculate the e.m.f that is generated between its rim and the centre. (03marks)

(c) Describe an experiment to demonstrate mutual induction (04marks)

(d) The diagram below shows the arrangement by which a laboratory balance is critically damped. The aluminium beam supporting the pan moves in the magnetic field of two powerful magnets.



(i) Exaplain how damping is caused (03marks)

(ii) What change would occur in the performance of the balance if the magnets were replaced by weaker ones? (01mark)

(e) (i) Define the ampere. (01mark)

(ii) Two parallel wires, P and Q of equal length 0.1m, each carrying a current of 10.0A are a distance 0.05m apart, with P carrying a current of 10A are a distance 0.05m apart, with P directly above Q. if P remains stationary, find the weight of P. (03marks)

Uneb 2010

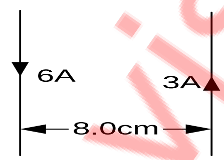
(a) Define the term magnetic flux density (01mark)

(b) Write the expression for the;

(i) Magnetic flux density at a perpendicular distance R from a long straight wire carrying current, I, in air. (01mark)

(ii) Force on a straight conductor of length, l (meter) carrying current, I(amperes) at an angle, θ to a uniform magnetic field of flux density, B (tesla) (01mark)

(c) Two straight long and parallel wire of negligible cross-sectioanl area carrying currents of 6.0A and 3.0A in opposite directions as shown below



If the wire are separated by a distance of 8.0cm, find the;

(i) Magnetic flux density at a point mid-way between the wires. (04marks)

(ii) Force per metre between the wire. (03marks)

(d) Define;

(i) Angle of dip (01mark)

(ii) Angle of declination (01mark)

(e) A straight conductor of length, l is perpendicular to a uniform ,agnetic field of flux density B. if the conductor moves with a velcotiy, U at an angle θ to the magnetic field, derive the expression for the e.m.f induced. (04marks)

(f) An air craft with the wing span of 20m moving horizontally from west to east at a velocity of $250ms^{-1}$ in a place where the angle of dip is 40°. The e.m.f induced across the tips of the wings is 6mV. Find the magnetic flux density of the earth field. (04marks)

Uneb 2010

(a) State the **laws** of **electromagnetic induction**. (02marks)

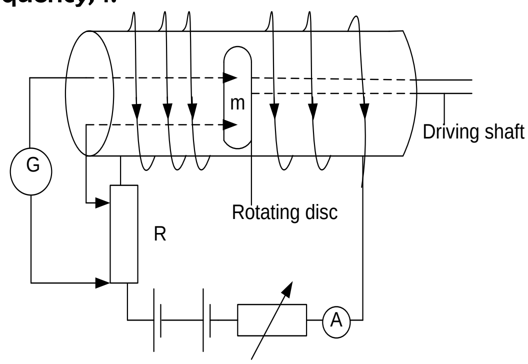
(b) The e.m.f generated in a coil rotating in a uniform magnetic field is given by $E_0 \sin \omega t$

(i) State the meaning of the symbols used and give their units (03marks)

(ii) Draw a diagram showing the relative position of the coil and the magnetic field when it is zero and when the e.m.f generated is E_0 (03marks)

(c) A rectangukar coil of 50 turns is 15.0cm wide and 30.0cm long. If it is rotated at a uniform rate of 2000 revolutions per minute about an axis parallel to its long side and at right angles to a uniform magnetic field of flux density 0.04T, find the peak value of the e.m.f induced in the coil. (03marks)

(d) A solenoid of, n, turns per metre, a resistor, R, an ammeter, A, and a rheostat are connected to the battery as shown below. A metal disc of radius, r, is mounted inside the solenoid with its axis coincident with that of the solenoid. The centre and rim of the disc are connected across, R. The disc is rotated with its plane perpendicular to the axis of the solenoid at a frequency, f.



The rheostat is adjusted until the galvanometer shows no deflection, and ammeter reads a current of I amperes

(i) Show that the e.m.f induced between the centre and the rim of the disc is $\pi B r^2 f$, where B is magnetis flux density inside the solenoid. (04marks)

(ii) Deduce an expression for the resistance of R in terms of n, f ,I and r (03marks)

(iii) State the two limitations of the set up in measurements of resistance. (02marks)

Uneb 2009

(a) Define the terms magnetic flux and magnetic flux density (02marks)

(b) A straight wire of length 20cm and resistacne 0.25Ω lies at right angles to a magnetic field of flux density 0.4T. The wire moves when a p.d of 2.0V is applied across its ends. Calculate the,

(i) Intial force on the wire. (02marks)

(ii) Force on the wire when it moves at a speed of 15ms^{-1} (02marks)

(iii) Maximum speed attained by the wire. (02marks)

(c) (i) Sketch the magnetic field patterns around a vertical straight wire carrying a current in the earth's magnetic field and use it to explain a neutral point in a magnetic field. (03marks)

(ii) Two long parallel wire palced 12cm apart in air carry currents of 10A and 15A respectively in the same direction. Determine the position where the magnetic flux density is zero. (04marks)

(d) Describe with the aid of a diagram an absolute method of determing resistance. (05mark)

Uneb 2009

(a) Derive an expression for the charge, Q induced in a coil of N turns when the magnetic flux though it changes. (04marks)

(b) (i) Describe how a ballistic galvanometer of an unknown charge sensitivity can be used to measure magnetic flux density in a region between the poles of a magnet. (05marks)

(ii) State the possible sources of errors in the above experiment (02marks)

(c) A flat circular coil with 2000 turns, each of radius 50cm, is rotated at a uniform rate of 600 revolutions per minuted about its diameter at right angles to a uinform magnetic flux density $5x10^{-4}T$. Calculate the amplitude of the induced e.m.f. (03marks)

(d) Describe with the aid of a diagram the structure and action of a hot wire ammeter. (06mark)

Uneb 2008

(a) Define the following terms

(i) tesla (01mark)

(ii) magnetic flux (01mark)

(b) Two infinitely long straight wires carrying currents, I_1 and I_2 respectively are placed parallel to each other I a vacuum at a distane, d meters apart. Derive an expression for the force per metre between the wires. (05mks)

(c) (i) Sketch the magnetic fields pattern due to a current flowing in a circular coil. (02marks)
(ii) Write an expression for magnetic flux density, B at the center of a circular coil of N turns each of radius r and carrying a current I. (01mark)
(iii) A wire of length 7.85m is wound into a circular coil of radius 0.05m. if a current of 2 A passes through the coil, find the magnetic flux density at the centre of the coil. (04marks)

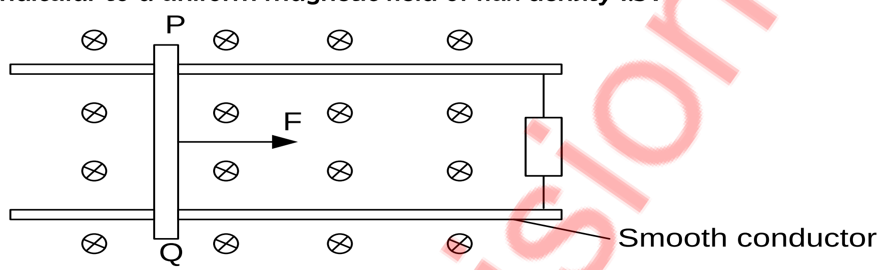
(d) (i) Explain the term **back e.m.f** in a d.c motor (02marks)
(ii) Show how the **back e.m.f** in a motor is related to the efficiency of the motor. (04marks)

Uneb 2008

(d) (i) Define **self-induction** and **mutual induction**. (02marks)
(ii) State **Faraday's law** of **electromagnetic induction**. (01mark)

(e) (i) Describe the structure and action f an a.c transformer (06marks)
(ii) Explain why voltage at a generating power station must be stepped up to a very high value for long distance transmission (03marks)

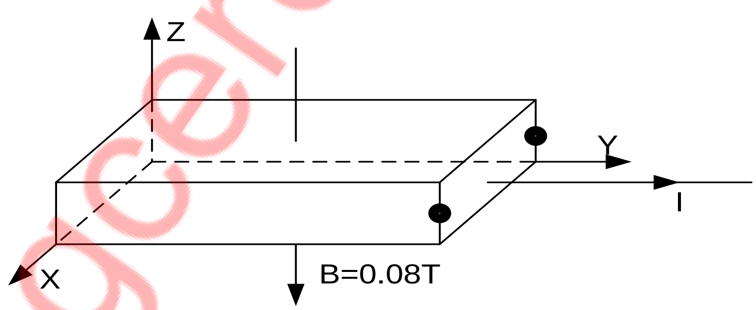
(f) The figure below shows connecting rod PQ of length 20mm rests on a smooth conducting frame to form a complete circuit of resistance 4.0Ω. When a force, F, is applied, the rod moves at a constant velocity of $6.0ms^{-1}$ perpendicular to a uniform magnetic field of flux density 1.5T



(iv) Explain why the rod PQ moves with a constant velocity (03marks)
(v) Calculate the magnitude of the induced e.m.f (02marks)
(vi) Calculate the magnitude of the force F (03marks)

Uneb 2007

(a) What is a magnetic field (01mark)
(b)



A magnetic field of flux density 0.08T is applied normally to a metal strip carrying current, I as shown above
(i) Account for the occurrence of potential difference (p.d) between points P and Q (03marks)
(ii) Calculate the electric field intensity between P and Q if the drift velocity if the conduction electrons is $4.0x10^{-4}m/s$. (03marks)

(c) (i) Describe with the aid of a diagram the structure and mode of operation of a moving coil galvanometer.
(ii) Explain how the design of a moving coil galvometercan be modified to produce a ballistic galvanometer

(d) A flat circular coil X of 30 turns and mean diameter $30cm$ is fixed in a vertical plane and carries a current of $3A$. Another coil Y of $2cmx2cm$ and having 200 turns is suspended in a vertical plane at the center of the circular coil. Initially the planes of the two coils coincide. Determine the torque on coil Y when a current of $2.0A$ is placed through it. (04marks)

Uneb 2007

(a) (i) Describe an experiment to demonstrate the damping effect of eddy current. (04marks)

(ii) Give two practical applications of this effect (01mark)

(b) What is meant by;

(i) **Self induction** ? (01mark)

(ii) **Mutual induction** ? (01mark)

(c) Discuss the factors which determine the maximum e.m.f generated by a dynamo. (04marks)

(d) A transformer has 2000 turns in the primary coil. The primary coil is connected to a 240V mains. A 12 V, 36W lamp is connected to the secondary coil. If the efficiency of the transformer is 90%, determine the;

(i) number of the turns in the secondary coil. (02marks)

(ii) Current flowing in the primary coil (03marks)

(e) Explain any two factors which lead to energy losses in the transformer (04marks)

Uneb 2006

(a) Define magnetic flux density and state its units (02marks)

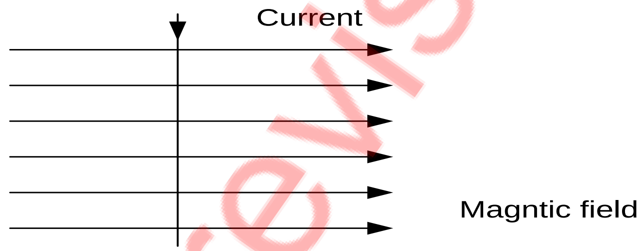
(b) Describe how the magnetic flux density between the poles of a powerful magnet can be determined. (05marks)

(c) (i) Explain with the aid of a sketch, the terms angle of dip and declination (04marks)

(ii) Explain what happens to the angle of dip as one moves along the same the same longitude from the equator to the north. (02marks)

(iii) Find the force per unit length on a straight horizontal wire carryng a curren of $2.0A$ in the direction North to south. If the angle of dip $70°$ and the earths horizontal field component is $1.6x10^{-5}T$. (03marks)

(d)



A wire is placed vertically in a horizontal magnetic field as shown above. Sketch the rsulstant magnetic field pattern. (03marks)

Uneb 2006

(a) (i) With the aid of a diagram, describe how a simple d.c motor works. (06marks)

(ii) Explain the significance of back e.m.f in the operation of a d.c motor (02mark)

(b) A motor of armature resistance 0.75Ω is operated from a 240V d.c supply;

(i) When the motor turns freely witout a load, the current in the armature is 4.0A and the motor makes 400 revolution per minute. Calculate the back e.m.f (02mark)

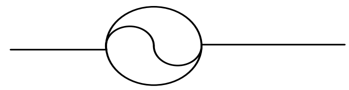
(ii) When the load is placed on the motor, the armature current increases to 60.0A. find the new speed of rotation of the motor (04mark)

(c) (i) A circular coil of 10 turns and radius 5.0cm carries a current of 1.0A. Find the magnetic flux density at its centre.

(ii) A copper wire of cross-sectional area 1.5mm^2 carries a current of 5.0A. The wire is placed perpendicular to a magnetic field of flux density 0.2T. if the density of free electrons in the wire is 10^{29}m^{-3}. Calculate the force on each electron (04marks)

ALTERNATING CURRENT CIRCUITS

An alternating current or voltage is one which varies periodically with time in magnitude and direction.
When an alternating p.d is applied to a conductor it causes the direction of the charge carriers to reverse many times per second (at the frequency of the alternating voltage).
An a.c source is represented by



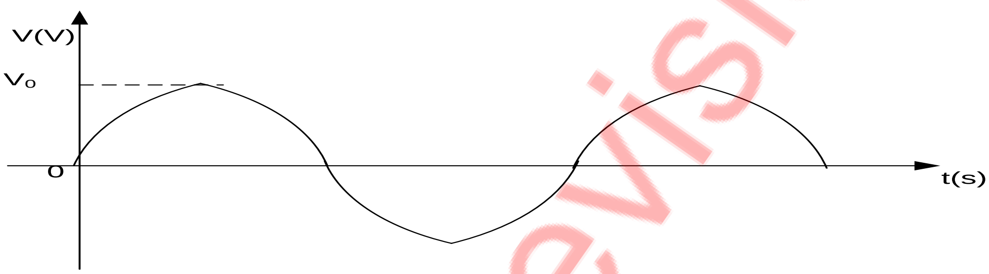
In a conductor it causes heating and produces around it a magnetic field that fluctuates with time. Since alternating voltages and currents vary in magnitude and direction, they are treated as vector quantities.

Sinusoidal alternating currents and voltages

Sinusoidal voltage is a periodic voltage whose time dependence is given by a sine function

The voltage is of the form $V = V_O sin\,\omega t$, where

where ***V*** is the voltage at time ***t***, V_O is the amplitudes or peak value of the voltage and ω the angular velocity given by $\omega = 2\pi f$ where f is the frequency of the alternating voltage



When applied to a circuit the alternating voltage produces an alternating current given by $I = I_O sin\,\omega t$ where ***I*** is the current at time t and I_O is the amplitudes or peak value of the alternating current.

Definitions

(i) Peak value of alternating current: this is the maximum value of alternating current
(ii) Peak value of alternating voltage: this is the maximum value of alternating voltage
(iii) Frequency f: this is the number of complete cycles made by an alternating current per second

$$\omega = 2\pi f$$

(iv) Period T: this the time taken for an alternating current to make one complete cycle

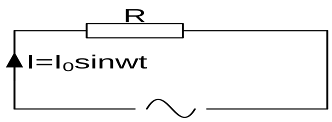
$$\omega = \frac{2\pi}{T}$$

Root mean square value of alternating current or voltage (I_{rms} or V_{rms})

The root mean square value of alternating current is the value of steady current which dissipates heat in a resistor at the same rate as the alternating current.
The root mean square value of alternating voltage is the value of steady voltage which dissipates electrical energy in a resistor at the same rate as the alternating voltage.

Relationship between I_O and I_{rms}

Consider a resistor in series with a.c source of electrical energy.



The instantaneous current I is given by,

$I = I_O \sin \omega t$

The instantaneous power dissipated in the resistor, $P = I^2R = \left(I_O{}^2 \sin^2 \omega t\right)R$

The average power over one cycle,

$\langle P\rangle_T = \langle I_O{}^2 \sin^2 \omega t\rangle_T\, R = \frac{I_O{}^2 R}{2}$

For direct current, I_{rms} passing through the same resistor,

power dissipated by the steady current $= I_{rms}^2 R$

$I_{rms}^2 R = \frac{I_O{}^2 R}{2}$

$$\boxed{I_{rms} = \frac{I_O}{\sqrt{2}}}$$

similarly, $V_{rms} = \frac{V_O}{\sqrt{2}}$

Alternative derivation

Consider the average value of I^2 over a cycle of the a.c.

$I_{r.m.s} = \sqrt{mean\ value\ of\, I^2}$

$$< I^2 >= \frac{1}{T-0}\int_0^T I_o^2 \sin^2\omega t dt$$

$< I^2 >= \frac{I_o^2}{T}\int_0^T I_o^2 \sin^2\omega t dt$

$$< I^2 >= \frac{\omega I_o^2}{2\pi}\int_0^T I_o^2 \sin^2\omega t dt$$

Recall the identity: $\cos 2\omega t = 1 - 2\sin^2\omega t$

$$\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

Since the period, $T = \frac{2\pi}{\omega}$ we can now write

$< I^2 >= \frac{\omega I_o^2}{4\pi}\int_0^{\frac{2\pi}{\omega}}(1 - \cos\omega t)dt$

$< I^2 >= \frac{\omega I_o^2}{4\pi}\int_0^{\frac{2\pi}{\omega}}(t-)dt$

$< I^2 >= \frac{\omega I_o^2}{4\pi}\left[t - \frac{1}{2\omega}\sin\omega t\right]_0^{\frac{2\pi}{\omega}}$

$< I^2 >= \frac{\omega I_o^2}{4\pi} \times \frac{2\pi}{\omega}$

But $I_{r.m.s} = \sqrt{< I^2 >} = \sqrt{\frac{I_o^2}{2}}$

Thus $I_{r.m.s} = \frac{I_o}{\sqrt{2}}$

Examples

1. The r.m.s value of the domestic mains p.d is 240 V. Determine the peak value of the mains p.d.

Solution

$V_{r.m.s} = 240\ V$

$V_{r.m.s} = \frac{V_0}{\sqrt{2}}$

$V_o = \sqrt{2} \times 240$

$V_o = 339.4\ V$

2. What is the peak value of the voltage from a 220V a.c mains?

solution

$V_{r.m.s} = 220$ V

$V_{r.m.s} = \frac{V_0}{\sqrt{2}}$

$V_o = \sqrt{2} \times 220$

$V_o = 311.127\ V$

3. An electric kettle draws 3000W from a 240V mains supply. Find the peak value of the current drawn by the kettle if the voltage is sinusoidal.

Solution

$$P = I_{r.m.s}V_{r.m.s}$$

$$I_{r.m.s} = \frac{3000}{240} = 12.5A$$

$$I_o = \sqrt{2}\, x I_{r.m.s}$$

$$I_o = \sqrt{2} \times 12.5 = 17.68A$$

4. An alternating p.d that varies sinusoidally is represented by $V = \sqrt{5000}\sin 1000t$

Determine,

(i) The r.m.s value of the voltage

(ii) Frequency of the p.d

Solution

(i) Comparing $V = \sqrt{5000} sin 1000t$

with $V = V_o sin\omega t$

$V_o = \sqrt{5000}$ V.

$$V_{r.m.s} = \frac{V_0}{\sqrt{2}} = \frac{\sqrt{5000}}{\sqrt{2}}$$

$V_{r.m.s} = 50\ V$

(ii) $\omega = 1000$ rad s^{-1}

$$f = \frac{\omega}{2\pi} = \frac{1000}{2\pi}$$

$f = 159.2$ Hz

Measurement of a.c

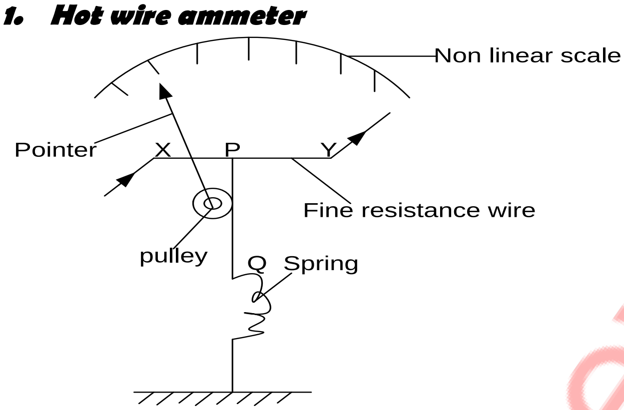
Moving -coil ammeter is not suitable for measureing a.c.

Explanation

When an a.c is passed through a moving – coil ammeter, the direction of the couple changes each time the current reverses. The pointer therefore vibrates at the frequecncy of the a.c about the zero position hence the value of the current can not be read.

Instruments used to measure alternating current

1. Hot wire ammeter

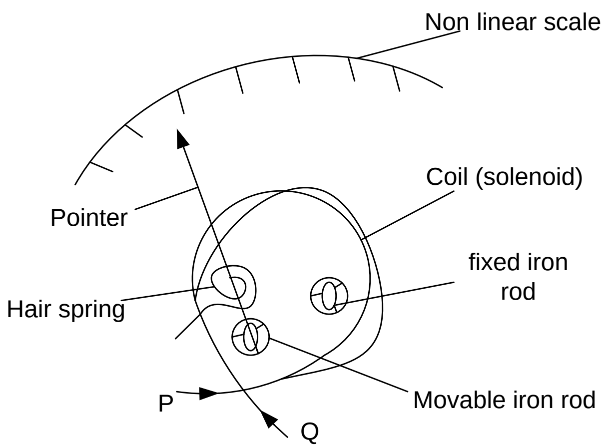


- When alternatin current is passed through the fine resistance wire XY, it heats up, expands and sags.
- The sag is taken up by the wire PQ, which is held taut by a spring.
- The pulley moves and hence pointer attached to the pulley turns over the scale.
- The deflection of the pointer is proportional to the rate at which heat is developed in the wire xy
- The deflection of the pointer is proportional to the average value of the square of the current in the coil.

Note:

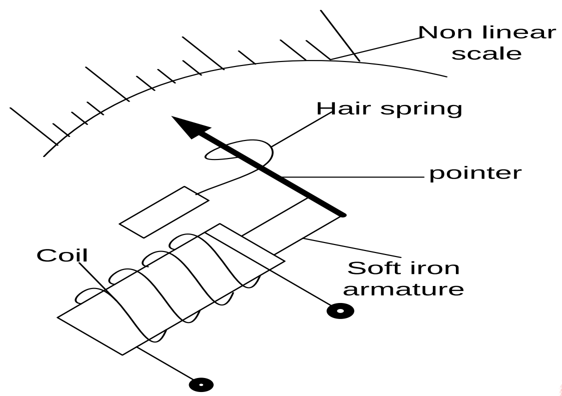
If current to be measure is passed through the wire XY, then the conclusion should be deflection is proportional to the square of the value of the current

2. Moving Iron Ammeter (Repulsion type)



- When the alternating current is passed through the coil via terminals P and Q, whatever the direction of the current , the iron rods are magnetized in the same sense, and so they repel each other.
- This causes the pointer to deflect over the scale until it is stopped by the restoring couple of the hair spring
- The force of repulsion is proportional to the average value of the square of the current in the coil.
- The deflection of the pointer is proportional to the average value of the square of the current in the coil.

3. *Moving Iron Ammeter (attraction type)*



- If alternating current is passed through the solenoid, the soft – iron armature gets magnetized and is attracted towards the coil whatever the direction of the current
- This causes the pointer to deflect over the scale until it is stopped by the restoring couple of the hair spring
- The attractive force is proportional to the average value of the square of the current.
- The deflection θ of the pointer is therefore proportional to the average value of the square of the current, $\theta \propto \langle I^2 \rangle$.

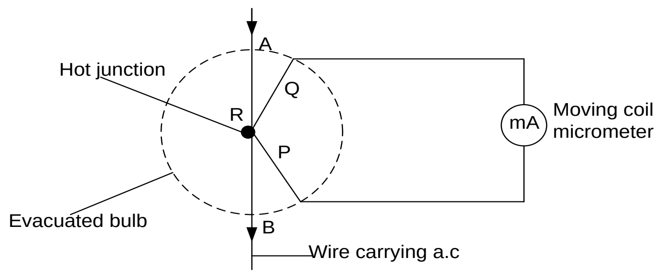
Advantages of moving iron ammeter

- they are cheap
- can be used to measure both a.c and d.c.

Disadvantages of moving iron ammeter

- they have non-linear scales.
- They are not sensitive like moving coil galvanometers
- Can easily be affected by stray magnetic fields like earth's magnetic field unless shielded by an iron case

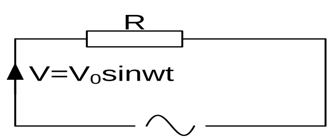
4. **Thermocouple meter**



- Pand Q are <u>different metals</u> joined at R
- Alternating current is passed through wire AB and it heats the junction R of the thermocouple.
- An electric e.m.f is generated causing a direct current to flow through the micro ammeter already calibrated to measure r.m.s value of current.

A.c through a Resistor

Consider a resistor in series with a.c source of electrical energy.



When an alternating p.d is applied to a pure resistor its magnitude rises to a maximum in the same time as that of the current. Thus at any instant in a resistor ***I*** and ***V*** are said to be in phase.

If the alternating voltage is $V = V_O \sin \omega t$, then the current I is given by

$$I = \frac{V}{R}$$

$$I = \frac{V_O \sin \omega t}{R}$$

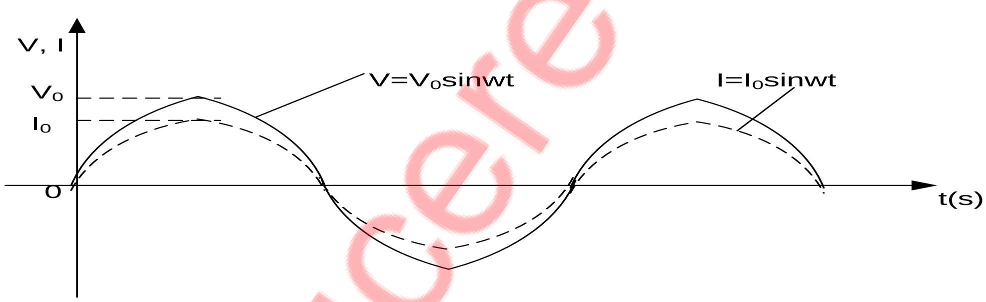
But $\frac{V_O}{R}$ is the maximum value of I i.e. $I_O = \frac{V_O}{R}$

Hence $I = I_O \sin \omega t$

For a resistor current and voltage are in phase

$I = I_O \sin \omega t$ and $V = V_O \sin \omega t$,

Variation of *V* and *I* with *t* in a resistor.



Phasor Diagrams/Vector Diagram for a resistor

Phasor a diagram is a vector diagram showing variation of voltage with current.

For a resistor voltage and current are in phase. When voltage is maximum currents also maximum

$\longrightarrow V_R$

$\longrightarrow I_R$

Power absorbed in a resistor

Instantaneous power absorbed, $P = VI$

$$P = (V_O \sin \omega t)(I_O \sin \omega t)$$

$$P = V_O I_O \sin^2 \omega t$$

Mean power (average power) $= V_O I_O \langle \sin^2 \omega t \rangle$

but $\langle \sin^2 \omega t \rangle = \frac{1}{2}$

$$\boxed{\langle P\rangle = \frac{V_0 I_0}{2}}$$

Since $V_0 = \sqrt{2}\ V_{rms}$

and $I_0 = \sqrt{2}\ I_{rms}$

$$\langle P\rangle = \frac{(\sqrt{2}\ I_{rms})(\sqrt{2}\ V_{rms})}{2}$$

$$\boxed{\langle P\rangle = I_{rms}\ V_{rms}}$$

Examples

1. A sinusoidal alternating voltage $V = 170sin120\pi t$ is applied across a resistor of resistance 100 Ω. Determine
   (i) the r.m.s value of the current which flows;
   (ii) the frequency of the current through the resistor

Solution

(i) Instantaneous current through R is given by

$I = \frac{V}{R}$

$$I = \frac{170sin120\pi t}{100}$$

$$\therefore I = 1.7sin120\pi t$$

Comparing $I = 1.7sin120\pi t$ with $I = I_o \sin\omega t$ we have;

$$I_o = 1.7\ A$$

From $I_{r.m.s} = \frac{I_o}{\sqrt{2}}$, it follows that

$$I_{r.m.s} = \frac{1.7}{\sqrt{2}} = 1.2\ A$$

(i) $\omega = 120\pi$ rad s^{-1}

But $\omega = 2\pi f$

$$2\pi f = 120\pi$$

$f = 60$ Hz

2. A sinusoidal a.c $I = 4sin100\pi t$ flows through a resistor of resistance 2.0 Ω. Find the mean power dissipated in the resistor. Hence deduce the r.m.s value of the current.

Solution

Mean power, $< P >= \langle(4sin100\pi t)^2\rangle \times R$

$< P >= \frac{4^2\times 2}{2} = 16$ W since $\langle sin100\pi t\rangle^2 = \frac{1}{2}$

$< P >= 16W$

OR

Average power $= \frac{I_0^2 R}{2}$, but$I_0 = 4A$

$< P >= \frac{4^2\times 2}{2} = 16$ W

Power dissipated by d.c$< P >= I_{rms}^2 R$

$$16 = 2xI_{rms}^2$$

$I_{rms}^2 = 8$

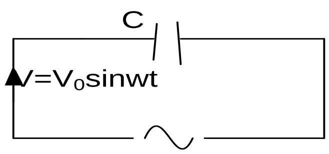
$$I_{rms} = 2\sqrt{2}A$$

Questions

1. A sinusoidal a.c $I = 3sin120\pi t$ flows through a resistor of resistance 2.5 Ω. Find the mean power dissipated in the resistor. **An(11.25W)**
2. A sinusoidal alternating voltage $V = 340sin120\pi t$ is applied across a resistor of resistance 40 Ω. Determine
   (i) Amplitude of the current which flows;
   (ii) the average power developed in the resistor **An[8.5A, 1445W]**
3. What is the rms value of alternating current which must pass through a resistor immersed in oil in a calorimeter so that the initial rate of rise of temperature of the oil is three times that produced when a direct current of 2A passes through the resistor under the same conditions? **An[3.46A]**
4. A current I = 8.0 sin 100 t amperes is maintained in a heating coil immersed in 20kg of water. The resistance of the coil is 50Ω. Find the temperature rise obtained in 5 minutes
5. An a.c circuit of a resistor of resistance R and it is connected to a source of p.d given by $V = V_0 sin\omega t$
   (a) Derive and expression for current at any time, t
   (b) Sketch the variation of current with time on the same axes and use your graph to explain the phase difference between I and V

Capacitors in a.c circuits

Consider a capacitor of capacitance C, connected to an a.c source so that the p.d V acroos the capacitor at time t is given by $V_O sin\ \omega t$



The charge *Q* on the plates at time *t* is given by

$Q\ =\ CV$

$Q = CV_O sin\ \omega t \ldots\ldots\ldots\ldots\ldots (1)$

At any time t, the current I, in the circuit is equal to the rate of flow of charge.

$$I = \frac{dQ}{dt}$$

$$I = \frac{d}{dt}(CV_O sin\ \omega t)$$

$$I = CV_O \omega cos\omega t \ldots\ldots\ldots\ldots\ldots . (2)$$

If I_O is the peak value of the current, then we can write

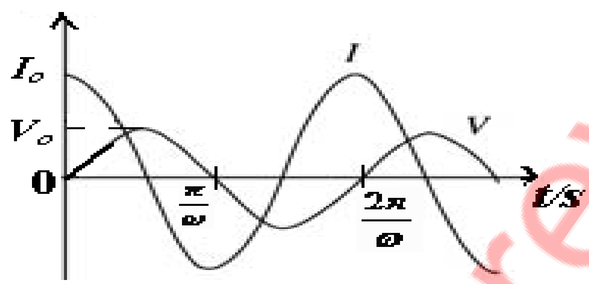
$$\boxed{I = I_O cos\omega t}$$

where. $I_O = CV_O\omega$

$$\boxed{I = I_O sin\left(\omega t + \frac{\pi}{2}\right)}$$

Thus the current (cosine curve) through a capacitor leads the p.d (sine curve) across the capacitor by a quarter of a cycle or $90°$ or $\frac{\pi}{2}$ radians. i.e the current reaches its maximum value one quarter of a cycle before the p.d reaches its peak value.

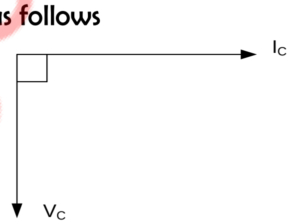
Variation of I and V of a capacitor with time



<u>**Phasor Diagrams/Vector Diagram for a capacitor**</u>

For a pure capacitor in an *a.c* circuit the current *I*, leads the applied p.d *V* by 90° or $\frac{\pi}{2}$ radians.

The vector diagram for a capacitor is as follows



<u>**Capacitive Reactance,** X_C</u>

The capacitive reactance is the non- resistive opposition to the flow of a.c through a capacitor.

Its symbol is X_C and is defined by the equation

$$X_C = \frac{V_O}{I_O}$$

Since $V_O = \sqrt{2}V_{rms}$ and $I_O = \sqrt{2}I_{rms}$, it follows that

$$X_C = \frac{V_{r.m.s}}{I_{r.m.s}}$$

From, $I_O = CV_O\omega = I_O = 2\pi f CV_O$ since $\omega = 2\pi f$

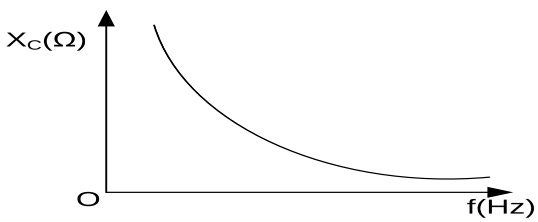
but $X_C = \frac{V_O}{I_O} = \frac{V_O}{2\pi f\ CV_O} = \frac{1}{2\pi f C}$

$$\boxed{X_C = \frac{1}{2\pi f C}}$$

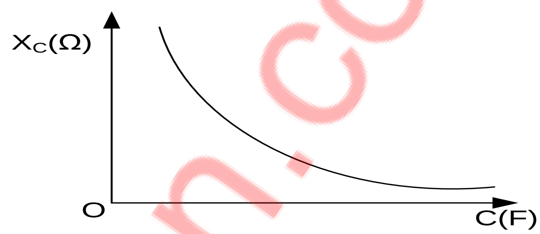
The S.I unit of X_C is the ohm (Ω) when f is in hertz and C in farad.

Thus for a given frequency, $X_C \propto \frac{1}{C}$ and for a given capacitance, $X_C \propto \frac{1}{f}$.

Variation of X_C with f



Variation of X_C with C



A.c power in a capacitor

In a pure capacitor ***V*** lags on ***I*** by $\frac{\pi}{2}$.

If the p.d applied to the capacitor is,
$V = V_O sin\ \omega t$ then the current ***I***, is given by $I = I_O cos\ \omega t$.

Instantaneous power absorbed, $P = VI$

$$P = (V_O sin\ \omega t)(I_O cos\ \omega t)$$

$$P = I_O\ V_O sin\ \omega t cos\ \omega t$$

But $sin2\ \omega t = 2 sin\ \omega t cos\ \omega t$

$$sin\ \omega t cos\ \omega t = \frac{sin2\ \omega t}{2}$$

$$P = \frac{I_O\ V_O sin2\ \omega t}{2}$$

Average power $\langle P \rangle = \frac{I_O\ V_O \langle sin2\ \omega t \rangle}{2}$ but $\langle sin2\ \omega t \rangle = 0$

Average power $\langle P \rangle = 0$

Thus mean power absorbed by a capacitor over a cycle is zero.

Explanation for zero power

- During the first quarter cycle, the capacitor charges and energy is drawn from the source and stored in the electric field of the capacitor.
- During the second quarter cycle, the capacitor discharges and energy is returned to the source.
- During the third cycle, the capacitor charges in the opposite direction, again energy is stored in the electric field in the capacitor
- In the last quarter, the capacitor discharges and energy returns to the source. Therefore in one cycle, there is no net energy stored in the capacitor.

Question: Explain why a capacitor allows the flow of a.c but not d.c.

- When a capacitors is connected to a d.c source, the capacitor charges and when fully charged current stops flowing
- When the capacitor is connected to an a.c source, the capacitor charges when the voltage is increasing, and the capacitor also discharges when the voltage is decreasing. Since increase and decrease in voltage is continuous, there is continuous flow of current (charge) in the circuit hence capacitor appears to allow flow of a.c

Examples

1. A capacitor of capacitance $1\mu F$ is used in a radio circuit where frequency is 1000Hz and current is 2mA. Calculate the voltage across capacitor.

Solution

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi x1000x1x10^{-6}} = 159\Omega$$

$$V = IX_C$$

$$V = 159x2x10^{-3} = 0.32V$$

2. A capacitor of capacitance $2\mu F$ is connected to an a.c source of current 4mA (r.m.s) and frequency 50Hz. Calculate the ;
   (i) Capacitive reactance
   (ii) Voltage across the plates of the capacitor

   Solution

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi x50x2x10^{-6}} = 1592\Omega$$

$$V_{r.m.s} = I_{r.m.s}X_C$$

$$V = 1592x4x10^{-3} = 6.37V$$

6. A capacitor of capacitance $2\mu F$ is connected to an a.c source of voltage 20V and frequency 50Hz. Calculate the Current which flows

   Solution

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi x50x2x10^{-6}} = 1592\Omega$$

$$V_{r.m.s} = I_{r.m.s}X_C$$

$$20 = 1592xI_{r.m.s}$$

$$I_{r.m.s} = 12.6mA$$

Exercise

1. A capacitor of capacitance $2\mu F$ is connected to an a.c source of current 2mA and frequency 100Hz. Calculate the ;
   (i) Capacitive reactance
   (ii) Voltage across the plates of the capacitor
   (iii) Reactance if the frequency is 140Hz

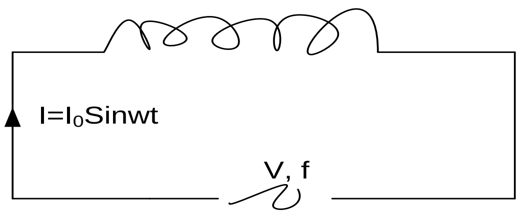
   An(795.8Ω, 1.6V, 568.4Ω)

2. A 240 V, 60Hz alternating voltage is applied across a capacitor of capacitance 10 μF. Calculate the
   (i) root mean square value of the current which flows
   (ii) power expanded **An(0.63A, 0W)**

3. A capacitor of capacitance C and infinite resistance is connected across a source of an a.c source whose p.d is given by $V = V_O cos\ \omega t$ and frequency f.
   (i) find the expression for the charge and current
   (ii) using the same axes, show how the applied voltage and current in the circuit may vary with time and comment on this variation
   (iii) find the expression for reactance of the circuit and sketch is variation with frequency

A.c through an inductor

A pure inductor is a coil of negligible resistance.



Consider an inductor of negligible resistance, inductance L, through which an alternating current I flows where

$I = I_O sin\ \omega t$ 1

The a.c produces a changing magnetic flux in the inductor and sets up a back e.m.f E_b in the coil.

The back e.m.f is given by $E_b = -L\frac{dI}{dt}$

$$= -L\frac{d}{dt}(I_O sin\ \omega t\ )$$

$$E_b = -LI_O\omega cos\omega t$$

But for finite current across a pure inductor

$V = -E_b$

$$V = -(-LI_O\omega cos\omega t)$$

$$V = LI_O\omega cos\omega t$$

$$\boxed{V = V_O cos\omega t}$$

Where $V_O = LI_O\omega$

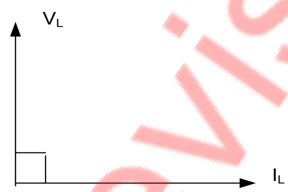
$$\boxed{V = V_O sin\left(\omega t + \frac{\pi}{2}\right)}$$

Thus the voltage (cosine curve) across an inductor leads the current (sine curve) through an inductor by $90°$ or $\frac{\pi}{2}$ radians.

Phasor Diagrams/Vector Diagram for an inductor

For a pure inductor in an *a.c* circuit the voltage leads the current by 90° or $\frac{\pi}{2}$ radians.

The vector diagram for a capacitor is as follows



Inductive Reactance, X_L

The Inductive reactance is the non- resistive opposition to the flow of a.c through an inductor.

Its symbol is X_L and is defined by the equation

$X_L = \frac{V_O}{I_O}$

Since $V_O = \sqrt{2}V_{rms}$ and $I_O = \sqrt{2}I_{rms}$, it follows that .

$$X_L = \frac{V_{r.m.s}}{I_{r.m.s}}$$

From, $V_O = L\ I_O\omega = V_O = 2\pi f L\ I_O$ since

$$\omega = 2\pi f$$

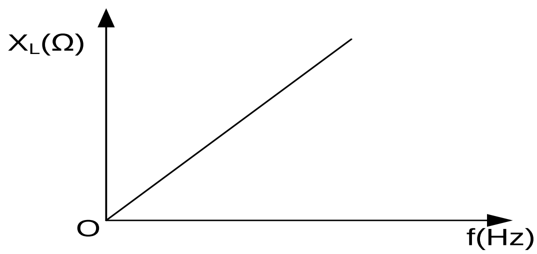
but $X_L = \frac{V_O}{I_O} = \frac{2\pi f\ L\ I_O}{I_O} = 2\pi f\ L$

$$\boxed{X_L = 2\pi f\ L}$$

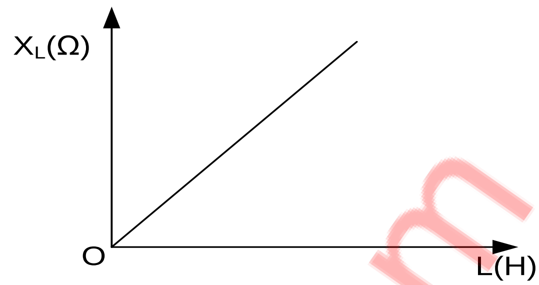
The S.I unit of X_L is the ohm (Ω) when f is in hertz and C in farad.

Thus for a given frequency, $X_L \propto L$ and for a given capacitance, $X_L \propto L$.

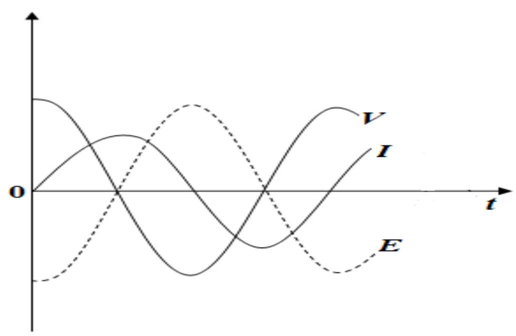
Variation of X_L with L



Variation of X_L with f



Variation of *V* and *I* with *t* for an inductor



Power in the inductive coil

In a pure inductor *V* leads *I* by $\frac{\pi}{2}$

If current through the inductor is, $I = I_O sin\ \omega t$.

then the p.d $V = V_O cos\ \omega t$

Instantaneous power absorbed, $P = VI$

$$P = (V_O cos\ \omega t)(I_O sin\ \omega t)$$

$$P = I_O\ V_O sin\ \omega t cos\ \omega t$$

But $sin2\ \omega t = 2 sin\ \omega t cos\ \omega t$

$$sin\ \omega t cos\ \omega t = \frac{sin2\ \omega t}{2}$$

$$P = \frac{I_O\ V_O sin2\ \omega t}{2}$$

Average power $\langle P \rangle = \frac{I_O\ V_O \langle sin2\ \omega t \rangle}{2}$ but

$\langle sin2\ \omega t \rangle = 0$

Average power $\langle P \rangle = 0$

Thus mean power absorbed in an inductor over a cycle is zero.

Explanation for zero power

- During the first quarter cycle, <u>current increases</u> and magnetic field linking the coil builds up to the maximum value. Energy supplied by the source is <u>stored</u> in the magnetic field associated with the coil
- During the second quarter cycle, the <u>current decreases</u>, magnetic field collapses and the energy that was stored in the magnetic field of the coil is <u>restored</u> back to the source
- During the third cycle, current supplied by the source increase in the reverse direction. Energy supplied by the generator is stored in the magnetic field.
- In the last quarter, an equal amount of energy is restored back to the source. Therefore power dissipated in an inductor over one cycle is <u>zero</u>.

Examples

1. An inductor of 2H and negligible resistance is connected to a 12V mains supply, frequency 50Hz. Find the current flowing.

Solution

$X_L = 2\pi f L$ $X_L = 2\pi x 50 x 2 = 628\Omega$	$V_{r.m.s} = I_{r.m.s}X_L$ $12 = 628xI_{r.m.s}$	$I_{r.m.s} = 0.019A$

2. A coil of inductance 5H has negligible resistance and is connected to a 12V (r.m.s) supply source of frequency 50HZ.
   (a) Calculate the inductive reactance of the coil
   (b) Find the maximum current which would flow when the inductance changes to 10H An **[1570.8Ω, 5.4mA]**
3. A pure inductor of self inductance 1H is connected across an alternating voltage of 115V and frequency 60Hz. Calculate the;
   (i) inductive reactance
   (ii) inductive current
   (iii) peak current
   (iv) average power consumed. An **[377Ω, 0.31A. 0.44A, zero]**
4. An ac circuit f an inductor of inductance L and it is connected to a source of p.d given by $V = V_0 sin\omega t$
   (c) Derive and expression for current at any time, t
   (d) Sketch the variation of current with time on the same axes and use your graph to explain the phase difference between I and V

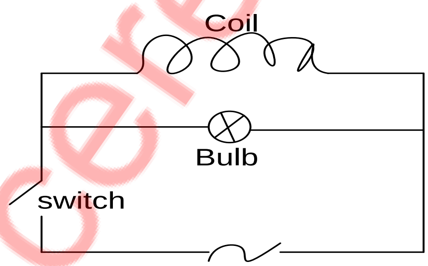
Self-Induction

This is the process of generating an emf in a coil due to changing current in the same coil

This e.m.f is called a back e.m.f and creates a current which opposes the flow of current in the coil its self.

Example

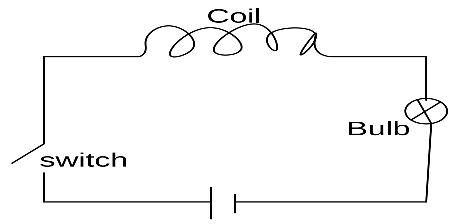
1. A coil of many turns of wire is connected in parallel with an electric bulb an a.c supply as shown below



Explain what happens to the bulb when the switch is;
   (i) Closed
   (b) Opened

- ❖ When switch is closed, the bulb will light brightly and then it will go dim after some time. This is because when the switch is closed, current begins to flow through the coil and the changing magnetic flux due to the changing current induces a back e.m.f in the coil which opposes the flow of current through it and hence most of the current passes through the bulb making it very bright.
- ❖ After a short time the back e.mf reduces to zero and all the current flows through the coil and hence bulb stops lighting.
- ❖ When the switch is opened, the bulb keeps the light for some time before going off. This is because as current decays from the coil. The back e.m.f is induced in it to oppose the decay. This e.m.f creates current which lights the bulb for some time

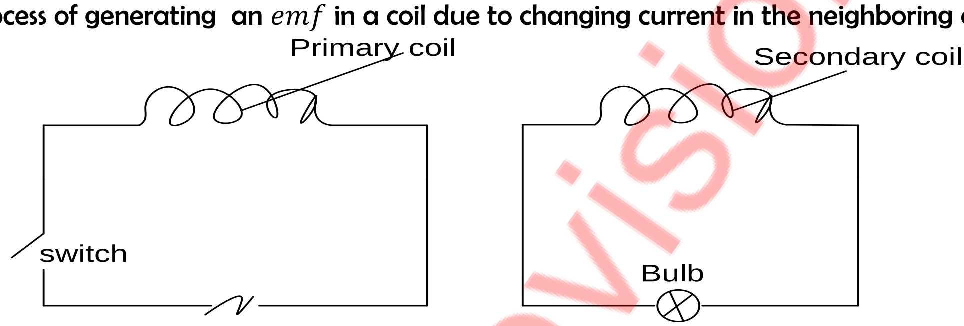
2. A bulb is concede in series with an inductive coil a d.c source as shown below



What happens to the brightness of the bulb when an iron core is inserted in the coil?
Solution
Bulb dims and then becomes bright again. When the iron core is introduced into, the magnetic flux linkage with the coil increases. The rate of change of magnetic flux linkage with the coil increase and hence an increase in the induced back e.m.f. This leads to a reduction in the current flowing though the bulb. Hence the bulb dims

Mutual Inductance

This is the process of generating an emf in a coil due to changing current in the neighboring coil



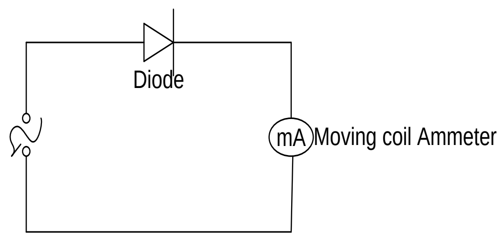
- When switch is closed, the bulb light momentarily. This is because the a.c in the primary coil creates a varying magnetic field in the primary.
- The magnetic field links up with the secondary coil. When this magnetic field changes, a back emf is induced in the secondary coil which opposes the varying magnetic field linking it. This e.m.f creates a current in the secondary which lights the bulb
- When the switch is opened, the bulb lights momentarily before going off. This because as the current decays in the primary coil, the back e.m.f is created to oppose the decay.
- The varying magnetic field lines then link the secondary. The back e.m.f produced in the secondary produces the current which lights up the bulb.

RECTIFICATION

Rectification is the process of converting Alternating current to Direct current.
This can be done by use of
- Thermionic diodes.
- Semiconductor diode

When a rectifier is connected to a supply its supposed to conduct and when it does so its said to be **forward biased**. And when connected in a reverse way it fails to conduct therefore its said to be **reverse-biased.**

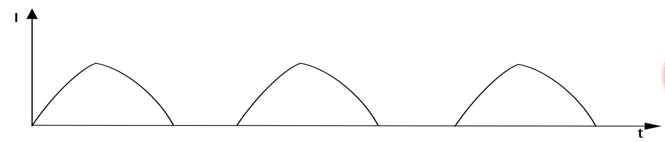
a) Half wave Rectification



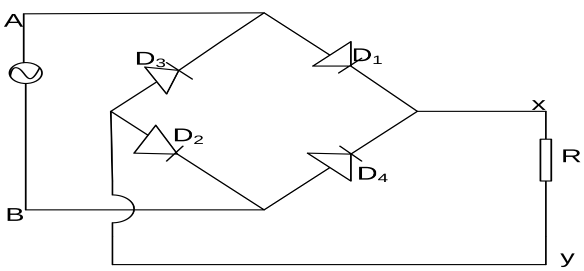
- Current to be measure is fed to the meter through the rectifier diode which conducts current in only one direction.
- So a direct current of varying magnitude flows through the meter.
- The moving coil meter is calibrated to measure the I_{rms}

N.B: The Arrow head in the rectifier symbol shows the direction of flow of current through the circuit.

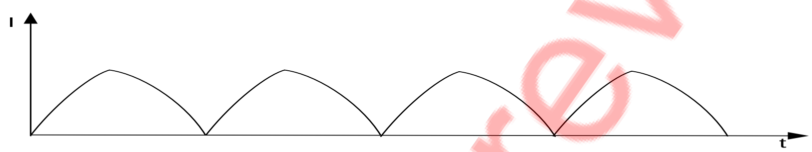
A graph of I against t is drawn



b) Full wave rectification



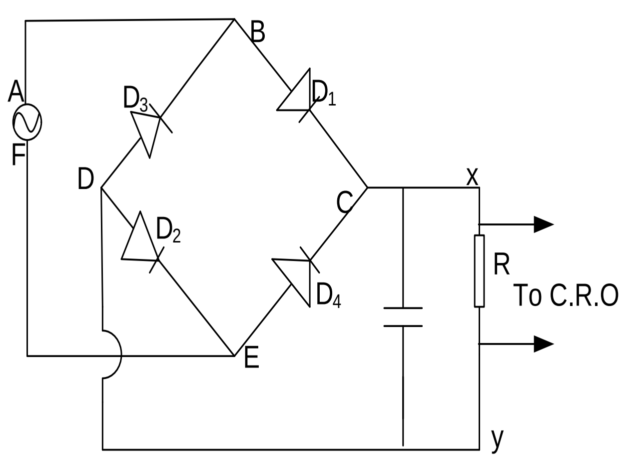
- In the half cycle when A is positive and B is negative, diodes D_1 and D_2 conduct and current flows through R in the direction x to y and diode D_3 and D_4 do not conduct in this half cycle
- During the next half cycle when B is positive and A is negative diodes D_3 and D_4 conduct while D_1 and D_2 do not conduct in this cycle and current (I) flows through R in the direction x to y.
- Hence current flows through R is in the same direction throughout.



Why rectify?

Many electrical appliances such as radios, televisions and computers require direct current for their operation. Alternating e.m.f.s are easy and cheap to generate and to use it for these devices it must be converted to direct current by the use of rectifiers.

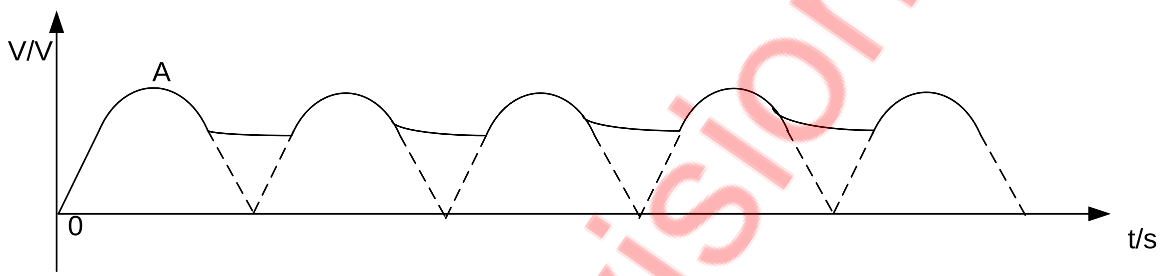
Capacitor Smoothing



A large electrolytic capacitor C is connected in parallel with the load resistor R such that its positive terminal is connected to the positive output of the bridge rectifier.

The voltage increases to a peak at A as the capacitor charges. When the p.d begins to drop, the capacitor supplies charge to the load, thus rising the p.d across the load. The resultant drop in p.d across the load is thus small. As the pd begins to increase as the capacitor charges and the cycle is repeated and thus fluctuation in p.d is effectively reduced

Smoothed output of a full – wave rectifier



Uneb 2016

(a) Define **root mean square(rms)** value of an alternating voltage. (01mark)

(b) A resistor of resistance 100Ω is connected across an alternating voltage $V = 20\,sin\,120\pi t$

(i) Find the frequency of the alternating voltage. (01mark)

(ii) Calculate the mean power dissipated in the resistor. (03marks)

(c) (i) Show that when an inductor is connected to an a.c supply voltage of $V = V_0\,sin\,2\pi ft$. The resulting current lags the current by 90°. (04marks)

(ii) Sketch on the same axes the variation with time of the voltage and current if a capacitor is connected to the voltage supply in (c) (i). (02marks)

(d) (i) Describe how a thermocouple meter works. (04marks)

(ii) Explain the precautionary measure taken in the desing of the thermocouple meter. (02marks)

(e) A capacitor of capacitance 60μF is connected to an a.c voltage supply of frequency 40Hz. An a.c ammeer connected in series with the capaciot reads 2.2A. find the p.d across the capacitor. (03marks)

Uneb 2015

(a) (i) Define **root mean square(rms)** current of an a.c. (01mark)

(ii) Derive an expression for capacitive reactance. (04marks)

(iii) Sketch on the same axes, the graphs showing variation of applied p.d and current when aan inductor is connected to an a.c supply. (02marks)

(b) (i) A capacitor of capacitance, **C** and an ammeter are connected in series across an alternating voltage, **V**, of frequency **f**. Explain why current apparently flows through the capacitor. (03marks)

(ii) a sinusoidal p.d of rms value of 20V and frequency 50Hz is applied across a 100μF capacitor. Calculate the capacitive reactance of the circuit. (02marks)

(c) Describe the mode of operation of a transformer. (04marks)

(d) A transformer connected to a.c supply of peak voltage 240V is to supply a peak voltage of 9.0V to a mini- lighting system of resistance 5Ω. Calculate the

(i) Rms current supplied to the lighting system (02marks)

(ii) Average power delivered to the lighting system. (02marks)

SECTION D: ELECTROSTATICS, CAPACITORS AND ELECTRICITY

Electrostatics (static electricity) is the study of electric charges at rest, the forces between them, and the electric fields associated with them.

Static electricity occurs when positive (+) or negative (-) electrical charges collect on an object's surface. There are several methods through which this condition can be caused.

One way is by rubbing certain materials together or pulling them apart. Another way is by bringing a charged material near to a neutral material, and also by sharing the charge on a body with another neutral insulated body when they come into contact with each other.

Electrification by friction / charging by rubbing or friction

- When two dissimilar bodies are rubbed together, heat is generated due to friction
- The heat is sufficient to make the material of lower work function to release some electron, which are taken up by other material.
- The one which lost electrons become positively charged while the one which gained electrons becomes negatively charged
- The number of electrons lost is equal to the number of electrons acquire therefore two insulating bodies rubbed together acquire equal and opposite charges.

Examples of charging by friction

- When a polythene rod (ebonite rod) is rubbed with fur (woolen duster), the ebonite rod becomes negatively charged while the duster becomes positively charged.
- If a glass rod (cellulose acetate) is rubbed with silk, a glass rod becomes positively charged while the silk becomes negatively charged.

Insulators, semiconductors and conductors

Conductor

This is a material with free electrons and it can allow electricity and heat to pass through it.
Examples: Copper, bronze

Insulator

This is a material without free electrons and it cannot allow electricity and heat to pass through it.
Examples: Dry wood, plastic

Semiconductors

These are materials which allow electric charges to pass through them with difficulty.
Examples: Moist air, paper

Law of electrostatics

Like charges repel each other and unlike charges attract each other.

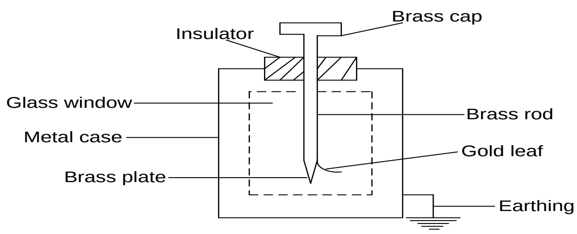
Precautions taken when carrying experiments in electrostatics

(i) Apparatus must be insulated
(ii) The surrounding must be free from dust and moisture

Attraction of neutral body by charged body

Consider the uncharged conductor being brought near a negatively charged ebonite rod. Negative charges on the ebonite rod repel the free electrons on the conductor to the remote end and positive charge is thus left near the end of the metal adjacent to the ebonite rod. So the conductor is now attracted by the ebonite rod.

GOLD LEAF ELETROSCOPE (GLE)



Uses of GLE

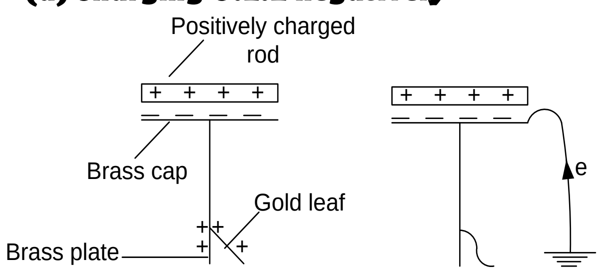
(i) Test for the presence of charge
(ii) Test the sign of the charge
(iii) To test the magnitude of charge
(iv) Measure $p.d$

Electrostatic induction

It's a phenomenon that describes the formation of charges on a conductor when a charged body is brought near it.
The charge acquired is opposite to that of inducing body.

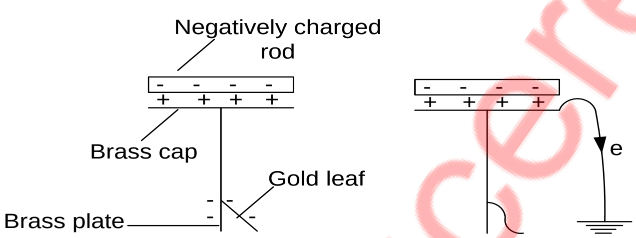
Charging a gold leaf electroscope by induction

(a) Charging G.L.E negatively



- A positively charged glass rod is brought near the cap of the G.L.E, negative charges are induced on the brass cap and positive charges on G.L and brass plate. The gold leaf diverges.
- With glass rod still in position, the G.L.E is earthed. Free electrons flow from the earth to the brass plate and gold leaf thus collapses.
- With the rod still in position, the earthing wire is removed.
- Glass rod is removed, the negative charges then redistribute themselves to the brass cap, plate and gold leaf thus causing he leaf to diverge. The electroscope is now negatively charged.

(b) Charging G.L.E negatively



- A negatively charged rod is brought near the cap of the G.L.E, positive charges are induced on the brass cap and negative charges on G.L and brass plate. The gold leaf diverges.
- With glass rod still in position, the G.L.E is earthed. Free electrons flow from the plate and leaf to the earth thus the leaf collapses.
- With the rod still in position, the earthing wire is removed.
- The rod is removed, the positive charges then redistribute themselves to the brass cap, plate and gold leaf thus causing he leaf to diverge. The electroscope is now positively charged

Testing for the sign of charge on a body

- Charge an electroscope negatively and the divergence noted. Bring the body under test near the cap of *GLE*. If the leaf divergence increases then that body is negatively charged, but if the divergence of the leaf decreases, then that body has either **positive** charge or it is **neutral body**
- To differentiate between the two alternatives, discharge the *GLE* and now charge it positively
- Bring the same body under test near the cap of appositively charged *GLE*. If the leaf divergence increases again, then that body has positive charges but if the leaf divergence decreases then that body is neutral.

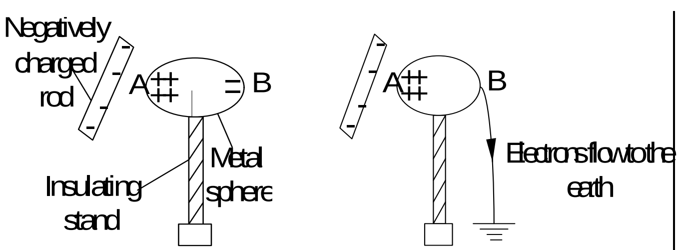
Note: Repulsion is the only confirmatory test for the sign of the charge

Summary

Charge on GLE	Charge brought near cap	Effect on leaf divergence
+	+	Increase(repulsion)
-	-	Increase(repulsion)
+	-	Decrease (attraction)
-	+	Decrease (attraction)
+ or -	Uncharged body	Decrease (attraction)

Charging a conductor by induction

a) Positively



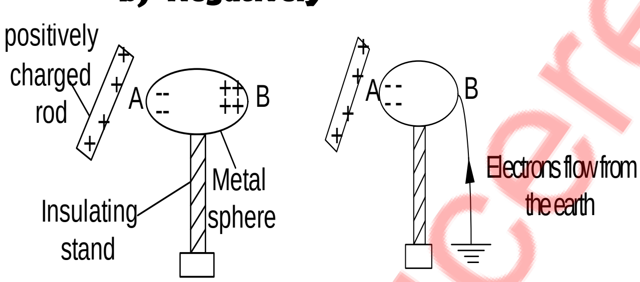
❖ Metal sphere on an insulating stand is placed near the negatively charged body. Free electrons in the metal sphere are repelled to the far end of the sphere.

❖ The sphere is earthed while the charged body is still in position. Free electrons move from the sphere to the earth.

❖ The earthing wire is removed while the charged rod is still in position

❖ The charged body is removed and charges distributes themselves all over the sphere. Hence the metal sphere is now positively charged.

b) Negatively



❖ Metal sphere on an insulating stand is placed near the positively charged body. Free electrons in the metal sphere are attracted to the near end of the sphere.

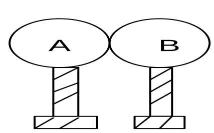
❖ The sphere is earthed while the charged body is still in position. Free electrons move from the earth to the sphere.

❖ The earthing wire is removed while the charged rod is still in position

❖ The charged body is removed and charges distributes themselves all over the sphere. Hence the metal sphere is now negatively charged.

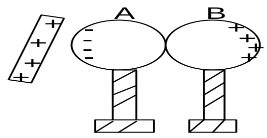
Separation of conductors

i)



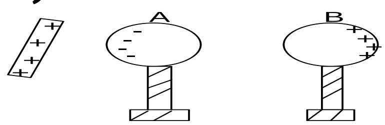
Two identical brass spheres A and B are placed together so that they touch one another.

ii)



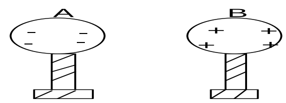
A positively charged rod is now brought near end A and as a result negative charge is induced at A and positive charges repelled to B.

iii)



Keeping the positive rod in position, sphere B is moved a short distance away from B

iv)



The charged rod is now removed and charges redistribute

Explain how two spherical conductors made of brass can be changed oppositely and simultaneously by induction.

How to distinguish a conductor and an insulator using an electroscope

- An electroscope is given charge and the divergence noted. The material is brought near the cap of the electroscope
- If there is no change in divergence, the material is an insulator. If the leaf divergences material is a conductor

Charging a body negatively at zero potential

- A positively charged glass rod is brought near end A of the conductor. Negative charges are induced at the near end and positive charges at the far end of the neutral body.
- With the glass rod still in position, body is earthed. Body is now negatively charged at zero potential

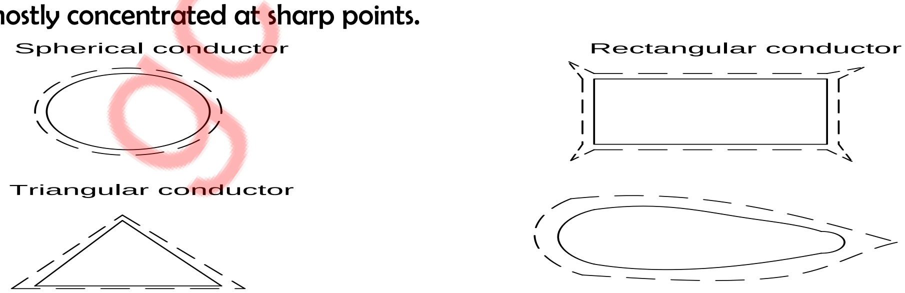
Electrophorus

This an instrument for produce unlimited supply of charge but it is not source of energy though converts mechanical energy to electrical energy

Distribution of charge over the surface of a conductor.

Surface charge density:

This is the quantity of charge per unit area over the surface of the conductor. Charge is mostly concentrated at sharp points.



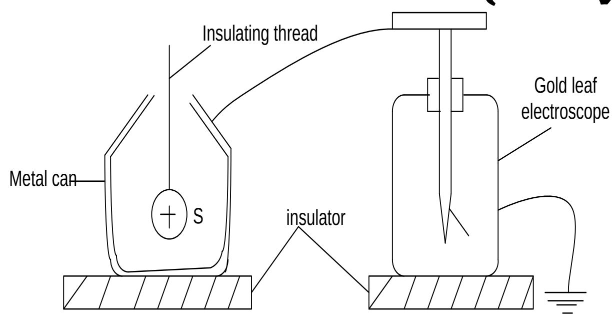
Note:

Charge only resides on the outside of a hollow conductor

Investigating charge distribution on a pear shaped conductor

- A proof plane is placed on the surface of the conductor. A sample of charge acquired by the proof plane is then transferred to a hollow metal can placed on the cap neutral electroscope and the deflection of the electroscope is noted
- The proof plane is then used to pick samples charges from different parts of the conductor and each time the deflection of the electroscope is noted
- The greatest deflection is obtained when the sample of charge are picked from the pointed end of the conductor. Therefore the surface charge density of charges is greatest where the curvature is greatest

Experiment to show distribution of charge in a hollow conductor.
(Faraday's ice pail experiment)



- A positively charged metal sphere, S is lowered into a metal can (without touching it) connected to a gold leaf electroscope. The leaf of the electroscope diverges
- S is withdrawn, the leaf of the electroscope collapses
- S is again lowered inside the metal (without touching it), the leaf of the electroscope diverges to the same extent as before.
- S is then allowed to touch the can. The divergence of the leaf remains unchanged
- S is withdrawn and on testing , it is found to have no charge
- There must have been charge inside the can equal and opposite to the charge on S. since the leaf remains diverged, the charge on the can must be residing on the out side of it. This charge is equal to that which was originally on S

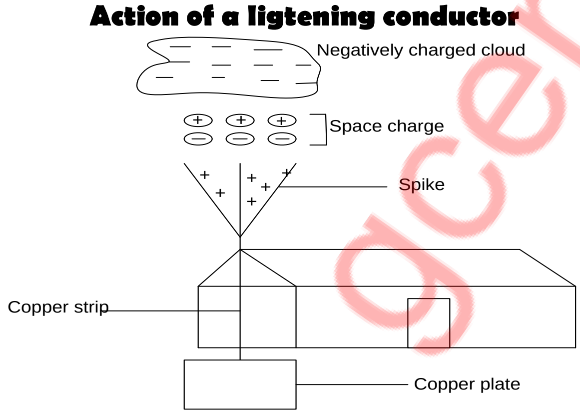
Action at sharp points [Corona discharge]

The high electric field intensity at the sharp points of a charged conductor, ionizes the air molecules around the sharp points. The ions of opposite charge are attracted to the sharp point and neutralize the charge there. This way the conductor loses charge and the process is called **corona discharge.**

Applications of action at sharp points

(a) Lightening Conductor

Action of a ligtening conductor

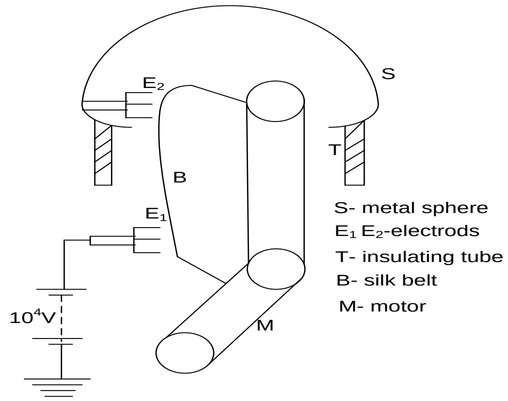


- When a charged cloud passes over a lightening conductor, it induces opposite charge on the spikes of the conductor which results to high electric field intensity
- The high electric field intensity on the spikes ionizes the air around it. Charge similar to those on the spikes is repelled to the clouds and neutralize charge on the cloud, while those opposite are attracted and discharged at the spikes
- This way charge from the cloud is safely conducted to the ground

Effect of Lightening

Clouds in relative motion become charged due to friction. The resulting charge builds up leading to a high p.d between the clouds and the earth. Large discharge currents through the building can cause them to burn

(b) Vander graaf generator



- The electrode E_1 is made 10^4V positive relative to the earth. The high electric field intensity at the sharp points of E_1 ionizes air around it repelling positive charges on to the belt.
- The belt driven by a motor carries this charge into the sphere. As it approaches E_2, it induces negative charge at the spikes of E_2 and positive charge on the sphere
- The high electric field intensity around E_2 ionizes air there, repelling negative charge onto the belt. The negative charge neutralizes positive charge on the belt before it goes over the upper pulley.
- The process is repeated many times until the potential of the sphere is about 10^6V relative to the earth

COULOMBS LAW OF ELECTROSTATICS

It states that the force between any two point charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance of separation of the charges.

$$\boxed{F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}}$$

For vacuum or air

$$\boxed{F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}}$$

$$\varepsilon_o = 8.85x10^{-12}Fm^{-1}$$

$$\frac{1}{4\pi\varepsilon_o} = 9x10^9$$

$$\boxed{F = \frac{9x10^9Q_1Q_2}{r^2}}$$

This law holds for all sign of charges. If Q_1 and Q_2 are unlike then the force is attractive but if they are like then the force is repulsive

Illustration

Example

1. Find the force between two point charges $+4\mu C$ and $-3\mu C$ placed 10cm apart

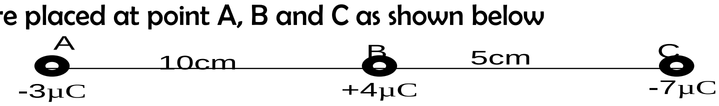
Solution

$$F = \frac{9x10^9Q_1Q_2}{r^2}$$

$$F = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.1^2}$$

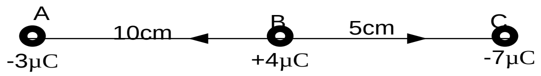
$F = 10.8N$ attractive

2. Three point charges are placed at point A, B and C as shown below



Find the resultant force on the charge at B

Solution



$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$

$F_A = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.1^2}$

$F_A = 10.8N$ attracted towards A

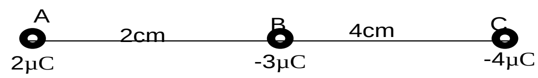
$F_c = \frac{9x10^9x4x10^{-6}x7x10^{-6}}{0.05^2}$

$F_c = 100.8N$ attracted towards C

$F_B = 100.8 - 10.8$

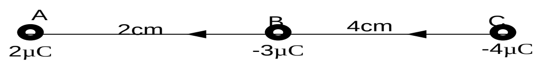
$F_B = 90N$ attracted towards C

3.



Calculate the force on $-3\mu C$

Solution



$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$

$F_A = \frac{9x10^9x2x10^{-6}x3x10^{-6}}{0.02^2}$

$F_A = 135N$ towards left

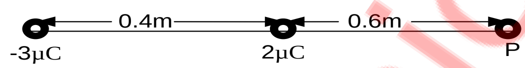
$F_c = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.04^2}$

$F_c = 67.5N$ towards left

$F_B = 135 + 67.5$

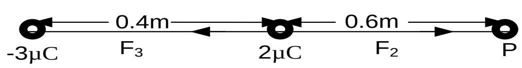
$F_B = 202.5N$ towards left

4.



Find the resultant force acting at point P if a charge of $1\mu C$ is placed at point P.

Solution



$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$

$F_3 = \frac{9x10^9x1x10^{-6}x3x10^{-6}}{1^2}$

$F_3 = 0.027N$ towards left

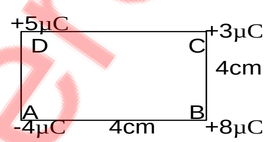
$F_2 = \frac{9x10^9x2x10^{-6}x1x10^{-6}}{0.6^2}$

$F_2 = 0.05N$ towards right

$F_P = 0.05 - 0.027$

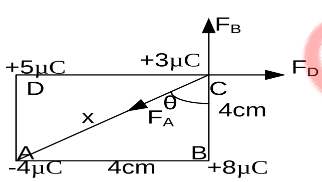
$F_P = 0.023$ towards right

5.



Find the resultant force at C

Solution



$x^2 = 4^2 + 4^2$

$x = \sqrt{32} = 5.66cm$

$tan\theta = \frac{4}{4}$

$\theta = tan^{-1}(1) = 45°$

$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$

$F_A = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.0566^2}$

$F_A = 33.75N$

$F_B = \frac{9x10^9x8x10^{-6}x3x10^{-6}}{0.04^2}$

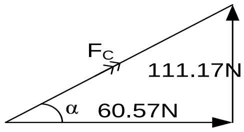
$F_B = 135N$upwards

$F_D = \frac{9x10^9x5x10^{-6}x3x10^{-6}}{0.04^2}$

$F_D = 84.375N$towards right

$F = \begin{pmatrix} -33.7cos45 \\ -33.7sin45 \end{pmatrix} + \begin{pmatrix} 0 \\ 135 \end{pmatrix} + \begin{pmatrix} 84.375 \\ 0 \end{pmatrix}$

$F = \begin{pmatrix} 60.57 \\ 111.17 \end{pmatrix}$

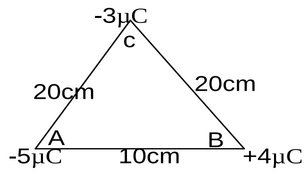


$$F^2 = 60.57^2 + 111.17^2$$
$$F = 126.59N$$

$$tan\alpha = \frac{111.17}{60.57}$$
$$\alpha = tan^{-1}\left(\frac{111.17}{60.57}\right) = 61.42°$$

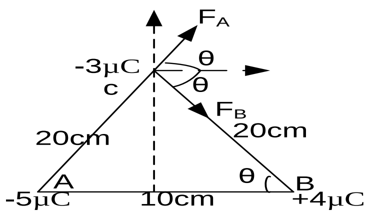
Resultant forces at C is 126.59N at $61.42°$ to the horizontal

6.



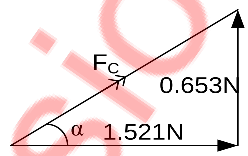
Find the resultant force at c

Solution



$$cos\theta = \frac{5}{20}$$
$$\theta = cos^{-1}\left(\frac{1}{4}\right) = 75.5°$$
$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$
$$F_A = \frac{9x10^9x5x10^{-6}x3x10^{-6}}{0.2^2}$$
$$F_A = 3.375N$$
$$F_B = \frac{9x10^9x4x10^{-6}x3x10^{-6}}{0.2^2}$$

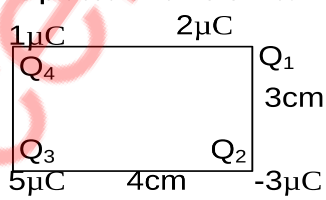
$$F_B = 2.7N$$
$$F = \begin{pmatrix} 3.375cos75.5 \\ 3.375sin75.5 \end{pmatrix} + \begin{pmatrix} 2.7cos75.5 \\ -2.7sin75.5 \end{pmatrix}$$
$$F = \begin{pmatrix} 1.521 \\ 0.653 \end{pmatrix}$$



$$F^2 = 1.521^2 + 0.653^2$$
$$F = 1.655N$$
$$tan\alpha = \frac{0.653}{1.521}$$
$$\alpha = 23.23°$$

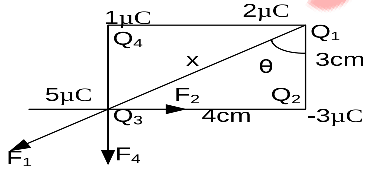
Resultant forces at C is 1.655N at $23.23°$ to the horizontal

7. Four point charges Q_1, Q_2, Q_3 and Q_4 are placed at different corners of rectangle



Find the resultant force at Q_3

Solution



$$x^2 = 4^2 + 3^2$$
$$x = \sqrt{25} = 5cm$$
$$tan\theta = \frac{4}{3}$$

$$\theta = 53.13°$$
$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$
$$F_4 = \frac{9x10^9x1x10^{-6}x5x10^{-6}}{0.03^2}$$

$F_4 = 50N$ **downwards**

$$F_2 = \frac{9x10^9x5x10^{-6}x3x10^{-6}}{0.04^2}$$

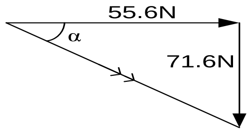
$F_2 = 84.4N$ **towards the right**

$$F_1 = \frac{9x10^9x5x10^{-6}x2x10^{-6}}{0.05^2}$$

$$F_1 = 36N$$

$$F = \begin{pmatrix} 0 \\ -50 \end{pmatrix} + \begin{pmatrix} 84.4 \\ 0 \end{pmatrix} + \begin{pmatrix} -36\sin 53.13 \\ -36\cos 53.13 \end{pmatrix}$$

$$F = \begin{pmatrix} 55.6 \\ -71.6 \end{pmatrix}$$



$$F^2 = 55.6^2 + 71.6^2$$

$$F = 90.66N$$

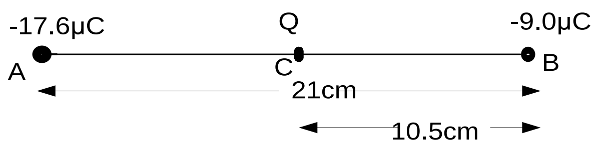
$$\tan\alpha = \frac{71.6}{55.6}$$

$$\alpha = 52.2°$$

Resultant forces at Q_3 is 90.66N at $52.2°$ to the horizontal

8. Two points charges A nad B of $-17.6\mu C$ and $-9.0\mu C$ respectively are placed in vacuum at a distance of 21cm apart. When a third charge C is placed mid way between A and B, the net force on B is zero
   (i) Determine the charge on C
   (ii) Sketch the electric field lines for the above charge distribution

Solution



$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$

$$F_A = \frac{9x10^9 x17.6x10^{-6} x9x10^{-6}}{0.21^2} (\leftarrow)$$

$$F_C = \frac{9x10^9 x9x10^{-6} xQ}{0.105^2} (\rightarrow)$$

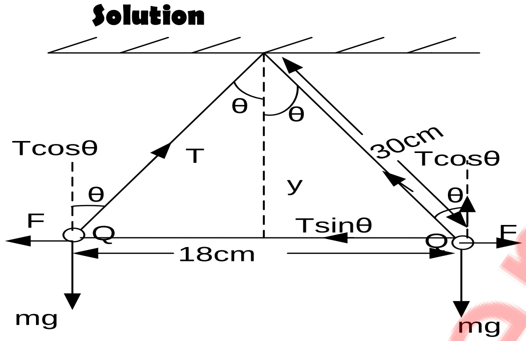
$$F_A = F_C$$

$$\frac{9x10^9 x17.6x10^{-6} x9x10^{-6}}{0.21^2} = \frac{9x10^9 x9x10^{-6} xQ}{0.105^2}$$

$$Q = 4.4x10^{-6}\text{C}$$

9. Two pith balls Pand Q each of mass 0.1g are separately suspended form the same point by threads 30cm long. When the balls are given equal charges, they repel each other and come to rest 18cm apart. Find the magnitude of each charge.

Solution



$$\sin\theta = \frac{9}{30}$$

$$\theta = 17.5°$$

$$(\uparrow)\ T\cos\theta = mg$$

$$T\cos 17.5° = 0.1x10^{-3} x9.81$$

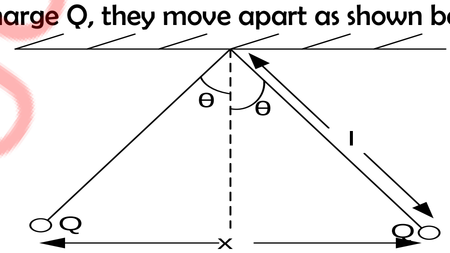
$$T = 1.029x10^{-3}N$$

$$(\rightarrow) T\sin\theta = \frac{Q^2}{4\pi\varepsilon_o x(0.18)^2}$$

$$1.029x10^{-3} x\frac{9}{30} = \frac{9x10^9 xQ^2}{(0.18)^2}$$

$$\boldsymbol{Q = 3.33x10^{-8}C}$$

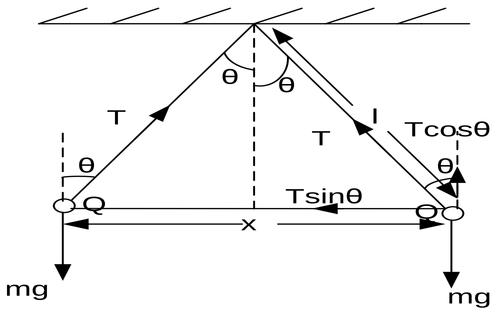
10. Two identical conducting balls of mass m are each suspended in air from a thick of length l, When the two balls are each given identical charge Q, they move apart as shown below



If at equilibrium each thread makes an angle θ with vertical and separated x is given by

$$x = \left(\frac{Q^2 l}{2\pi\varepsilon_o mg}\right)^{1/3}$$

Solution



$$Tsin\theta = \frac{Q^2}{4\pi\varepsilon_o x^2}............(1)$$

$$Tcos\theta = mg\ ...\ ...\ ...\ ...\ ..\ (2)$$

$$eqn1 \div eqn2$$

$$\frac{Tsin\theta}{Tcos\theta} = \frac{\left(\frac{Q^2}{4\pi\varepsilon_o x^2}\right)}{mg}$$

$$x^2 = \frac{Q^2}{4\pi\varepsilon_o mgtan\theta}$$

But for small angles in radians $tan\theta \approx sin\theta$

$$sin\theta = \frac{x}{2}\Big/_{l} = \frac{x}{2l}$$

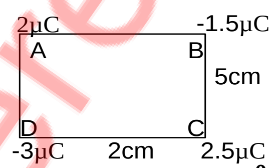
$$x^2 = \left(\frac{Q^2}{4\pi\varepsilon_o mg\frac{x}{2l}}\right)$$

$$x^3 = \left(\frac{Q^2 l}{2\pi\varepsilon_o mg}\right)$$

$$x = \left(\frac{Q^2 l}{2\pi\varepsilon_o mg}\right)^{1/3}$$

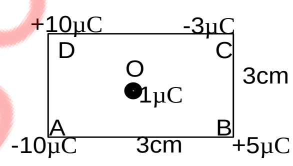
Exercise

1. Two points charges A nad B of $47.0\mu C$ and $24.0\mu C$ respectively are placed in vacuum at a distance of 30cm apart. When a third charge C of $-35.0\mu C$ is placed between A and B at a distance of 20cm from A. find the net force on C **An(-643.2N)**
2. Two point charges of 5μC and 2μC are placed in liquid of relative permittivity 9 at distance 5cm apart. Calculate the force between them. **An(3.998N)**
3. Two insulating metal spheres each of charge $5x10^{-8}C$ are separated by distance of 6cm. What is the force of repulsion if;
   (a) The spheres are in air **An(0.00625N)**
   (b) The spheres are in air with the charge in each sphere doubled and their distance apart is halved **An(0.1N)**
   (c) The two sphere are placed in water whose dielectric constant is 81 **An($7.7x10^{-5}N$)**
4.



Find the resultant force at charge B. **An($12.58N$ at 88.5^0 to the horizontal)**

5.



Calculate the resultant force at charge O, where O is the mid- point of the square **An($523.166N$ at 46.8^0 to the horizontal)**

ELECTRIC FIELD

An electric field is a region within which an electric force is experienced.
Electric fields can be represented by electric field line.

Definition

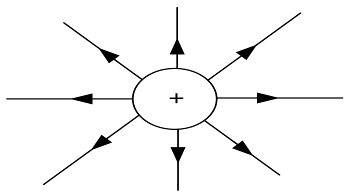
An electric field line is the path taken by a small positive charge placed in the field

Properties of electric field lines

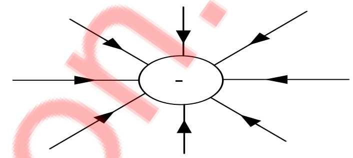
- They originate from positive and end on negative.
- they are in a state of tension which causes them to shorten
- they repel one another side ways
- they travel in straight lines and never cross each other
- the number of filed lines originating or terminating on a charge is proportional to the magnitude of the charge

Field patterns

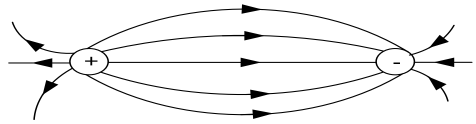
i) Isolated positive charge



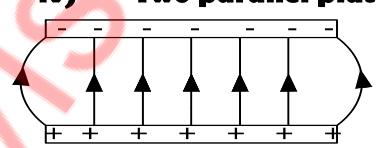
ii) Isolated negative charge



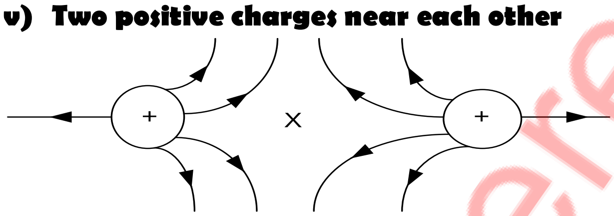
iii) Two equal opposite charges



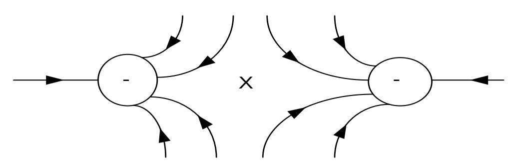
iv) Two parallel plates



v) Two positive charges near each other



vi) Two negative charges near each other



X is Neutral point;

Neutral point is the point in the electric field where the resultant electric force is zero.

Explain what happens to the potential energy as two point charges of the same sign are brought together

- Like charges repel. Work has to be done against the repulsive forces between them to bring them closer
- This work is stored as electric potential energy of the system
- The potential energy of the two like charges therefore increases when the charges are brought closer together

ELECTRIC FIELD INTENSITY/ ELECTRI C FIELD STRENGTH

Electric field intensity at a point is the force experienced by a positive one coulomb charge placed in an electric field.

$E = \frac{F}{Q}$

Generally

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

But in air

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$\varepsilon_o = 8.85x10^{-12}Fm^{-1}$$

$$\frac{1}{4\pi\varepsilon_o} = 9x10^9$$

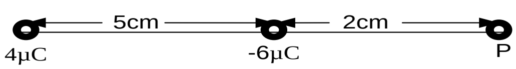
$$E = \frac{9x10^9 Q}{r^2}$$

S.I unit of electric field intensity is NC^{-1} or Vm^{-1}

Electric field intensity is a vector quantity and therefore direction is important. The direction of E is radially outwards if the point charge is positive and radially inwards if the pint charge is negative

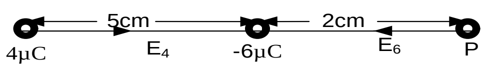
Examples

1.



Find Electric field intensity at P

Solution



$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_4 = \frac{9x10^9 x4x10^{-6}}{0.07^2}$$

$E_4 = 7.347x10^6 NC^{-1}$ towards the right

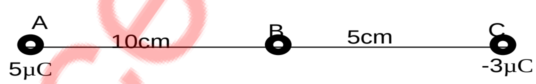
$$E_6 = \frac{9x10^9 x6x10^{-6}}{0.02^2}$$

$E_6 = 1.35x10^8 NC^{-1}$ towards the left

$$E = 1.35x10^8 - 7.347x10^6$$

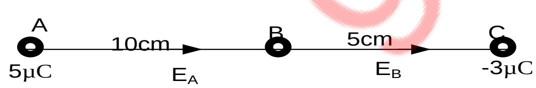
$E = 1.2765x10^6 NC^{-1}$ Towards left

2.



Find electric field intensity at B

Solution



$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_A = \frac{9x10^9 x5x10^{-6}}{0.1^2}$$

$E_A = 4.5x10^6 NC^{-1}$ towards the right

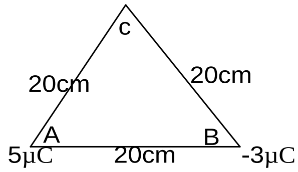
$$E_C = \frac{9x10^9 x3x10^{-6}}{0.05^2}$$

$E_C = 10.8x10^6 NC^{-1}$ towards the right

$$E = 10.8x10^6 + 4.5x10^6$$

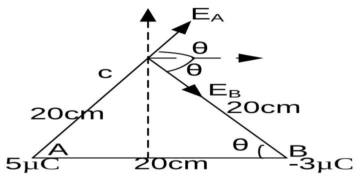
$E = 15.3x10^6 NC^{-1}$ Towards Right

3.



Find electric field intensity at B

Solution



$$cos\theta = \frac{10}{20}$$

$$\theta = cos^{-1}\left(\frac{1}{2}\right) = 60°$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_A = \frac{9x10^9 x5x10^{-6}}{0.2^2}$$

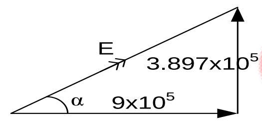
$$E_A = 1.125x10^6 NC^{-1}$$

$$E_B = \frac{9x10^9 x3x10^{-6}}{0.2^2}$$

$$E_B = 6.75x10^5 NC^{-1}$$

$$E = \begin{pmatrix} 1.125x10^6 cos60 \\ 1.125x10^6 sin60 \end{pmatrix} + \begin{pmatrix} 6.75x10^5 cos60 \\ -6.75x10^5 sin60 \end{pmatrix}$$

$$E = \begin{pmatrix} 9.0x10^5 \\ 3.8971x10^5 \end{pmatrix}$$



$$E^2 = (9.0x10^5)^2 + (3.8971x10^5)^2$$

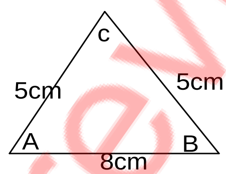
$$E = 9.81x10^5 NC^{-1}$$

$$tan\alpha = \frac{3.8971x10^5}{9.0x10^5}$$

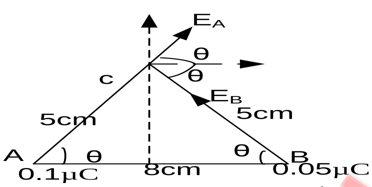
$$\alpha = 23.4°$$

Resultant electric field is $9.81x10^5 NC^{-1}$at $23.4°$ to the horizontal

4. Two point charges A and B of charges $0.10\mu C$and $0.05\mu C$ respectively placed 8cm apart as shown below



Solution



$$cos\theta = \frac{4}{5}$$

$$\theta = cos^{-1}\left(\frac{4}{5}\right) = 36.9°$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$E_A = \frac{9x10^9 x0.1x10^{-6}}{0.05^2}$$

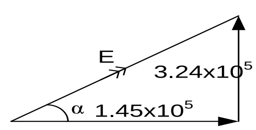
$$E_A = 3.6x10^5 NC^{-1}$$

$$E_B = \frac{9x10^9 x0.05x10^{-6}}{0.05^2}$$

$$E_B = 1.8x10^5 NC^{-1}$$

$$E = \begin{pmatrix} 3.6x10^5 cos36.9 \\ 3.6x10^5 sin36.9 \end{pmatrix} + \begin{pmatrix} -1.8x10^5 cos36.9 \\ 1.8x10^5 sin36.9 \end{pmatrix}$$

$$E = \begin{pmatrix} 1.45x10^5 \\ 3.24x10^5 \end{pmatrix}$$



$$E^2 = (1.45x10^5)^2 + (3.24x10^5)^2$$

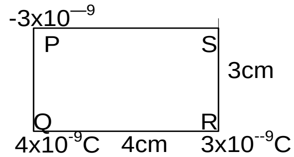
$$E = 3.55x10^5 NC^{-1}$$

$$tan\alpha = \frac{3.24x10^5}{1.45x10^5}$$

$$\alpha = 65.89°$$

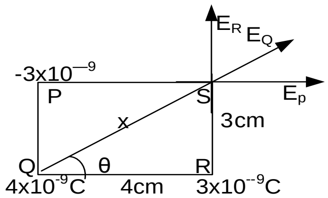
Resultant electric field is $3.55x10^5 NC^{-1}$ at $65.89°$ to the horizontal

5. Three charges$-3x10^{-9}C$, $3x10^{-9}C$ and $4x10^{-9}C$ are placed in a vacuum at the vertices PRQ respectively at rectangle PQRS of sides 3cm by 4cm as shown below



Calculate the resultant electric field strength at S

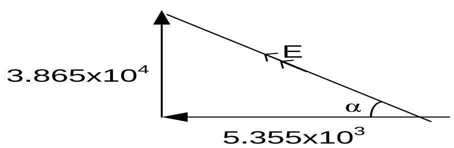
Solution



$$x^2 = 4^2 + 3^2$$
$$x = \sqrt{25} = 5cm$$

$$tan\theta = \frac{3}{4}$$
$$\theta = 36.9°$$

$$E = \begin{pmatrix} -5.355x10^3 \\ 3.865x10^4 \end{pmatrix}$$



$$E^2 = (-5.355x10^3)^2 + (3.865x10^4)^2$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$
$$E_P = \frac{9x10^9 x3x10^{-9}}{0.04^2}$$
$E_P = 1.687x10^4 NC^{-1}$ towards left
$$E_R = \frac{9x10^9 x3x10^{-9}}{0.03^2}$$
$E_R = 3x10^4 NC^{-1}$ upwards
$$E_Q = \frac{9x10^9 x4x10^{-9}}{0.05^2}$$
$$E_Q = 1.44x10^4 NC^{-1}$$

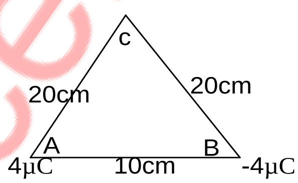
$$E = \begin{pmatrix} -1.687x10^4 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3x10^4 \end{pmatrix} + \begin{pmatrix} 1.44x10^4 cos36.9 \\ 1.44x10^4 sin36.9 \end{pmatrix}$$

$$E = 3.9x10^4 NC^{-1}$$
$$tan\alpha = \frac{3.865x10^4}{5.355x10^3}$$
$$\alpha = 82.11°$$

Resultant electric field is $3.9x10^4 NC^{-1}$ at $82.11°$ to the horizontal

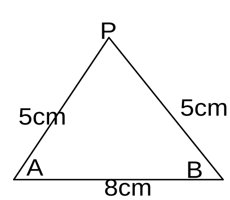
Exercise

1. The electric field intensity at the surface of the earth is about $1.2x10^2 Vm^{-1}$at points towards the centre of the earth. Assuming that the earth is sphere of radius $6.4x10^6 m$. Find the charge held by the earth surface **An($5.46x10^5 C$).**
2. Two point charges $+4\mu C$ and $-4\mu C$ are placed 10cm apart in air.



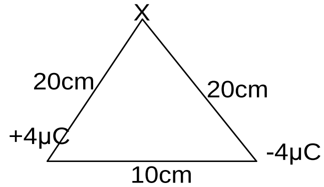
Find the electric field intensity at point C which is a distance of 20cm form each charge.
An($4.5x10^5 NC^{-1}$).

3.



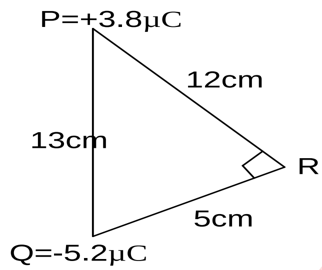
The point charges A and B of charges + 0.10 μc and +0.05μ c are separated by a distance of 8.0 cm along the horizontal as shown above. Find the electric field intensity at P.
An($3.55x10^5 NC^{-1}$ $at\ 66°\ to\ horizontal$).

4. Two point charges +4.0μC and – 4 .0μC are separated by 10.0 cm in air as shown below



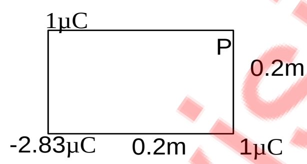
Find the electric field intensity at point x a distance of 20.0 cm from each charge . **An(**$4.5x10^5 NC^{-1}$ **at** $75.52°$ **to the horizontal)**

5. Two point charges $+3.8\mu C$ and $-5.2\mu C$ are in air at points P and Q as shown below**.**



Find the electric field intensity at R. **An(**$1.89x10^7 NC^{-1}$ **at** $7.2°$ **to the horizontal)**

6. Find the electric field strength at P

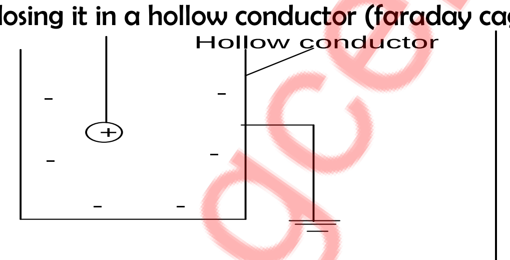


An($142.8NC^{-1}$ **at** $45°$ **to the horizontal)**

7. The electric intensity at the surface of the earth is about 1.2×10^2 V m $^{-1}$ and points towards the centre of the earth. Assuming that the earth is a sphere of radius 6.4×10^6 m, find the charge held by the earth's surface.

ELECTROSTATIC SHEILDING OR SCREEENING

It is the creation of an electrically neutral space in the neighborhood of an electric field however strong it is by enclosing it in a hollow conductor (faraday cage)



- The charged body is enclosed in a hollow conductor which is earthed.
- Equal but opposite charge is induced on the inner walls of the hollow conductor
- Electric field outside will not affect the charged body inside the conductor

ELECTRIC FLUX $\emptyset$

This is the product of electric field strength at any point and area normal to the field

$$\emptyset = AE$$

TOTAL ELECTRIC FLUX

Consider a spherical surface of radius concentric with point charge

$E = \dfrac{Q}{4\pi\varepsilon r^2}$

But $\emptyset = AE$

$$\emptyset = A\frac{Q}{4\pi\varepsilon r^2}$$

But $A = 4\pi r^2$

$$\emptyset = 4\pi r^2 \frac{Q}{4\pi\varepsilon\, r^2}$$

$\boxed{\emptyset = \frac{Q}{\varepsilon}}$ This is called Guass's law of electrostatics

Guass's theorem of electrostatic states that the total electric flux passing normally through a closed surface, whatever its shape is always constant

Electric field intensity due to hollow charged sphere

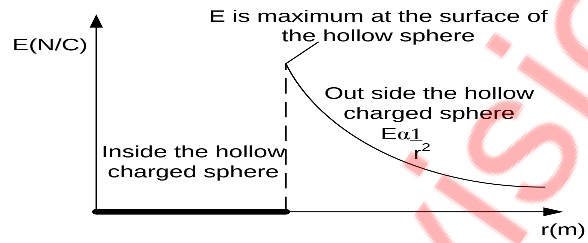
(i) Outside the sphere

Since $E = \frac{Q}{4\pi\varepsilon_o r^2}$ there $E \propto \frac{1}{r^2}$

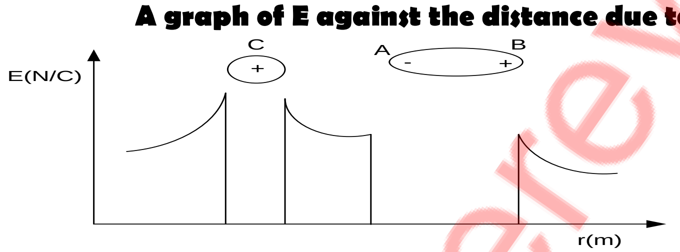
(ii) Inside the sphere

No charge resides on the inside of a hollow conductor therefore $E = 0$

A graph of E against the distance of a charge from a hollow charged sphere



A graph of E against the distance due to charges

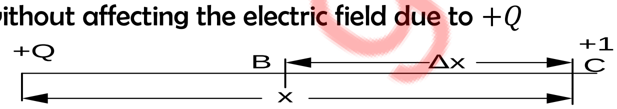


ELECTRIC POTENTIAL

This is the work done in moving a positive one coulomb charge from infinity to a point against an electric field.

Expression for electric potential

Consider $+1C$ charge xm away from $+Q$ being moved from C to B through a small displacement Δx without affecting the electric field due to $+Q$



Force on 1C of charge, $F = \frac{Q}{4\pi\varepsilon x^2}$

Work done to move the charge through Δx against the field is $\Delta w = -F\Delta x$

Total work done to bring the charge form infinity to a point a distance r from the charge of

$$w = \int_{\infty}^{r} -F dx$$

$$= \int_{\infty}^{r} -\frac{Q}{4\pi\varepsilon x^2} dx$$

$$= -\frac{Q}{4\pi\varepsilon}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= -\frac{Q}{4\pi\varepsilon}\left(-\frac{1}{r} - -\frac{1}{\infty}\right)$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon r}}$$

Note: Electric potential is a scalar quantity

Examples:

1. A hollow spherical conductor of diameter $21.4\ cm$ carrying a charge of $6.9\ x\ 10^{-10}$ C is raised to a potential of $50V$. Find the permitivity of the surrounding medium

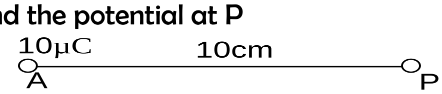
Solution

$$V = \frac{Q}{4\pi\varepsilon r}$$

$$\varepsilon = \frac{6.9x10^{-10}}{4\pi x 10.7x10^{-2}x50}$$

$$\boldsymbol{\varepsilon} = 1.026x10^{-11}Fm^{-1}$$

2. Find the potential at P



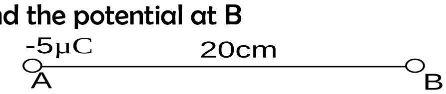
Solution

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \frac{10x10^{-6}x9x10^{9}}{0.1}$$

$$\boldsymbol{V} = 9x10^{5}V$$

3. Find the potential at B



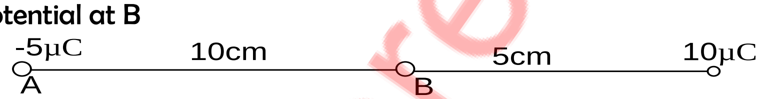
Solution

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \frac{-5x10^{-6}x9x10^{9}}{0.2}$$

$$\boldsymbol{V} = -2.25x10^{5}V$$

4. Find the potential at B

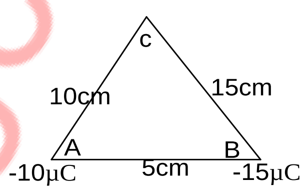


$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = 9x10^{9}x10^{-6}x\left(\frac{-5}{0.1} + \frac{10}{0.05}\right)$$

$$\boldsymbol{V} = 1.35x10^{6}V$$

5.



Find;

(a) Electric potential at C

(b) The potential energy at C if a charge of 10μC is placed at C

(c) Work done to move $-10\mu C$ charge from A to C

Solution

(a) $V = \frac{Q}{4\pi\varepsilon_0 r}$

$$V_c = 9x10^{9}x10^{-6}x\left(\frac{-15}{0.1} - \frac{15}{0.05}\right)$$

$$\boldsymbol{V_c} = -1.8x10^{6}V$$

(b) $\boldsymbol{W = QV}$

$$\boldsymbol{P.e} = 10x10^{-6}\boldsymbol{x} - 1.8x10^{6}$$

$$\boldsymbol{P.e} = -18J$$

(c)

$$V_A = \frac{9x10^9 x - 15x10^{-6}}{0.05}$$

$$V_A = -2.7x10^6 V$$
$$\boldsymbol{W = QV_{AB}}$$
$$\boldsymbol{w} = -10x10^{-6}\boldsymbol{x}(-2.7x10^6 - -1.8x10^6)$$
$$\boldsymbol{W = 9J}$$

Potential due to hollow charged sphere

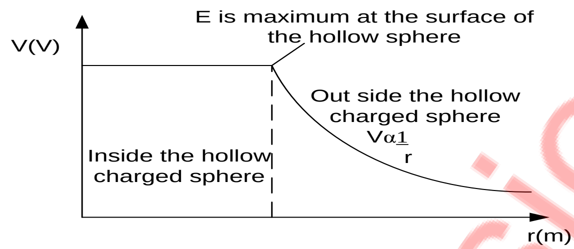
(i) Outside the sphere

Since $V = \frac{Q}{4\pi\varepsilon_o r}$ there $V \propto \frac{1}{r}$

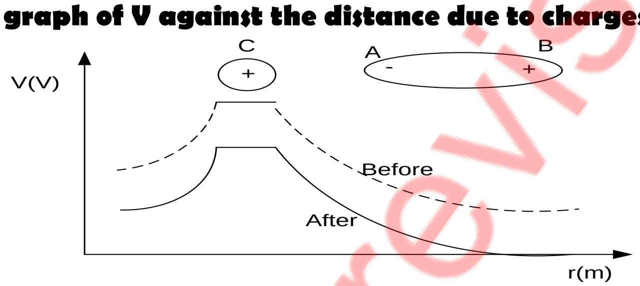
(ii) Inside the sphere

No charge resides on the inside of a hollow conductor$E = 0$, therefore there no work done is done to transfer a charge from the suffice of the sphere to inside hence potential remains constant

A graph of V against the distance of a charge from a hollow charged sphere



A graph of V against the distance due to charges

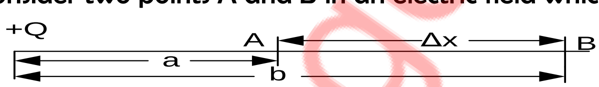


ELECTRIC POTENTIAL DIFFERENCE

Electric potential difference between two points is the work done to transfer +1C of charge form one point to the other against an electric field

Expression for electric potential

Consider two points A and B in an electric field which are Δxm apart



Force on 1C of charge,$F = \frac{Q}{4\pi\varepsilon x^2}$

Work done to move the charge through Δx against the field is $\Delta w = -F\Delta x$

Total work done to move the charge from point A to B

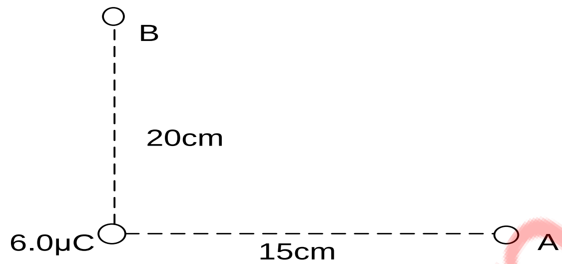
$$w = \int_a^b -F dx$$
$$= \int_a^b -\frac{Q}{4\pi\varepsilon x^2} dx$$
$$= -\frac{Q}{4\pi\varepsilon}\left[-\frac{1}{x}\right]_{\boldsymbol{a}}^{\boldsymbol{b}}$$

$$= -\frac{Q}{4\pi\varepsilon}\left(-\frac{1}{b} - -\frac{1}{a}\right)$$

$$\boxed{V_{AB} = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{b} - \frac{1}{a}\right)}$$

Examples

1. Consider two points A and B at distances of 15.0 cm and 20.0 cm respectively, from a point charge of 6.0 μC as shown below



(i) Find the electric potential difference between A and B

(ii) Calculate the energy required to bring a charge of + 1.0 μC from infinity to point A

Solution

(i) $V = \frac{Q}{4\pi\varepsilon_o r}$

$$V_A = \frac{6.0x10^{-6}x9x10^9}{0.15} = 3.6x10^5V$$

$$V_B = \frac{6.0x10^{-6}x9x10^9}{0.2} = 2.70x10^5V$$

$$\boldsymbol{V_{AB} = V_A - V_B}$$

$$\boldsymbol{V_{AB}} = 3.6x10^5 - 2.7x10^5$$

$$\boldsymbol{V_{AB}} = 9.0x10^4V$$

(ii) $\boldsymbol{W = QV_A}$

$$\boldsymbol{W} = 1.0x10^{-6}x3.6x10^5$$

$$\boldsymbol{W = 0.36J}$$

2. Consider two point charges 9.8 μC and -6.4 μC, are placed as in figure below in airow

Find the potential energy of a charge of 2.5 μC placed at P

Solution

$$x^2 = 6^2 + 8^2$$

$$x = 10cm$$

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

$$V_{9.8} = \frac{9.8x10^{-6}x9x10^9}{0.1} = 8.82x10^5V$$

$$V_{6.4} = \frac{-6.4x10^{-6}x9x10^9}{0.06} = -9.6x10^5V$$

$$\boldsymbol{V_P} = V_{6.4} + V_{9.8}$$

$$\boldsymbol{V_P} = -9.6x10^5 + 8.82x10^5$$

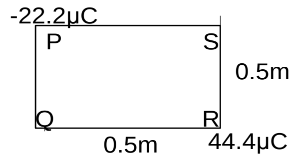
$$\boldsymbol{V_P} = -7.8x10^4V$$

$$\boldsymbol{P.e = QV_P}$$

$$\boldsymbol{P.e = 2.5}x10^{-6}x - 7.8x10^4$$

$$\boldsymbol{P.e = -0.195J}$$

3. The figure below shows point charges $44.4\ \mu C$ and $-22.2\mu C$ placed at the corners of a square of side 0.5m as shown below



Calculate;;

(i) Electric potential at S

(ii) Potential energy of 10μC charge placed at S

Solution

(i) $V = \frac{Q}{4\pi\varepsilon_0 r}$

$$V_R = \frac{44.4x10^{-6}x9x10^9}{0.5} = 7.99x10^5 V$$

$$V_P = \frac{-22.2x10^{-6}x9x10^9}{0.5} = -3.99x10^5 V$$

$$\boldsymbol{V_S = V_R + V_P}$$

$$\boldsymbol{V_S} = 7.99x10^5 + -3.99x10^5$$

$$\boldsymbol{V_S} = 4.0x10^5 V$$

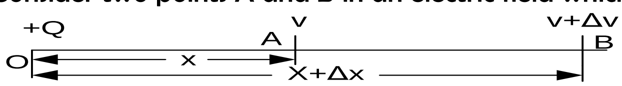
(iii) $\boldsymbol{P.e = QV_S}$

$$\boldsymbol{P.e} = 10x10^{-6}x4.0x10^5$$

$$\boldsymbol{P.e = 4J}$$

ELECTRIC POTENTIAL GRADIENT (relation between E and V)

Consider two points A and B in an electric field which are Δxm apart



If the potential at A is v and that at B is $v + \Delta v$. Then potential difference between A and B is

$$V_{AB} = V_A - V_B$$

$$V_{AB} = v - (v + \Delta v)$$

$$V_{AB} = -\Delta v \ldots\ldots\ldots\ldots\ldots..(1)$$

Work done to move 1C of change from A to B is equal to p.d and is given by

$$V_{AB} = E\Delta x \ldots\ldots\ldots\ldots\ldots..(2)$$

$$E\Delta x = -\Delta v$$

$$E = \frac{-\Delta v}{\Delta x}$$

Limit as $\Delta x \rightarrow 0$

$$\boxed{E = -\frac{dv}{dx}}$$

EQUIPOTENTIAL SURFACES

An equipotential surface is any two dimensional surface over which the electric potential is constant and work done moving charge from one point on surface to another is zero

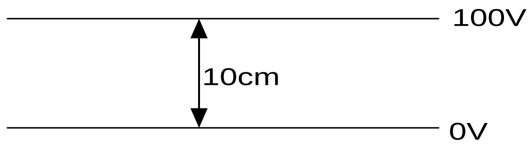
The direction of force is always at right angles to equipotential surfaces. This implies that the is no component of electric field inside the surface

Properties of equipotential surface

- Work done along an equipotential surface is zero
- Electric field intensity along surfaces is zero
- The surfaces are at right angles to the line of force

Examples

1. Calculate the electric field intensity between plates

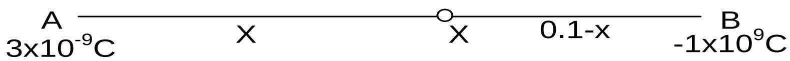


Solution

$$E = \frac{dv}{dx} \quad \Big| \quad E = \frac{100 - 0}{0.1} \quad \Big| \quad E = 1000Vm^{-1}$$

2. Points A and B are 0.1m apart, a point charge of $3x10^{-9}C$ is placed at A and another point charge $-1x10^{-9}C$ is placed at B. X is a point on straight line through A and B but between A and B where electric potential is zero. Calculate the distance AX

Solution



$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

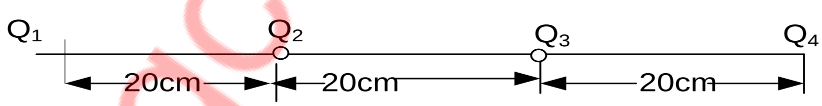
$$0 = 9x10^9x10^{-9}x\left(\frac{3}{x} - \frac{1}{0.1 - x}\right)$$

$$\boldsymbol{x = 0.075m}$$

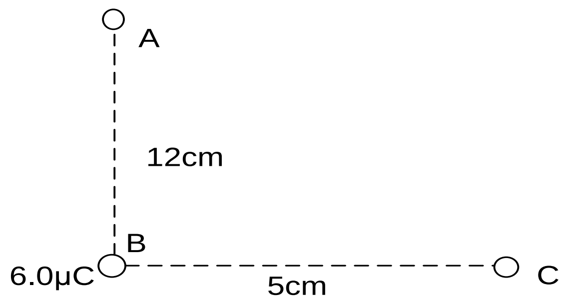
Exercise

1. Two point charge $3x10^{-9}C$ and $-1x10^{-9}C$ are placed at points A and B respectively. A and B are 0.2m apart and x is a point on a straight line through A and B but between A and B. Calculate distance BX for which electric potential at x is zero. **An($0.15m$)**
2. The figure shows charges Q_1, Q_2, Q_3 and Q_4 of $-1\mu C, 2\mu C, -3\mu C$ and $4\mu C$ are arranged on a straight line in vacuum

   (a) Calculate potential energy at Q_2 **An($-1.8x10^{11}J$)**
   (b) what is the significance of the sign of the potential energy above
3. Alpha particles of charge 2e each having kinetic energy $1.0x10^{-12}J$ are incident head on, on a gold nuclide of charge 79e in a gold foil. Calculate the distance of closest approach of an alpha particle and gold foil.($e = 1.6x10^{-19}C$) **An($3.64x10^{-14}m$)**
4. The figure shows charges Q_1, Q_2 and Q_3 of $5\mu C, 6\mu C$,and $-20\mu C$ are arranged on a straight line in vacuum

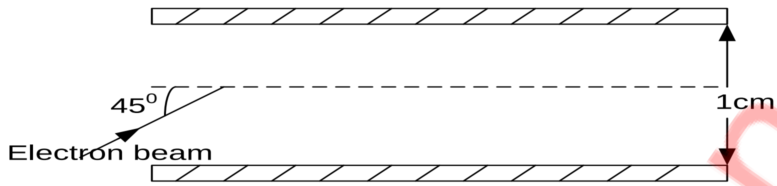


   (a) Calculate electric field intensity midway between Q_1 and Q_2 **An($4.44x10^6Vm^{-1}$)**
   (b) Calculate electric potential midway between Q_1 and Q_2 **An($7.85x10^5V$)**
5. Consider two points A and C at distances of 12.0 cm and 5.0 cm respectively, from a point charge of 6.0 μC situated at B as shown below



Calculate the energy required to bring a charge of + 2.0 μC from A to point C. **An (1.26J)**

6. Two large oppositely charged plates are fixed 1.0 cm apart as shown below. The p.d between the plates is 50V.



An electron beam enters the region between the plates at an angle of 45° as shown. Find the maximum speed the electrons must have in order for them not to strike the upper plate

[Mass of an electron = $9.11\ x\ 10^{-31}$ kg.]

7. A conducting sphere of radius 9.0 cm is maintained at an electric potential of 10kV. Calculate tahe charge on the sphere. **An**$(\mathbf{1x10^{-7}C})$

Uneb 2016

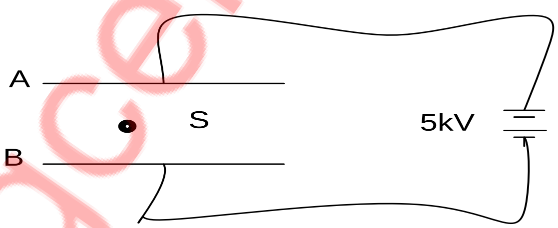
(a) (i) Explain an equipotential surface. (04marks)

(ii) Give an example of an equipotential surface. (01mark)

(b) (i) State **coulomb's law.** (01mark)

(ii) With the aid of a sketch diagram, explain the variation of electric potential with distance from the centre of a charged metal sphere. (03marks)

(iii) Two metal plates A and B, 30cm apart are connected to a 5kV d.c supply as shown below



When a small charged sphere, S, of mass $9.0x10^{-3}kg$ is placed between the plates, it remains stationary. Indicate the forces acting on the sphere and determine the magnitude of the charge on the sphere. (04marks)

(c) (i) Define **electric field intensity** (01mark)

(ii) With the aid of a diagram, explain electrostating shielding. (04marks)

(d) Explain briefly a neutral metal body is attracted to a charged body when broguth near it. (02marks)

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(a) (i) Define **electric potential** (01mark)

(ii) Derive an expression for the electric potential at a point of a distance r, from a fixed charge. (04marks)

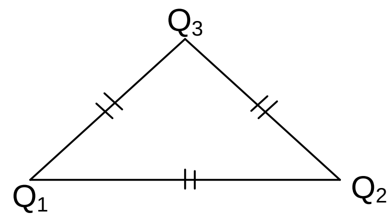
(b) With reference to a charged pear-charged conductor.

(i) Describe an experiment to show the distribution of charge on it. (03marks)

(ii) Show that the surface of the conductor is an equipotential surface. (03marks)

(c) Explain how a lightning conductor protects a house from lightning. (04marks)

(d) Three charges Q_1, Q_2 and Q_3 of magnitude $2\mu C$, $-3\mu C$, and $5\mu C$ respectively are situated at corners of an equilateral triangle of sides 15cm as shown below.



Calculate the net force on Q_3. (05marks)

CAPACITORS

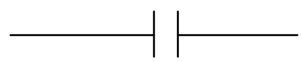
A capacitor is a device which stores charge

A capacitor consists of a pair of oppositely charged plates separated by an insulator called a <u>dielectric.</u>

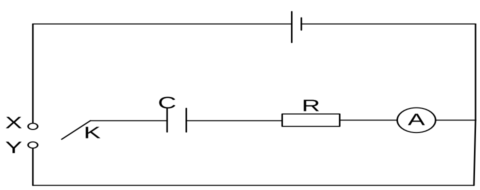
A dielectric is an insulator which breaks down when the potential difference is very high

The dielectric is can be air, oil, glass or a paper

The symbol of a capacitor is

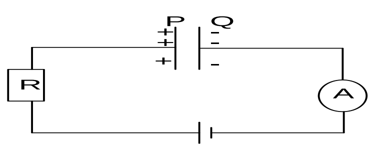


Charging and Discharging process



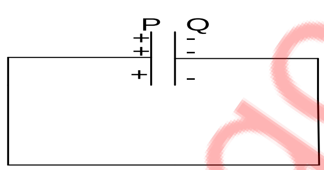
- When switch k is brought to contact x, the capacitor, c charges. Current flowing through the ammeter is initially high but slowly comes to zero with time when the capacitor is fully charged
- If switch k is brought in contact with y, capacitor c is discharged. The current is initially high but eventually comes to zero and in opposite direction to that when the capacitor is being charged.

Explanation of charging process

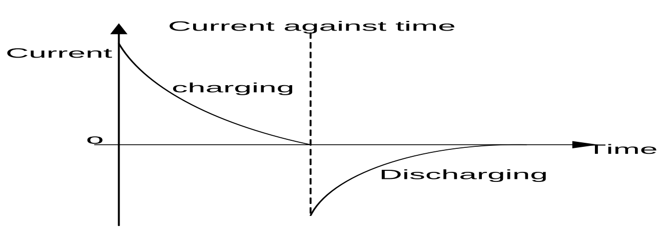


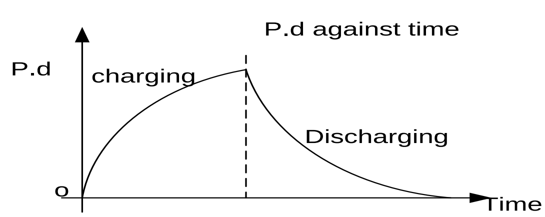
- When the capacitor is connected to a battery, electrons flow from the negative terminal of the battery to the adjacent plate of the capacitor and at the same rate electrons flow from plate P of the capacitor towards the positive terminal of the battery leaving positive charges at P.
- Positive and negative charges therefore appear on the plate and oppose the flow of electrons that cause them
- As charge accumulates the p.d between the plates increase and charge current falls to zero when the p.d between the plates of the capacitor is equal to battery voltage

Explanation of discharging process



Connect a wire from the positive plate to the negative plate. Electrons flow form the negative plate to positive plate through wire until the p.d is zero. The capacitor is fully discharged





Note

Energy changes in charging a capacitor include Chemical energy is changed to heat and electrical energy which is stored in the plates of the capacitor.

Capacitance of capacitor

This is the ratio of the magnitude of charge on either of the plates of a capacitor to the p.d between the plates of the capacitor

$$C = \frac{Q}{V}$$

The S.I unit of capacitance is farad, F

Definition

The farad is the capacitance of the capacitor when one coulomb of charge changes its potential difference by one volt.

Examples

Given the capacitance of capacitor of 4μF and charge on the plate is 5μC. Find the p.d across the plate.

Solution

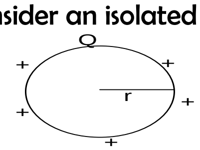
$$C = \frac{Q}{V}$$

$$V = \frac{5x10^{-6}}{4x10^{-6}}$$

$$V = 1.25V$$

Capacitance of an isolated sphere

Consider an isolated sphere of radius r. If the conductor is given charge Q, then its p.d is



$$V = \frac{Q}{4\pi\varepsilon_O r}$$

Where ε_O – is permittivity of free space

$$4\pi\varepsilon_O r = \frac{Q}{V}$$

$$\boxed{C = 4\pi\varepsilon_O r}$$

Example

Calculate the capacitance of the earth given that the radius of the earth is $6.4x10^6$m

Solution

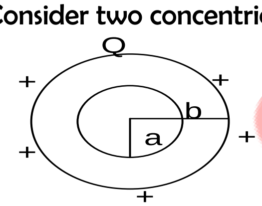
$C = 4\pi\varepsilon_O r$

$C = 4x3.14x8.85x10^{-12}x6.4x10^6$

$C = 7.12x10^{-4}F$

Capacitance of concentric spheres

Consider two concentric sphere A and B each of radius a and b respectively.



$$V = \frac{Q}{4\pi\varepsilon_O}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\frac{V}{Q} = \frac{1}{4\pi\varepsilon_O}\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{V} = 4\pi\varepsilon_O\left(\frac{ab}{b-a}\right)$$

$$C = 4\pi\varepsilon_O\left(\frac{ab}{b-a}\right)$$

Example

1. Find the capacitance of concentric spheres of radius 9cm and 10cm. Given that $\varepsilon_O = 8.85x10^{-12}Fm^{-1}$

   Solution

   $$C = 4\pi\varepsilon_O\left(\frac{ab}{b-a}\right)$$

   $$C = 4x3.14x8.85x10^{-12}\left(\frac{0.1x0.09}{0.1-0.09}\right)$$

   $$C = 1.0x10^{-10}F$$

2. Given two concentric sphere of radius 5cm and 2cm separated by material of permittivity $8.0x10^{-11}Fm^{-1}$. Calculate it capacitance

Solution

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$
$$C = 4x3.14x8.0x10^{-11}\left(\frac{0.02x0.05}{0.05-0.01}\right)$$
$$C = 3.352x10^{-11}F$$

Capacitance of a parallel plate capacitor

Consider two parallel plate of capacitors each having charge Q and an area A separated by a distance d by a dielectric of permittivity ε .

Total electric flux $\emptyset$ through the surface is given by: $\emptyset = AE$......................(1)

Where E is electric field intensity

From Gauss law $\emptyset = \frac{Q}{\varepsilon}$..................................(2)

Equating (1) and (2) $\frac{Q}{\varepsilon} = AE$

But $E = \frac{V}{d}$

$$\frac{Q}{\varepsilon} = A\frac{V}{d}$$
$$\frac{Q}{V} = \frac{\varepsilon A}{d}$$
$$\boxed{C = \frac{\varepsilon A}{d}}$$

For parallel plate capacitor placed in vacuum or air $\boxed{C = \frac{\varepsilon_0 A}{d}}$

Example

4. Calculate the capacitance of a parallel capacitor whose plates are 10 cm by 10 cm separated b an air gap of 5 mm

Solution

$$C = \frac{\varepsilon_0 A}{d}$$
$$C = \frac{8.85x10^{-12}x0.1x0.1}{0.005}$$
$$C = 1.77x10^{-11}F$$

5. A parallel plate capacitor consists of two separate plates each of size 25cm and 3.0mm apart. If a p.d of 200V is applied to the capacitor. Calculate the charge in the plates

Solution

$$C = \frac{\varepsilon_0 A}{d}$$
$$C = \frac{8.85x10^{-12}x0.25x0.25}{0.003}$$
$$C = 1.854x10^{-10}F$$
$$C = \frac{Q}{V}$$
$$Q = 1.854x10^{-10}x200$$
$$Q = 3.708x10^{-8}C$$

6. The plates of a parallel plate capacitor each of area 2.0 cm^2 are 5 mm apart. The plates are in vacuum and a potential difference of 10,000V is applied across the capacitor. Find the magnitude of the charge on the capacitor.

RELATIVE PERMITIVITY / DIELECRTIC CONSTANT

It is defined as the ratio of capacitance of a capacitor when the insulating material (dielectric) between its plates to the capacitance of the same capacitor with a vacuum between it plates

$\varepsilon_r = \frac{C}{C_0}$................(1)

$$\boxed{C = \varepsilon_r C_0}$$

But

$C = \frac{\varepsilon A}{d}$ and $C = \frac{\varepsilon_0 A}{d}$ put into (1)

$$\varepsilon_r = \frac{\left(\frac{\varepsilon A}{d}\right)}{\left(\frac{\varepsilon_0 A}{d}\right)}$$

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$............(2)

$$\boxed{\varepsilon = \varepsilon_r\varepsilon_0}$$

Relative permittivity can also be defined as the ratio of the permittivity of a material to permittivity of free space

Examples

1. A parallel plate capacitor was charged to 100V and then isolated. When a sheet of a dielectric is inserted between its plates, the p.d decreases to 30V. Calculate the dielectric constant of the dielectric.

Solution

By law of conservation of charge

$Q_0 = Q$

$C_0V_0 = CV$

$\frac{V_0}{V} = \frac{C}{C_0}$

$\varepsilon_r = \frac{100}{30}$

$\varepsilon_r = 3.33$

2. A $2\mu F$ capacitor that can just withstand a p.d of 5000V uses a dielectric with a dielectric constant 6 which breaks down if the electric field strength in it exceeds $4x10^7 Vm^{-1}$. Find the;
   (i) Thickness of the dielectric
   (ii) Effective area of each plate
   (iii) Energy stored per unit volume of dielectric

Solution

(i) $E = \frac{V}{d}$

$4x10^7 = \frac{5000}{d}$

$d = 1.25x10^{-4}m$

(ii) $C = \frac{\varepsilon A}{d}$

$\varepsilon = \varepsilon_r\varepsilon_o$

$2x10^{-6} = \frac{6x\ 8.85x10^{-12}xA}{1.25x10^{-4}}$

$A = 4.71m^2$

(iii) $\frac{Enegry}{volume} = \frac{\frac{1}{2}CV^2}{Ad}$

$= \frac{\frac{1}{2}x2x10^{-6}x5000^2}{4.71x1.25x10^{-4}}$

$= 4.246x10^4 Jm^{-3}$

3. A parallel plate capacitor has an area of $100cm^2$, plate separation of 1cm and charged initially with the p.d of 100V supply, it is disconnected and a slab of dielectric 0.5cm thick and relative permittivity 7 is then placed between plates.
   (a) Before the slab was inserted calculate;
      (i) Capacitance
      (ii) Charge on the plates
      (iii) Electric field strength in the gap between plates
   (b) After the dielectric was inserted, find;
      (i) Electric field strength
      (ii) P.d between the plates
      (iii) capacitance

Solution

(a) (i) $C = \frac{\varepsilon_o A}{d}$

$C = \frac{8.85x10^{-12}x100x10^{-4}}{1x10^{-2}}$

$C = 8.85x10^{-12}F$

(ii) $Q = CV$

$Q = 8.85x10^{-12}x100$

$Q = 8.85x10^{-10}C$

(iii) $E = \frac{V}{d}$

$E = \frac{100}{1x10^{-2}}$

$E = 1x10^4 Vm^{-1}$

(b) (i) $E = \frac{Q}{\varepsilon A}$

$\varepsilon = \varepsilon_r\varepsilon_o$

$E = \frac{8.85x10^{-10}}{7x8.85x10^{-12}x100x10^{-4}}$

$E = 1.43x10^3 Vm^{-1}$

(ii) $E = \frac{V}{d}$

$1.43x10^3 = \frac{V}{0.5x10^{-2}}$

$V = 7.15V$

(iii) $C = \frac{\varepsilon A}{d}$

$\varepsilon = \varepsilon_r\varepsilon_o$

$C = \frac{7x\ 8.85x10^{-12}x100x10^{-4}}{0.5x10^{-2}}$

$C = 1.24x10^{-10}F$

DIELECRTIC STRENGHT

It is the maximum electric field intensity an insulator can with stand without conducting
Or Is the maximum potential gradient an insulator can with stand without conducting

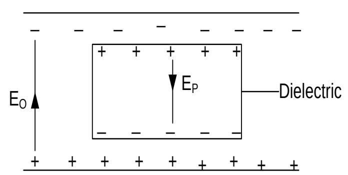
USES OF DIELECTRIC

- It should increase the capacitance of a capacitor
- It is used to separate the plates of a capacitor
- It reduces the chance of dielectric breakdown

QUALITIES OF GOOD DIELECTRIC

- It should have a large dielectric constant
- It should have high dielectric strength

ACTION OF DIELETRIC



- The molecules of the insulator get polarized. Charge inside the material cancel each other's influence but the surfaces adjacent to the plates develop charge opposite to that on the near plate.
- Since charges are bound, electric field intensity, E_P develops between the opposite faces of the insulator in opposition to the applied field, E_O
- The resultant electric field intensity between the plates is thus reduced. But electric field intensity, $E = \frac{V}{d}$ thus p.d between the plates reduces, since capacitance, $C = \frac{Q}{V}$ hence capacitance increases

Note:

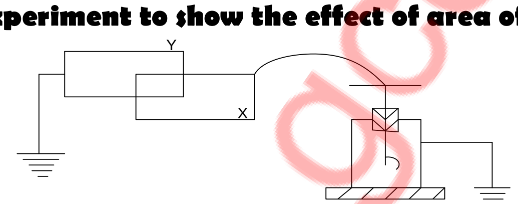
If a conductor instead a dilectric is palced between the plates of a charged capacitor, charge reduces to zero on the plates. This is because electrons move form the negative palet to the positive plate to neutralize the positive charge

FACTORS THAT AFFECT CAPAITANCE OF A CAPACITOR

Capacitance of a capacitor is affected by;

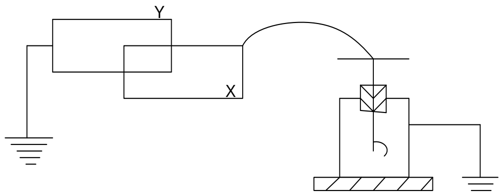
(i) Area of overlap of the plates
(ii) Distance of separation of the plates
(iii) Dielectric

Experiment to show the effect of area of overlap on capacitance



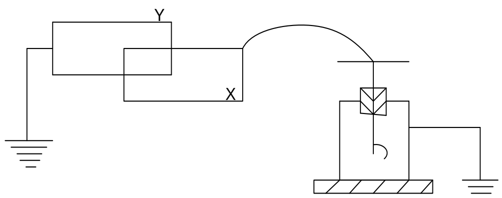
- Plate x is charged and divergence of the leaf of the electroscope noted
- Plate y is then displaced upwards relative to x and the divergence of the leaf of the electroscope is seen to increase
- The p.d between the plates has increased. Since $C = \frac{Q}{V}$, capacitance has decreased with decrease in area and $C \propto A$

Experiment to show the effect of plate separation on capacitance



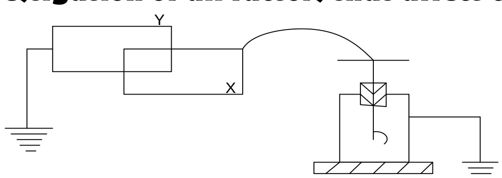
- XY are metal plates near each other but not touching.
- Plate x is charged and divergence of the leaf of the electroscope noted
- Plate y is then moved closer to x and the divergence of the leaf of the electroscope is seen to decrease
- The p.d between the plates has decreased. Since $C = \frac{Q}{V}$, capacitance has increased with decrease plate separation and $C \propto \frac{1}{d}$

Experiment to show the effect of dielectric on capacitance



- XY are metal plates near each other but not touching.
- Plate x is charged and divergence of the leaf of the electroscope noted
- Insert a dielectric between the plates and the divergence of the leaf of the electroscope is seen to decrease
- The p.d between the plates has decreased. Since $C = \frac{Q}{V}$, capacitance has increased and $C \propto \varepsilon$

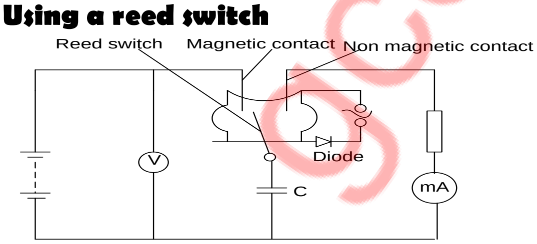
Investigation of all factors that affect capacitance of a parallel plate capacitor



- Plate x is charged and divergence of the leaf of the electroscope noted
- Plate y is then displaced upwards relative to x and the divergence of the leaf of the electroscope is seen to increase. The p.d between the plates has increased. Since $C = \frac{Q}{V}$, capacitance has decreased with decrease in area and $C \propto A$
- Plate y is now restored to its intial position. Plate y is then moved closer to x and the divergence of the leaf of the electroscope is seen to decrease. The p.d between the plates has decreased. Since $C = \frac{Q}{V}$, capacitance has increased with decrease plate separation and $C \propto \frac{1}{d}$
- The plates are restored, an insulator Inserted between the plates. Divergence of the leaf decreases. Since $C = \frac{Q}{V}$, capacitance has increased and $C \propto \varepsilon$

Measurement of capacitance

(a) Using a reed switch



- The circuit is connected as above
- The switch is closed, the microameter reading I is taken together with the voltmeter reading
- Knowing the frequency f of the A.C in the read switch circuit, the capacitance of the capacitor is calculated form

$$C = \frac{I}{fV}$$

Example

1. A capacitor filled with a dielectric is charged and then discharged through a milliameter. The dielectric is then withdrawn half way and the capacitor charged to the same voltage, and discharged through the milliammeter again, show the relative permitivity, ε_r of the dielectric is given by

$$\varepsilon_r = \frac{I}{2I^1 - I}$$

Where I, and I^1 are the readings of the milliammeter respectively

Solution

$$I = cfV$$

$$I = \frac{\varepsilon A}{d} fV \dots\dots\dots\dots (i)$$

When the dielectric is withdrawn half way, the area is halved and both portions one with a dielectric and the other with out a dielectric cont1ribute to current.

$$I^1 = \frac{\varepsilon AfV}{2d} + \frac{\varepsilon_0 A fV}{2d}$$

$$2\,I^1 = \frac{\varepsilon AfV}{d} + \frac{\varepsilon_0 A fV}{d}$$

$$2\,I^1 = I + \frac{\varepsilon_0 A fV}{d}$$

$$2\,I^1 - I = \frac{\varepsilon_0 A fV}{d} \dots\dots\dots\dots\dots (ii)$$

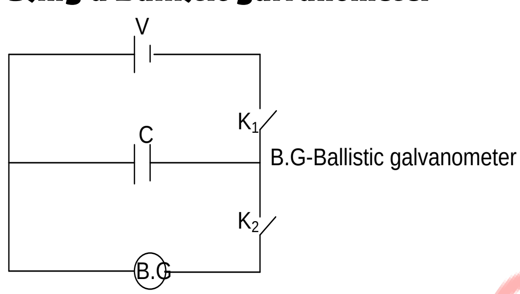
$$eq\ (i)\ \div (ii)$$

$$\frac{I}{2\,I^1 - I} = \frac{\varepsilon A f V}{d} \Big/ \frac{\varepsilon_0 A fV}{d}$$

$$\frac{I}{2\,I^1 - I} = \varepsilon / \varepsilon_0 = \varepsilon_r$$

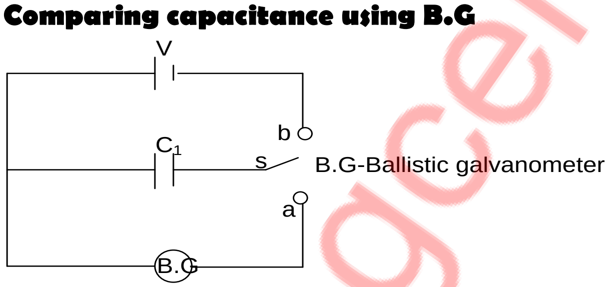
$$\varepsilon_r = \frac{I}{2I^1 - I}$$

(b) Using a Ballistic galvanometer



- The circuit is connected as shown above
- First with a standard capacitor of capacitance, C_s switch K_1 is closed and after a short time it is opened
- Switch K_2 is now closed and the deflection of B.G, θ_s is noted
- The capacitor is replaced with the test capacitor of capacitance C and the procedure repeated. The deflection, θ of B.G is noted
- Capacitance, C is calculated from $C = \frac{\theta}{\theta_s} C_s$

Comparing capacitance using B.G

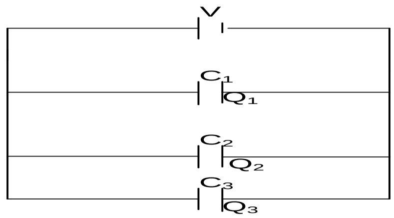


- The capacitor of capacitance C_1 is charged by connecting s to b. After sufficiently charging, s is now connected to a. The deflection θ_1 of the B.G is noted
- The capacitor of capacitance C_1 is then replaced by one of capacitance C_2
- The capacitor is charged by connecting s to b. It is then discharged through a B.G by connecting s to a. The deflection θ_2 of the B.G is noted

$$Then\ \frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

CAPACITOR NETWORKS

(a) Capacitors in parallel



For capacitors connected in parallel p.d across the plate of capacitors is the same

$Q = Q_1 + Q_2 + Q_3$

But $Q_1 = C_1V, Q_2 = C_2V$ and $Q_3 = C_3V$

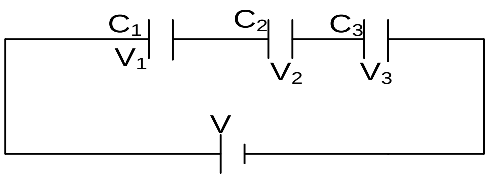
$Q = C_1V + C_2V + C_3V$

$\frac{Q}{V} = C_1 + C_2 + C_3$

$but\ \frac{Q}{V} = C$

$$\boxed{C = C_1 + C_2 + C_3}$$

(b) Capacitors in series



For capacitors connected in series charge stored on the plates of capacitor is the same

$$V = V_1 + V_2 + V_3$$

But $V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}$ and $V_3 = \frac{Q}{C_3}$

$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

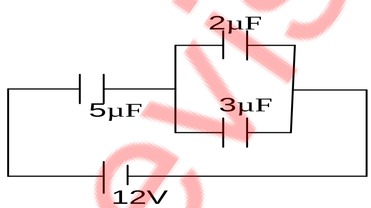
$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$but\ \frac{V}{Q} = \frac{1}{C}$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Examples

1.



A battery of emf 12V is connected across a system of capacitor. Calculate the total energy stored in capacitor network.

Solution

$C_P = 2\mu F + 3\mu F$

$C_P = 5\mu F$

$\frac{1}{C} = \frac{1}{C_5} + \frac{1}{C_P}$

$\frac{1}{C} = \frac{1}{2} + \frac{1}{5}$

$\frac{1}{C} = \frac{5 + 2}{2x5}$

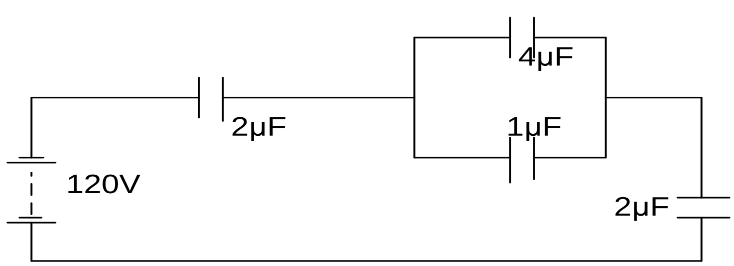
$C = 2.5\mu F$

$Energy\ stored = \frac{1}{2}CV^2$

$Energy\ stored = \frac{1}{2}x2.5x10^{-6}x12^2$

$Energy\ stored = 1.8x10^{-4}J$

2.



The diagram above shows a network of capacitors connected to a 120V supply. Calculate the;

(i) Charge on the 4μF capacitor

(ii) Energy stored in 1 μF capacitor

Solution

$$C_P = 4\mu F + 1\mu F$$
$$C_P = 5\mu F$$
$$\frac{1}{C} = \frac{1}{C_2} + \frac{1}{C_P} + \frac{1}{C_2}$$
$$\frac{1}{C} = \frac{1}{2} + \frac{1}{5} + \frac{1}{2}$$
$$\frac{1}{C} = \frac{6}{5}$$
$$C = \frac{5}{6}\mu F$$

Total charge flowing in the circuit, $Q = CV$

$$Q = \frac{5}{6}x10^{-6}x120$$
$$Q = 1.0x10^{-4}C$$

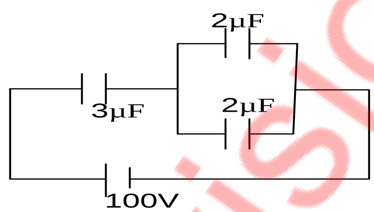
p.d across the paralle combination; $V_p = \frac{Q}{C}$

$$V = \frac{1.0x10^{-4}}{5x10^{-6}}$$
$$V = 20V$$

Charge on 4μF Capacitor $Q_4 = CV$

$$Q_4 = 4x10^{-6}x20$$
$$Q_4 = 8.0x10^{-5}C$$
$$Energy\ stored\ 1\mu F\ Capacitor = \frac{1}{2}CV^2$$
$$Energy\ stored = \frac{1}{2}x1x10^{-6}x20^2$$
$$Energy\ stored = 2x10^{-4}J$$

3.



Calculate energy stored in a system of capacitors, if the space between the 3μF is filled with an insulator of dielectric constant 3 and capacitors are fully charged

Solution

$$C_P = 2\mu F + 2\mu F$$
$$C_P = 4\mu F$$
$$C_3{}' = \boldsymbol{\varepsilon_r} C_0$$
$$C_3{}' = 3x3\mu F$$
$$C_3{}' = 9\mu F$$
$$\frac{1}{C} = \frac{1}{C_3{}'} + \frac{1}{C_P}$$
$$\frac{1}{C} = \frac{1}{9} + \frac{1}{4}$$
$$\frac{1}{C} = \frac{4+9}{9x4}$$
$$C = \frac{36}{13}\mu F$$
$$Energy\ stored = \frac{1}{2}CV^2$$
$$Energy\ stored = \frac{1}{2}x\frac{36}{13}x10^{-6}x12^2$$
$$Energy\ stored = 13.8x10^{-3}J$$

4. **A** 47μF capacitor is used to power the flash gun of a camera. The average power output of the gun is 4.0kW for a duration of the flash which is 2.0ms. Calculate the;

(i) Potential difference between the terminals of the capacitor immediately before a flash

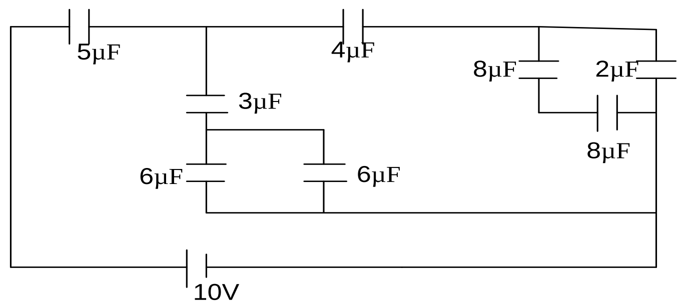
(ii) Maximum charge stored by the capacitor

(iii) Average current provided by the capacitor during a flash

Solution

$$\frac{1}{2}CV^2 = pt$$
$$\frac{1}{2}x47x10^{-6}x(V)^2 = 4x10^3x2x10^{-3}$$
$$\boldsymbol{V = 583.5V}$$
$$\boldsymbol{Q = CV}$$
$$\boldsymbol{Q} = 47x10^{-6}x583.5$$
$$\boldsymbol{Q} = 2.74x10^{-2}C$$
$$\boldsymbol{I = \frac{Q}{t}}$$
$$\boldsymbol{I} = \frac{2.74x10^{-2}}{2x10^{-3}}$$
$$\boldsymbol{I = 13.7A}$$

5.



The figure above shows a network of capacitors connected to a 10V battry. Calculate the total energy stored in the network.

Solution

$8\mu F$ and $8\mu F$ are in series

$\frac{1}{C} = \frac{1}{8} + \frac{1}{8}$

$\frac{1}{C} = \frac{8+8}{8x8}$

$C = 4\mu F$

$2\mu F$ is in parallel with $4\mu F$

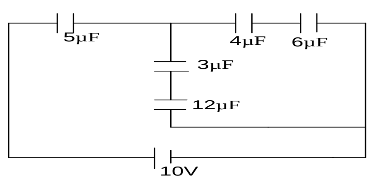
$C = 4 + 2$

$C = 6\mu F$

$6\mu F$ is in parallel with $6\mu F$

$C = 6 + 6$

$C = 12\mu F$



$4\mu F$ and $6\mu F$ are in series

$\frac{1}{C} = \frac{1}{4} + \frac{1}{6}$

$\frac{1}{C} = \frac{4+6}{4x6}$

$C = \frac{12}{5}\mu F$

$3\mu F$ and $12\mu F$ are in series

$\frac{1}{C} = \frac{1}{3} + \frac{1}{12}$

$\frac{1}{C} = \frac{3+12}{3x12}$

$C = \frac{12}{5}\mu F$

$C_P = \frac{12}{5}\mu F + \frac{12}{5}\mu F$

$C_P = \frac{24}{5}\mu F$

$\frac{1}{C} = \frac{1}{C_5} + \frac{1}{C_P}$

$\frac{1}{C} = \frac{1}{5} + \frac{1}{\left(\frac{24}{5}\right)}$

$\frac{1}{C} = \frac{\frac{24}{5} + 5}{\frac{24}{5}x5}$

$C = \frac{120}{49}\mu F$

$Energy\ stored = \frac{1}{2}CV^2$

$E = \frac{1}{2}x\frac{120}{49}x10^{-6}x12^2$

$E = 1.224x10^{-4}J$

ENERGY STORED IN A CAPACITOR

Suppose the p.d between the plates at some instant was V. When a small charge of $+\delta q$ is transferred from the negative plate to the positive plate, the p.d increases by δv.

Work done to transfer charge,

$$\delta w = (V + \delta v)\delta q$$

$$\delta w \approx V\delta q$$

But $V = \frac{q}{C}$

$$\delta w = \frac{q}{C}\delta q$$

Total work done to charge the capacitor to Q is

$$W = \int_0^Q \frac{q}{C}dq$$

$$= \frac{1}{2}\frac{Q^2}{C}$$

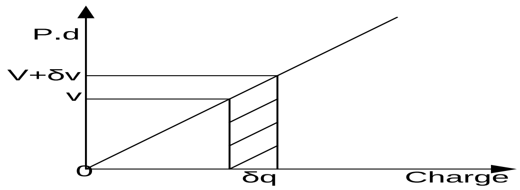
$$but\ Q = CV$$

$$\boxed{W = \frac{1}{2}CV^2}$$

ALTERNATIVELY

From $q = CV$

V is proportional to q, giving a graph of v against q



Area of the shaded part $= \frac{1}{2}(V + V + \delta v)\delta q$

=work done to increase charge on the capacitor from $q = 0$ to $q = Q$

Work done w= $average\ voltage\ x\ charge$

$$= \frac{1}{2}(0 + V)Q$$

$$= \frac{1}{2}QV$$

$$but\ Q = CV$$

$$\boxed{W = \frac{1}{2}CV^2}$$

JOINING TWO CAPACITORS

When two capacitors are joined together;

- Charge flows until p.d across the capacitors is the same
- Total charge on the circuit is conserved
- Capacitors are in parallel ie $\boldsymbol{C = C_1 + C_2}$

NOTE

There is loss of energy when capacitors are joined together. This is because charge flows until the p.d across the capacitor is the same. The flow of charge results in heating of the wire and hence loss in energy

Examples

b. A 5μF capacitor is charged by a 40V supply and then connected to an un charged 20μF capacitor. Calculate;

(i) Final p.d across each capacitor
(ii) Final charge on each
(iii) Energy lost

Solution

(i) $C = C_1 + C_2$

$C = 5x10^{-6} + 20x10^{-6}$

$C = 2.5x10^{-5}F$

Charge before= charge after connection

$\boldsymbol{Q_1 + Q_2 = Q}$

$\boldsymbol{C_1V_1 + C_2V_2 = CV}$

$5x10^{-6}\boldsymbol{x40} + 20x10^{-6}x0 = 2.5x10^{-5}V$

$V = 8V$

(ii) $\boldsymbol{Q = Q_1 + Q_2}$

$\boldsymbol{Q = C_1V_1 + C_2V_2}$

$Q = 5x10^{-6}\boldsymbol{x40} + 20x10^{-6}x0$

$\boldsymbol{Q = 2x10^{-4}C}$

OR

$\boldsymbol{Q = CV}$

$Q = 2.5x10^{-5}x8$

$Q = \boldsymbol{2x10^{-4}C}$

(iii) Energy lost = energy before−energy after

$= \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) - \frac{1}{2}CV^2$

$= \left(\frac{1}{2}x5x10^{-6}x\boldsymbol{40^2} + \frac{1}{2}x20x10^{-6}\boldsymbol{x0^2}\right)$

$- \left(\frac{1}{2}x2.5x10^{-5}x\boldsymbol{8^2}\right)$

$= 0.0032J$

c. A capacitor of 20μF is connected across 50V battery supply. When it has fully charged it is then disconnected and joined to capacitor of 40 μF having a p.d of 100V. Calculate;

(i) Effective capacitance after joining
(ii) The p.d on each capacitor
(iii) Energy lost

Solution

(i) $C = C_1 + C_2$

$C = 20x10^{-6} + 40x10^{-6}$

$C = 6.0x10^{-5}F$

(ii) Charge before= charge after connection

$\boldsymbol{Q_1 + Q_2 = Q}$

$\boldsymbol{C_1V_1 + C_2V_2 = CV}$

$20x10^{-6}\boldsymbol{x50} + 40x10^{-6}x100 = 6.0x10^{-5}V$

$V = 83.33V$

(iii) Energy lost = energy before−energy after

$= \left(\frac{1}{2}C_1V_1{}^2 + \frac{1}{2}C_2V_2{}^2\right) - \frac{1}{2}CV^2$

$= \left(\frac{1}{2}x20x10^{-6}x\mathbf{50^2} + \frac{1}{2}x40x10^{-6}\mathbf{x100^2}\right) - \left(\frac{1}{2}x6.0x10^{-5}x\mathbf{83.33^2}\right)$

$= 0.017J$

d. A paralle plate air-capacitor is charged to a potential difference of 20V. It is then connected in parallel with an uncharged capacitor of similar dimensions but having ebonite as its dieelectric medium. The potential difference of the combination falls to 15V. Calculate the dielectric constant of the ebonite

Solution

Charge before= charge after connection

$Q_0 = Q$

$C_0V_0 = CV$

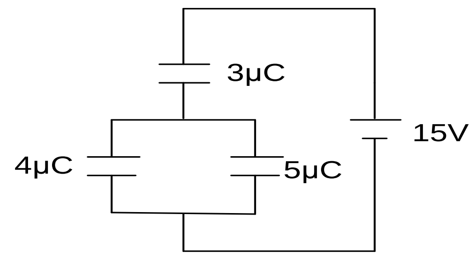
$C_0V_0 = (C_0 + \varepsilon_rC_0)V$

$29 = (1 + \varepsilon_r)15$

$\varepsilon_r = 0.33$

Exercise

1. A capacitor is charged by a 30V d.c supply. When the capacitor is fully charged, it is found to carry a charge of 6.0μC. find the;
   (i) Capacitance of the capacitor
   (ii) Energy stored in the capacitor
   An((i) $2.0x10^{-7}F$, **(ii)** $9.0x10^{-5}J, 1.5x10^{-2}J$**)**
2. Two capacitors of 2μF and 3μF are charged to a p.d of 50V and 1000V respectively. Calculate;
   (i) Charge stored in each
   (ii) Energy stored in each
   (iii) Suppose capacitors are now joined with plate of the same charge connected together. Find the energy lost in the circuit
   An((i) $1x10^{-4}C, 3x10^{-4}C$ **(ii)** $2.5x10^{-3}J, 1.5x10^{-2}J$**(iii)** $1.5x10^{-3}J$**)**
3. A 100 μF is charged from a supply of 1000V. it is then disconnected and then connected to an uncharged 50 μFcapacitor. Calculate;
   (i) Total energy stored initially and finally in the two capacitors
   (ii) Energy lost **An(50J, 0.333J, 49.667J)**
4. A 20 μF capacitor was charged to 1000V and then connected across an uncharged 60 μF capacitor. Calculate the p.d across a 60 μF capacitor **An(10V)**
5. A 10 μF capacitor was charged to 300V and then connected across an uncharged 60 μF capacitor. Calculate the total energy stored in both capacitors before and after connection. **An(0.45J, 0.064J)**
6. A 60 μF capacitor was charged from a 100V supply and then connected across an uncharged 15 μF capacitor. Calculate the final p.d across the combination and the energy lost **An(80V, 23.7J)**
7. A 20μF capacitor is charged to 40V and then connected across an uncharged 60μF capacitor. Calculate the potential difference across the 60μF capacitor. **An(10V)**
8. An air capacitor of capacitance 400μF. is charged to 180V and the connected across un charged capacitor of capacitance 500μF.
   (i) Find the energy stored in the 500μF capacitor
   (ii) With the two capacitors still connected, a dielectric of dielectric constant 1.5 is inserted between the plates of the 400 μF. capacitor. If the separation between the plates remains the same, find the new p.d across the two capacitors. **An(116J, 65.5J)**
9. A battery of e.m.f 15V is connected across a system of capacitors as shown below .

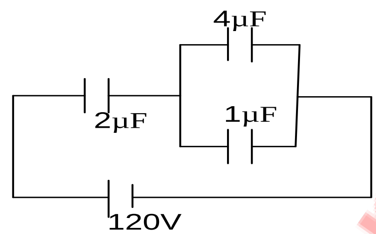


Find the

(i) charge on the 4μF capacitor

(ii) energy stored in the 3μF capacitor

10. Figure 9 shows a network of capacitors connected to a d.c. supply of 120 V.



Calculate the

(i) charge on 4 μF capacitor

(ii) energy stored in 1μF capacitor

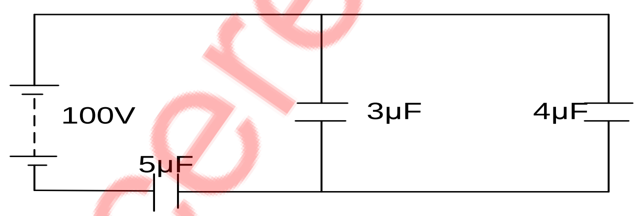
11. A 20 μF capacitor is charged to 40V and then connected across an uncharged 60 μF capacitor. Calculate the potential difference across the 60 μF capacitor

12. A 60μF capacitor is charged from a 100V supply. It is then connected across the terminals of a 15μF uncharged capactiro. Calculate

(i) the final p.d across the combination **An(80V)**

(ii) the difference in the initial and final energies stored in the capacitors and comment on the difference **An(0.06J)**

13.

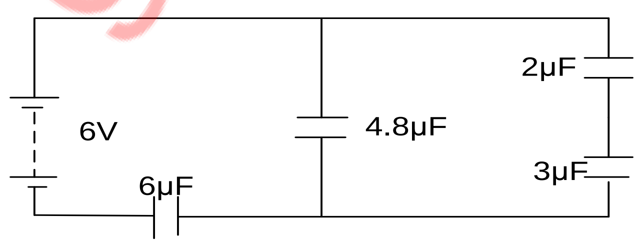


(i) Find the resultant capacitance in the circuit

(ii) Calculate the charge stored in wach capacitor

An($\frac{35}{12}\mu F$, $\frac{35}{12}x10^{-4}C, 1.25x10^{-4}C, 1.67x10^{-4}C$**)**

14.

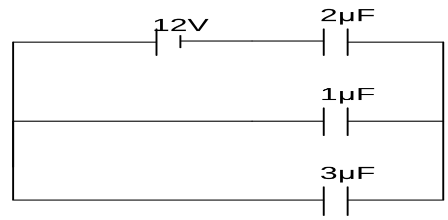


Find;

(i) Effective capacitance

(ii) Energy stored in the 2 capacitor **An(** $3\mu F$, $3.24x10^{-6}J$**)**

15.



Find the energy stored in the capacitor of capacitance 3μF shown in figure above, when the capacitor is fully charged

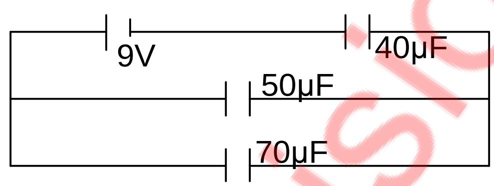
Uneb 2016

(a) (i) What is meant by capacitance of a capacitor. (01mark)

(ii) A parallel plate capacitor is connected across a battery and charged fully. When a dielectric material is now inserted between its plate, the amount of charge stored in the capacitor changes. Explain the change. (04marks)

(iii) Describe an experiment to determine the relative permittivity of a dielectric. (04marks)

(b) A network of capacitors of capacitance 40μF, 50μF, and 70μF is connected to a battery of 9V as shown below.



Calculate;

(i) Charge stored in the $50\mu F$ capacitor (05marks)

(ii) Energy stored in the $40\mu F$ capacitor (03marks)

(c) Explain **corona discharge**. (03marks)

Uneb 2015

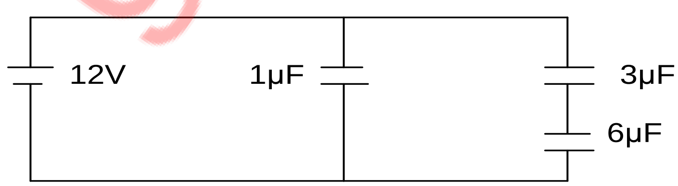
(a) (i) Define **capacitance** of a capacitor. (01mark)

(ii) Describe briefly an experiment to show the effect of placing a sheet of glasss or mica between the plates of a capacitor on capacitance. (05marks)

(b) Describe how capacitance eof a capacitor can be measured using a ballistic galvanometer. (04marks)

(c) Explain briefly how a charged capacitor can be fully discharged. (02marks)

(d) A network of capacitors of capacitance 3μF, 6μF, and 1μF is connected to a battery of 12V as shown below.



Calculate;

(i) Charge stored by each capacitor (05marks)

(ii) Energy stored in the $6\mu F$ capacitor when fully charged (03marks)

CURRENT ELECTRICITY

Current is the rate of flow of electric charge.

If charge Q, coulombs flows through a circuit in a time t seconds, then the current I, amperes is given by

$$I = \frac{Q}{t}$$

$$\boxed{Q = I\,t}$$

The S.I unit of current is Amperes(A) and current is measured using an instrument called an Ammeter.

Submultiples of A:

(i)	$1\,mA = 1\,x\,10^{-3}\,A$	(iii)	$1\,nA = 1\,x\,10^{-9}A$
(ii)	$1\,\mu A = 1\,x\,10^{-6}A$	(iv)	$1\,pA = 1\,x\,10^{-12}A$

Ampere;

Ampere is the current which, if flowing in two straight parallel wires of infinite length placed one meter apart in a vacuum, will produce on each of the wires a force of $2\,x\,10^{-7}$Nm^{-1}.

The S.I unit of charge is coulomb.

Coulomb;

Is the quantity of electricity which passes any point in a circuit in 1 second when a steady current of 1 ampere is flowing.

Examples

1. A charge of 180C flows through a lamp every 2 minutes. What is the electric current in the lamp.

Solution

$$I = \frac{Q}{t} \qquad I = \frac{180}{2\,x\,60} \qquad I = 1.5A$$

2. A charge of $20\,kC$ crosses two sections of a conductor in 1minute. Find the current through the conductor.

Solution

$$I = \frac{Q}{t} \qquad I = \frac{20\,x\,1000}{1\,x\,60} \qquad I = 333.33A$$

POTENTIAL DIFFERENCE ($P.d$)

$P.d$ is defined as the work done in moving one coulomb charge from one point to another a cross a conductor.

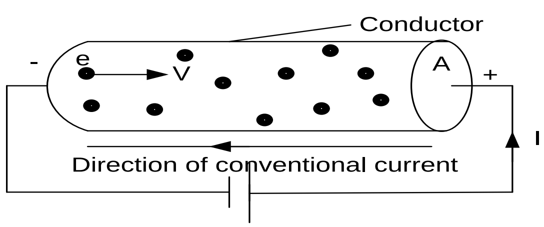
Current will flow through a conductor if there **<u>a potential difference</u>** between the ends of a conductor.

The $S.I$ unit of $P.d$ is volt and $P.d$ is measured using an instrument called a voltmeter

A volt;

A volt is the potential difference between two points when one joule of work is done in transferring one coulomb of charge from one point to another.

Mechanism of electrical conduction in metals



- In metals there are free electrons in random motion. When a battery (cell) is connected across the ends of a metal, an electric field is set up between its ends.
- The conduction electrons are accelerated by the field and gain velocity and energy. On collision with atoms vibrating about fixed

mean positions, they give up some of their energy to the atoms.

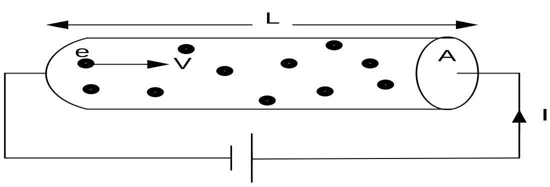
- ❖ The amplitude of vibration of the atoms increases and the temperature of the metal rises. The electric field continuously accelerates the free electrons and on average the electrons drift in the direction of the field with a mean speed.
- ❖ The continuous drift of the electrons in the same direction constitutes an electric current. The current produced is direct current (d.c) because the direction of flow is constant.

Heating Effect of Current

- ❖ When a p.d is applied across the wire, the conduction electrons gain kinectic enrgy form the applied field.
- ❖ As the electrons drift along the wire in a direction opposite to the applied field, they collide with the ions and lose their kinectic energy to the ions.
- ❖ The inos vibrate with increased amplitudes as a result temperature of the wire rises due to collision

Drift velocity of electrons

Consider a conductor of length ***l*** and cross –sectional area ***A*** having ***n*** free electrons per unit volume each carrying a charge, ***e***.



Volume of the conductor = Al

Number of free electrons in the conductor = nAl.

Total charge, ***Q*** of free electron = $nAle$.

Suppose a battery connected across the ends of the conductor causes a total charge *Q* to pass through the conductor in time *t* with average drift velocity, *v*.

The resulting steady current I, is given by

$$I = \frac{Q}{t}$$

$$I = \frac{nAle}{t}$$

But the mean speed, $v = \frac{l}{t}$

$$\boxed{I = nAve}$$

From $I = nAve$

we can write $\frac{I}{A} = nve$. The quantity $\frac{I}{A}$ is called the current density and is denoted by J.

Thus $J = \frac{I}{A}$

Definition: *Current density is the current flowing through a conductor of cross sectional area 1 m².*

Example

(e) A current of 10A flows through a copper wire of area 1mm^2. The number of free electrons per m^3 is 10^{29}. Find the drift velocity of the electron.

Solution

$$v = \frac{I}{nAe}$$

$$v = \frac{10}{10^{29}x1x10^{-6}x1x10^{-6}}$$

$$v = 6.25x10^{-4}m/s$$

(f) A metal wire contains $5x10^{22}$ electrons per cm^3 and has cross sectional area off 1mm^2. If the electrons move along the wire with a mean drift velocity of 1mms^{-1}, Calculate:

(i) current density
(ii) current in the wire.

Solution

(i) current density $= \frac{I}{A} = nev = 5x10^{22}x10^{6}x10^{-3}x1.6x10^{-19} = 8x10^{6}Am^{-2}$

(ii) current = current density x Area, $= 8x10^{6}x10^{-6}$

Conductivity, σ

The conductivity of a material is the reciprocal of its resistivity. It is denoted by σ.

$$\sigma = \frac{1}{\rho}$$

The S.I unit of conductivity is $\Omega^{-1}m^{-1}$

Example:

(a) Calculate the drift velocity of the free electrons in a copper wire of cross-sectional area 1.0 mm^2 when the current flowing through the wire is $2.0\ A$. (Number of free electrons in copper is $1\ x\ 10^{29}\ m^{-3}$)

(b) Explain how the drift velocity of free electrons in a metal conductor carrying a steady current changes when

(i) The p.d between the ends of the conductor is increased

(ii) The temperature of the conductor increases but the current remains unchanged.

Solution:

(a) Using the equation, $I = nAve$

Drift velocity, $v = \frac{I}{nAe}$

$$= \frac{2.0}{(1.0 \times 10^{29})(1.0 \times 10^{-6})(1.6 \times 10^{-19})}$$

$v = 1.25 \times 10^{-4}$ m s^{-1}

(b) (i) The drift velocity increases as the higher p.d would accelerate the free electrons to a higher velocity.

(ii) The drift velocity decreases as the temperature increases. At a higher temperature, the ions in the metal lattice vibrate with greater amplitude, the mean free path decreases and the free electrons are unable to accelerate to a higher velocity. This results in increase of resistance to current flow.

OHM'S LAW

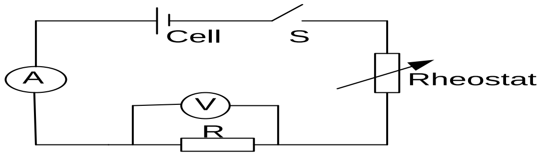
It states that the current flowing through a conductor is proportional to the potential difference across it's ends provided temperature and other physical conditions remain constant.

Ie $V \propto I$ at constant temperature

$$\boxed{V = I\,R}$$

R is resistance , V is p.d, I is current

Verification of ohm's law



A-ammeter, V- voltmeter, R-resistor, S- switch

- ❖ Arrange the apparatus as shown above.
- ❖ Switch is closed and rheostat adjusted so that A and V read suitable values of I and V respectively
- ❖ Ammeter reading I and voltmeter reading V are note and recorded
- ❖ The procedures above are repeated to obtain several values of I and V
- ❖ Tabulate the results in a suitable table
- ❖ A graph of V against I is plotted.
- ❖ A straight line through the origin verifies Ohms law

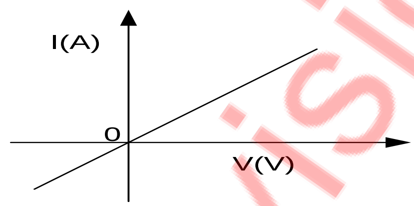
Limitations of ohm's law

- ➢ It does not apply to semiconductors and gases.
- ➢ It is only obeyed if physical conditions like temperature are constant.

Ohmic and non ohmic conductors

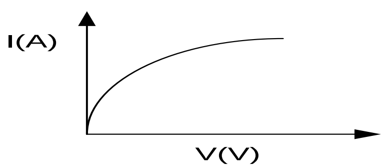
An ohmic conductor is one which obeys ohm's law.
Non ohmic conductor s one which doesnot obey ohm's law.
When we plot I against V between ends of a conductor, the shape of the curve is known as the characteristic of the conductor.

a) Ohmic conductors eg a metal

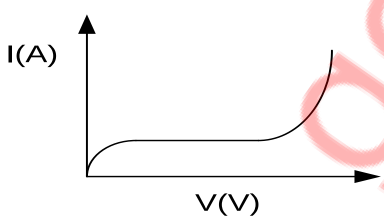


b) Non ohmic conductor

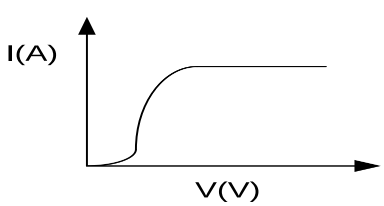
i) Filament lamp



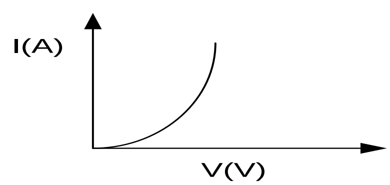
ii) Gas discharge tube



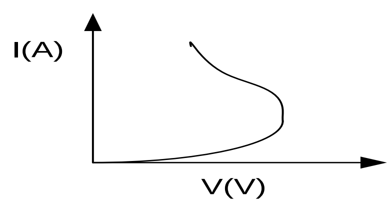
iii) Thermionic diode



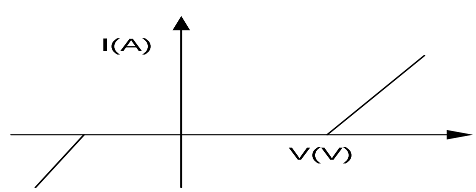
iv) Junction diode



v) Thermistor



vi) Electrolyte eg dilute sulphuric acid

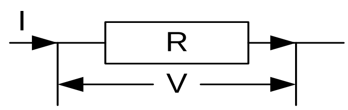


Resistance

This is the opposition of a conductor to the flow of current.
It is measured in ohms (Ω)

OR : Resistance of a conductor is the ratio of the potential difference across its ends to the current flowing through it.

A good conductor has low resistance while a good insulator has high resistance



$R = \frac{V}{I}$ 1

The S.I unit of electrical resistance is the ohm, symbol Ω.

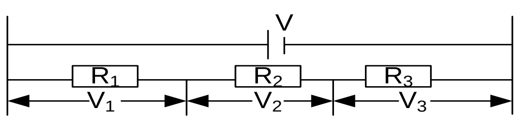
Definition

An ohm (Ω)

is defined as the resistance of a conductor if a current of 1A flows through when the p.d across it is 1V.
(1 Ω = 1 V A^{-1})

ARRANGEMENT OF RESISTORS

a) Resistors in series



When resistors are in series, <u>current</u> flowing through them is the <u>same</u>.

Total $P.d, V = V_1 + V_2 + V_3$

$V_1 = I\,R_1, V_2 = I\,R_2$ and $V_3 = I\,R_3$

$V = I\,R_1 + I\,R_2 + I\,R_3$

$V = I\,(R_1 + R_2 + R_3)$

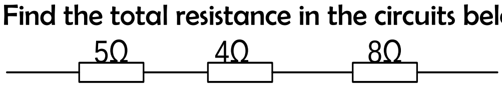
$\frac{V}{I} = R_1 + R_2 + R_3$ but $\frac{V}{I} = R$

$\boxed{R = R_1 + R_2 + R_3}$

Where R is equivalent reisisitance

Example

Find the total resistance in the circuits below

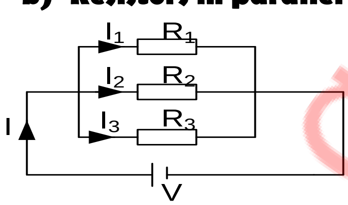


Solution

R = $(5 + 4 + 8)\,\Omega$

$R = 17\Omega$

b) Resistors in parallel



When resistors are in parallel, <u>p.d</u> across the ends is the <u>same</u>.

Total current, I = $I_1 + I_2 + I_3$

Where $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$ and $I_3 = \frac{V}{R_3}$

$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$ but $\frac{I}{V} = \frac{I}{R}$

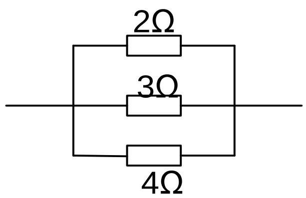
$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

Where R is equivalent reisistance

Example

a) Find the effective resistance of the following circuit

1.



Solution

$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

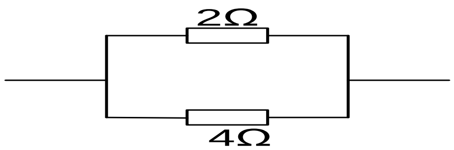
$$\frac{1}{R} = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)$$

$$\frac{1}{R} = \frac{13}{12}$$

$$R = \frac{12}{13}$$

R = 0.92Ω

2.



Solution

$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

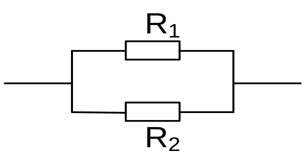
$$\frac{1}{R} = \left(\frac{1}{2} + \frac{1}{4}\right)$$

$$\frac{1}{R} = \frac{3}{4}$$

$$R = \frac{4}{3}$$

R = 1.33Ω

Note for two resistors in parallel



$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

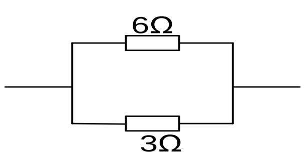
$$\frac{1}{R} = \frac{R_1 + R_2}{R_1\, R_2}$$

$$R = \frac{R_1\, R_2}{R_1 + R_2}$$

$$R = \frac{product\ of\ resisstance}{sum\ of\ resistance}$$

b) Calculate the effective resistance in each of the following circuits

(i)



Solution

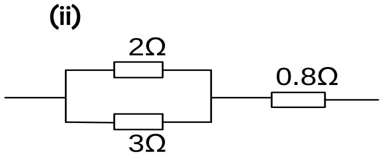
$$R = \frac{product\ of\ resisstance}{sum\ of\ resistance}$$

$$R = \frac{6x3}{6+3}$$

$$R = \frac{18}{9}$$

R = 2Ω

(ii)



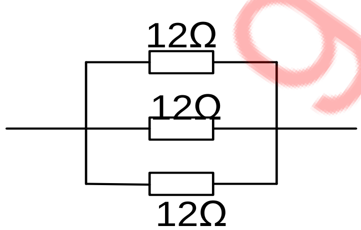
Solution

$$R = 0.8 + \frac{2x3}{2+3}$$

$$R = 0.8 + \frac{6}{5}$$

R = 2Ω

(iii)



Solution

$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

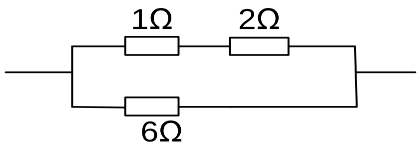
$$\frac{1}{R} = \left(\frac{1}{12} + \frac{1}{12} + \frac{1}{12}\right)$$

$$\frac{1}{R} = \frac{3}{12}$$

$$R = \frac{12}{3}$$

R = 4Ω

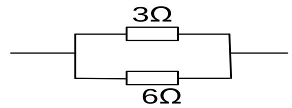
(iv)



Solution

For series

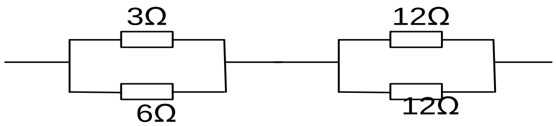
R = $(1+2)\Omega \quad =3\Omega$



$$R = \frac{6x3}{6+3}$$

$$R = \frac{18}{9} = 2\Omega$$

(v)



Solutions

For the first set of parallel resistors

$$R = \frac{6x3}{6+3}$$

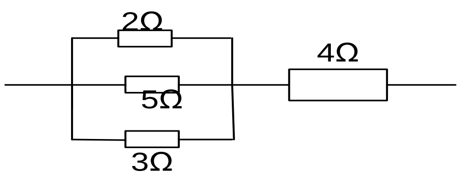
R = 2Ω

For the second set of parallel resistors

$$R = \frac{12x12}{12+12} = 6\Omega$$

Total resistance $=2+6=8\Omega$

(vi)



Solution

For parallel combination

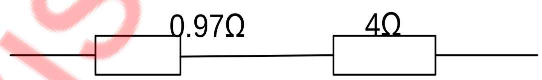
$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R} = \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{3}\right)$$

$$\frac{1}{R} = \frac{31}{30}$$

$$R = \frac{30}{31}$$

R = 0.97Ω

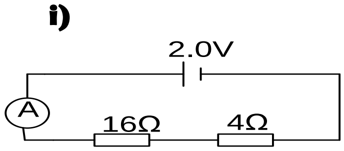


Total resistance $=0.97+4=4.97\Omega$

Further examples

1. Find the ammeter readings in each of the circuits below

i)



Solution

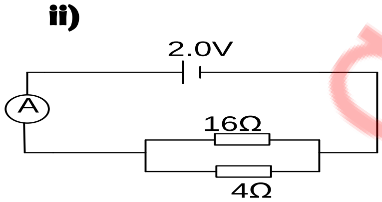
$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{2}{16+4}$$

$$I = 0.1\ A$$

ii)



Solution

$$V = IR$$

$$I = \frac{V}{R}$$

$$R = \frac{product\ of\ resisstance}{sum\ of\ resistance}$$

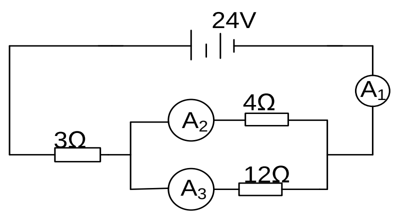
$$R = \frac{16x4}{16+4}$$

$$R = 3.2\Omega$$

$$I = \frac{2}{3.2}$$

$$I = 0.625\ A$$

iii)



Solution

$A_1 = A_2 + A_3$

A_1 reads current in the whole circuit

$V = IR$

$I_1 = \frac{V}{R}$

Total R= $\left[3 + \left(\frac{4x12}{4+12}\right)\right]$

$R = 3\Omega + 3\Omega$

$R = 6\Omega$

$I_1 = \frac{24}{6}$

$I_1 = 4A$

To find A_2 and A_3, we need to first find voltage across parallel combination

$V = IR_P$

I is the current through the parallel combination and R_P is total resistance of the parallel combination

$V = 4x\left(\frac{4x12}{4+12}\right)$

$V = 4x3$

$V = 12V$

Note : For any resistors in parallel, they have the same $p.d$

Current in A_2: $I_2 = \frac{V}{R}$

$I_2 = \frac{12}{4}$

$I_2 = 3A$

Current in A_3: $I_3 = \frac{V}{R}$

$I_3 = \frac{12}{12}$

$I_3 = 1A$

To quickly confirm the currents

Current in A_2: $I_2 = \frac{R_3}{R_2+R_3}xI$

$I_2 = \frac{12}{16}x4$

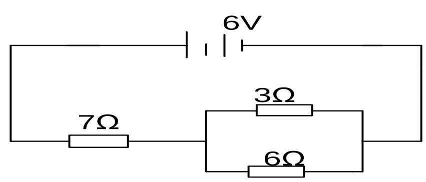
$I_2 = 3A$

Current in A_3: $I_3 = \frac{R_2}{R_2+R_3}xI$

$I_3 = \frac{4}{16}x4$

$I_3 = 1A$

2.



In the figure above find;

(i) Current through the circuit

(ii) Current across 3Ω and 6Ω resistor

(iii) $P.d$ across the 7Ω resistor

(iv) $P.d$ across the 3Ω and 6Ω resistor

Solution

i) Total resistance, R= $\left[7 + \left(\frac{6x3}{6+3}\right)\right]$

$R = 7\Omega + 2\Omega$

$R = 9\Omega$

$V = IR$

$I = \frac{V}{R}$

$I = \frac{6}{9}$

$I = 0.667\ A$

Current in the circuit is $0.667\ A$

ii) Voltage across the parallel combination

$V = IR_P$

$V = 0.667x\left(\frac{6x3}{6+3}\right)$

$V = 0.667x2$

$V = 1.334V$

Note : For any resistors in parallel, they have the same $p.d$

Current in 3Ω $resistor$: $I = \frac{V}{R}$

$I = \frac{1.334}{3}$

$I = 0.445A$

Current in 6Ω $resistor$: $I = \frac{V}{R}$

$I = \frac{1.334}{6}$

$I = 0.223A$

To quickly confirm the currents

Current in 3Ω $resistor$: $I = \frac{6}{6+3}x0.667$

$I = \frac{6}{9}x0.667$

$I = 0.445A$

Current in 6Ω $resistor$: $I = \frac{3}{6+3}x0.667$

$I = \frac{3}{9}x0.667$

$I = 0.223A$

iii) $P.d$ across the 7Ω resistor

$0.6A$ Passes through the 7Ω resistor

$V = I\ R$

$V = 7x\ 0.667$

$V = 4.669V$

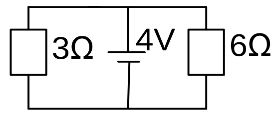
iv) $P.d$ across the 3Ω resistor and 6Ω resistor

$V = (6 - 4.669)V$

$V = 1.33V$

since the two resistors are in parallel therefore, they have the same $p.d$ of $1.33V$

3.



Two resistors of 3Ω and 6Ω are connected across a battery of emf of $4V$ as show, find

i) the combined resistance

ii) the current supplied by the battery

Solution

i) $R = \frac{product\ of\ resisstance}{sum\ of\ resistance}$

$R = \frac{6x3}{6+3}$

$R = 2\Omega$

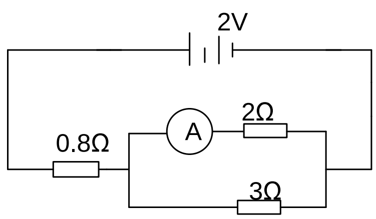
ii) $I = \frac{V}{R}$

$I = \frac{4}{2}$

$I = 2A$

Exercise

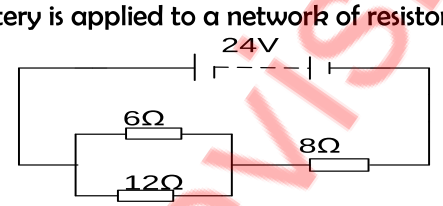
1.



Find the ammeter reading

[0.6A]

2. A $p.d$ of 24V from a battery is applied to a network of resistors as shown below

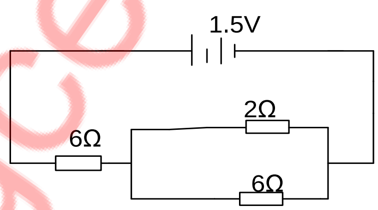


i) find the current through the circuit **[2A]**

ii) find the p.d across the 8Ω resistor **[16V]**

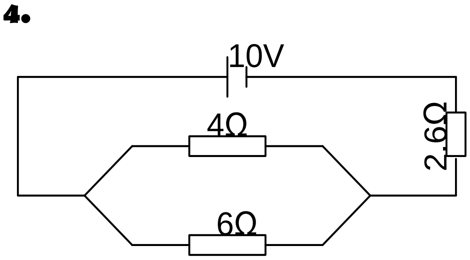
iii) find the current through the 6Ω resistor **[1.3A]**

3.



Find the current through the 2Ω resistor

4.



A battery of emf 10V and negligible internal resistance is connected across a network of resistors as shown above. calculate the current through the 6Ω resistor.

[0.8A]

Factors that affect resistance

a) Temperature

- Condution in metals is by free electorns. The drift electorns however are obstracted by atoms in their lattice positions.
- When temperature of the metal inceases, the atoms vibrate with a larger amplitude thus reducing the means free path of the free electorns reducing the drift velocity of free electorns hence increae in resistance

b) Length

The longer the conductor, the higher the resistance and the shorter the conductor the lower résistance. Free electrons collide morefrequently with atoms, at each collision they lose some kinetic energy to atoms vibrating at fixed mean positions. This leads to a decrease in the drift velocity of the electrons and hence an incease in resistance

c) Cross sectional area

The thinner the conductor, the higher the resistance and the thicker the conductor, the lower the resistance. When there is an increase in the cross sectional area the nuber of free electrons htat drift along the conductor also increases. This leads to an increase in current hence a decrease in ressitance.

The above factors can be combined as;

$$\rho \propto {}^{L}/_{A}$$

$$\boxed{R = \frac{\rho\, L}{A}}$$

Where ρ is resistivity

Definition

Electrical Resistivity is the resistance across <u>opposite faces</u> of a 1m-cube of 1a material
Resistivity is the electrical resistance across <u>opposite faces</u> of a 1m-cube of 1a material

Examples

1. A conductor of length $20cm$ has a cross sectional area of $2x10^{-4}\ m^2$. Its resistance at $20°\text{C}$ is 0.6Ω. find the resistivity of the conductor at 20°C.

Solution

$$R = \frac{\rho\, L}{A}$$

$$\rho = \frac{RA}{L}$$

$$\rho = \frac{0.6\ x\ 2x10^{-4}}{0.2}$$

$$\rho = 6x10^{-4}\ \Omega\, m$$

2. A wire of diameter 14mm and length 50cm has its resistivity as $1.0\ x10^{-7}\ \Omega\, m$. What is the resistance of the wire at room temperature?

Solution

$$d = 14mm, r = \frac{14}{2} = 7mm$$

$$l = 50cm, l = 0.5m$$

$$A = \pi\, r^2$$

$$A = \frac{22}{7} x \left(\frac{7}{1000}\right)^2$$

$$A = 1.54x\ 10^{-4}\ m^2$$

$$R = \frac{\rho\, L}{A}$$

$$R = \frac{0.5\ x\ 1x10^{-7}}{1.54x\ 10^{-4}}$$

$$R = 3.25x10^{-4}\ \Omega$$

3. A steady uniform current of 5mA flows along a metal cylinder of cross sectional area of 0.2mm^2, length, 5m and resistivity 3x10$^{-5}\Omega$m. find the p.d across the ends of the cylinder.

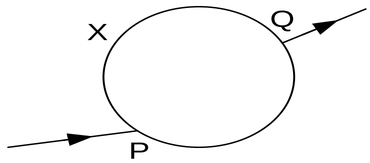
Solution

$$R = \frac{\rho\, L}{A}$$

$$R = \frac{5\ x\ 3x10^{-5}}{2x\ 10^{-7}}$$
$$R = 750\ \Omega$$

$$V = IR$$
$$V = 5x10^{-3}x750 = 3.75V$$

4. A wire of diameter ***d***, length ***l*** and resistivity ***ρ*** forms a circular loop. Current enters and leaves at points **P** and **Q**.



Show that the resistance R of the wire is given by $R = \frac{4\ \rho X(l-X)}{\pi d^2 l}$

Solution

Let R_1 and R_2 be resistance of portion x and $l - x$ of the wire respectively.

$R_1 = \frac{\rho x}{A}$ and $R_2 = \frac{\rho(l-x)}{A}$

The two portions are in parallel, hence $R = \frac{R_1 R_2}{R_1 + R_2}$

$$R = \frac{\left(\frac{\rho x}{A}\right)\left(\frac{\rho(l-x)}{A}\right)}{\left(\frac{\rho x}{A}\right) + \left(\frac{\rho(l-x)}{A}\right)} = \frac{\rho x(l-x)}{Al}$$

But $A = \frac{\pi d^2}{4}$

$$R = \frac{\rho x(l-x)}{Al} = \frac{4\ \rho x(l-x)}{\pi d^2 l}$$

Question

A p.d of 4.5V is applied to the ends of a 0.69m length of a wire of cross sectional area $6.6x10^{-7}m^2$. Calculate the drift velocity of electrons across the wire. (ρ of wire is $4.3x10^{-7}\Omega$m, number of electrons per m^3 is 10^{28} and electronic charge is $1.6x10^{-19}$C)

Temperature cofficient of resistance (α),

The temperature coefficient of resistance of a material is the fractional change in the resistance at 0 °C per degree celcius rise in temperature.

If a material has resistance R_o at 0 °C and its resistance increases to R_θ when heated through a temperature θ, then

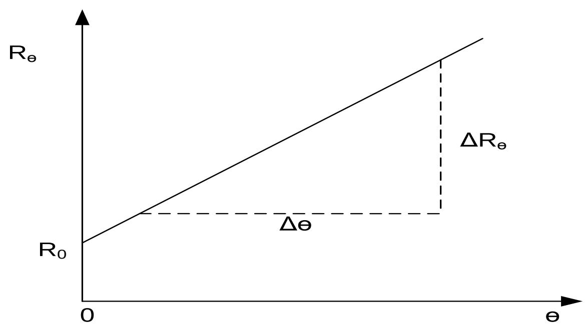
$$\alpha = \frac{R_\theta - R_0}{R_0 \theta}$$

The S.I unit of α is K^{-1}or $°C^{-1}$

The above equation can be rearranged;

$$\boxed{R_\theta = R_0(1 + \alpha\theta)}$$

A graph of R_θ against θ is a straight line whose intercept on the R_θ - axis is equal to R_0 and the slope = $\alpha\ R_0$.



$$S = \frac{\Delta R_\theta}{\Delta \theta} = \alpha\, R_0$$

$$\alpha = \frac{S}{R_0}$$

If R_1 and R_2 are the resistances of a conductor at temperatures θ_1 and θ_2, then

$R_1 = R_0(1 + \alpha\theta_1)$................................i

$R_2 = R_0(1 + \alpha\theta_2)$................................ii

Divide i by ii $\boxed{\frac{R_1}{R_2} = \frac{1+\alpha\theta_1}{1+\alpha\theta_2}}$

Note:

Super conductors are materails whose resistance vanishes when colled to about $-273°C$

Why metals have positive temperature coefficient of resistance

When the temperature of the metal increase, the amplitude of vibration of its atoms increase. This reduces the means free path for the conduction electrons. Thus fewer electrons now flow per second through the metal and hence less which increases the resistance.

Why semi-conductors have negative temperature coefficient of resistance

Semi-conductors have few electrons available for conduction at room temperature. When current is passed through it, the material heats is up. As the temperature increases, loosely bound electrons are released for conduction thus current increase hence resistance reduces

Examples

1. The resistivity of mild steel is $15 \times 10^{-8}\ \Omega$ m at 20 °C and its temperature coefficient of resistance is 50×10^{-4} K^{-1}. Calculate the resistivity at 60 °C.

Solution:

Since the resistance of a conductor when heated is proportional to resistivity, it follows that the resistivity at temperature θ is given by

$$\rho_\theta = \rho_0(1 + \alpha\theta)$$

Where α is the temperature coefficient of resistivity.

At 20 °C,

$$15 \times 10^{-8} = \rho_0(1 + 50 \times 10^{-4} \times 20)$$

$$15 \times 10^{-8} = 1.1\rho_0$$

$$\rho_0 = \frac{15 \times 10^{-8}}{1.1} = 13.64 \times 10^{-8}\Omega\ m$$

At 60 °C,

$$\rho_{60} = 13.64 \times 10^{-8}(1 + 60 \times 50 \times 10^{-4})$$

$\rho_{60} = 17.7 \times 10^{-8}\ \Omega$ m

2. A coil of wire has resistances of 30 Ω at 20 °C and 34.5 Ω at 60 °C. Calculate
   (i) The temperature coefficient of resistance of the wire
   (ii) The resistance of the wire at 0 °C.

Solution:

(i) At 20 °C,

$R_{20} = 30\ \Omega$

$30 = R_o(1 + 20\alpha)$ 1

At 60 °C,

$R_{60} = 34.5\ \Omega$

$R_{60} = R_o(1 + 60\alpha)$ 2

Divide equation 1 by 2

$$\frac{30}{34.5} = \frac{R_o(1 + 20\alpha)}{R_o(1 + 60\alpha)}$$

$$\frac{30}{34.5} = \frac{1 + 20\alpha}{1 + 60\alpha}$$

$$30 + 1800\alpha = 34.5 + 690\alpha$$

$$1110\alpha = 4.5$$
$$\alpha = 4.05 \times 10^{-3} K^{-1}$$

(ii) Substitute for α in equation 1 or 2 to solve for R_o

$$30 = R_o + 20 \times 0.00405 \times R_o$$
$$R_o = 27.75\Omega$$

3. The resistance of a nichrome element of an electric fire is 50.9 Ω at 20 °C. When operating on at 240 V supply, the current flowing in it is 4.17 A. Calculate the steady temperature reached by the electric fire if the temperature coefficient of resistance of nichrome is $1.7 \times 10^{-4} K^{-1}$.

Solution:

At 20 °C, R_{20} = 50.9 Ω

From $R_\theta = R_o(1 + \alpha\theta)$
$$50.9 = R_o(1 + 20 \times 1.7 \times 10^{-4})$$
$$R_0 = \frac{50.9}{1.0034} = 50.7275\ \Omega$$

Let the unknown temperature to which the element heats up be, β.

At β °C,

$V = 240\ V, I = 4.14\ A$
$$R_\beta = \frac{240}{4.17} = 57.554\ \Omega$$
$$R_\beta = R_o(1 + \alpha\beta)$$
$$57.554 = 50.7275(1 + 1.7 \times 10^{-4}\beta)$$
$$6.8265 = 8.6237 \times 10^{-3}\beta$$
$\beta = 791.6$ °C

4. An electric heater consists of 5.0m of nichrome wire of diameter 0.58mm. When connected to a 240V supply, the heater dissipates $2.5kW$ and the temperature of the heater is found to be 1020℃. If the resistivity of the nichrome at 10℃ is $1.02x10^{-6}\Omega m$. Calculate;

(i) The resistance of nichrome at 10℃

(ii) The mean temperature coefficient of reistance of nichrome between 10℃ and 1020℃

Solution

(i) $R_{10} = \frac{\rho l}{A} = \frac{1.02x10^{-6}x5}{\pi\left(\frac{0.58x10^{-3}}{4}\right)} = 19.3\ \Omega$

(ii) **At** 1020℃, $P_{1020} = \frac{V^2}{R_{1020}} = 2.5kW$
$$R_{1020} = \frac{240^2}{2.5x10^3} = 23.04\ \Omega$$

$23.04 = R_o(1 + 1020\alpha)$ 1

At 10℃,

$19.3 = R_o(1 + 10\alpha)$ 2

Eq 2÷Eq 1
$$\frac{23.04}{19.3} = \frac{1 + 1020\alpha}{1 + 10\alpha}$$
$$\alpha = 1.92x10^{-4}K^{-1}$$

Exercise

3. The resistance of a nichrome element of an electric fire is 50 Ω at 20 °C. When operating on a 240 V supply, the current flowing in it is 4.A. Calculate the steady temperature reached by the electric fire if the temperature coefficient of resistance of nichrome is $2.0 \times 10^{-4} K^{-1}$. **An($1024.1$℃)**

4. The resistivity of a certain wire is $1.6 \times 10^{-7}\ \Omega$ m at 30 °C and its temperature coefficient of resistance is 6.0×10^{-3} K^{-1}. Calculate the resistivity at 80 °C. **An($2.01x10^{-7}\Omega m$)**

5. A nichrome wire of length 1.0m and diameter 0.72mm at 25° C, is made into a coil. The coil is immersed in 200cm^3 of water at the same temperature and a current of 5.0 A is passed through the coil for 8 minutes until when the water starts to boil at 100°C.

Find:

(i) the resistance of the coil at 25°C

(ii) the electrical energy expended assuming all of it goes into heating the water.

(iii) the mean temperature coefficient of resistance of nichrome between 0° C and 100°C.

5. Two wires A and B have lengths which are in the ratio $4 : 5$, diameters which are in the ratio $2 : 1$, and resistances in the ratio of $3 : 2$. If the wires are arranged in parallel and current of $1.0\ A$ flows through the combination, find the:
   (i) ratio of resistance of wire A to that of wire B
   (ii) current through wire A
6. A battery of e.m.f 12V and negligible internal resistance is connected to the ends of a metallic wire of length 50 cm and cross-sectional area 0.1 mm^2. If the resistivity of a material of the wire is $1.0\ x\ 10^{-6}\Omega$ m at what rate is heat generated in the wire?
7. The table below shows the resistance of a nichrome wire at various temperatures.

Temperature (°C)	75	120	150	250	300
Resistance (Ω)	103	103.8	104.4	105.9	106.8

Plot a suitable graph and use it to determine the temperature coefficient of resistance of nichrome

Electromotive force and internal resistance

The e.m.f of a cell is the energy supplied by the cell to transfer 1C of charge round a complete circuit in which the cell is connected.

It may also be defined as the p.d across the terminals of the cell when on an open circuit.

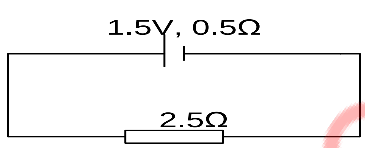
Internal resistance of a cell is the opposition in series with the external circuit which accounts for energy losses inside the cell when the cell is supplying current.
Internal resistance is represented by r.

$$\boxed{E = I\ (R + r)}$$

Examples

1. A battery of emf 1.5V and internal resistance 0.5Ω is connected in series with 2.5Ω resistor. Find;
   i) current through the circuit
   ii) $p.d$ of the 2.5Ω resistor

Solution



i) $E = I\ (R + r)$

$$I = \frac{E}{(R + r)}$$

$$I = \frac{1.5}{(2.5 + 0.5)}$$

$$I = \frac{1.5}{3}$$

$$I = 0.5\ A$$

ii) $V = IR$

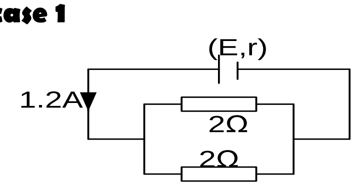
$V = 0.5x2.5$

$V = 1.25V$

2. A cell can supply a current of $1.2A$ through two 2Ω resistors connected in parralle. When they are connected in series the value of current is $0.4A$. Clculate the internal resistance and emf of the cell.

Solution

case 1



$E = I\ (R + r)$

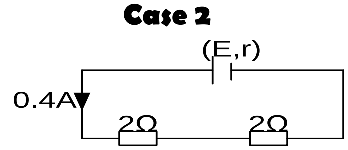
$$R = \frac{2x2}{2 + 2}$$

$R = 1\Omega$

$E = 1.2\ (1 + r)$

$E = 1.2 + 1.2\ r$........[1]

Case 2



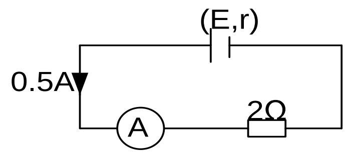
$E = I\ (R + r)$

$R = 2 + 2$

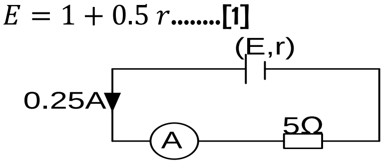
$R = 4\Omega$
$E = 0.4\ (4 + r)$
$E = 1.6 + 0.4\ r$........[2]
Equating 1 and 2
$1.2 + 1.2\ r = 1.6 + 0.4\ r$
$1.2\ r - 0.4\ r = 1.6 - 0.4$
$0.8r = 0.4$

$r = \frac{0.4}{0.8}$
$r = 0.5\Omega$
Also $E = 1.2 + 1.2\ r$
$E = 1.2 + 1.2\ x\ 0.5$
$E = 1.2 + 0.6$
$E = 1.8\ V$

3. An ammeter connected in series with a cell and a 2Ω resistor reads $0.5A$. When the 2Ω resistor is replaced by a 5Ω resistor, the ammeter reading drops to $0.25A$. Calculate the internal resistance and the emf of the cell.
Solution



$E = I\ (R + r)$
$E = 0.5\ (2 + r)$
$E = 1 + 0.5\ r$........[1]

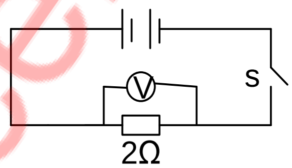


$E = I\ (R + r)$

$E = 0.25\ (5 + r)$
$E = 1.25 + 0.25\ r$........[2]
Equating 1 and 2
$1 + 0.5\ r = 1.25 + 0.25\ r$
$0.5\ r - 0.25\ r = 1.25 - 1$
$0.25r = 0.25$
$r = \frac{0.25}{0.25}$
$r = 1\ \Omega$
Also $E = 1 + 0.5\ r$
$E = 1 + 0.5\ x\ 1$
$E = 1 + 0.5$
$E = 1.5\ V$

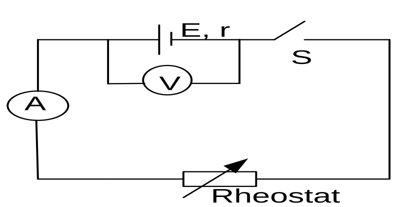
Exercise

1. A battery of e.m.f and internal resistance, r is connected across a variable resisto, when the resistor is set at 21Ω, the current through it is 0.48A. when it is set at 36Ω, the current is 0.30A. find E and r. **[$4\Omega, 12V$]**
2. A cell is joined in series with a resistance of 2Ω and a current of $0.25A$ flows through it. When a second resistance of 2Ω is connected in parallel with the first, the current through the cell is$0.3A$. Calculate the internal resistance and emf of the cell. **[$4\Omega, 1.5V$]**
3. Two cells each of e.m.f 1.5 V and internal resistance 0.5 Ω are connected in series with a resistor of 2 Ω as in the figure below.



The reading of the voltmeter V when S is closed is? **[2V]**

Measurement of E.m.f and internal resistance of a cell

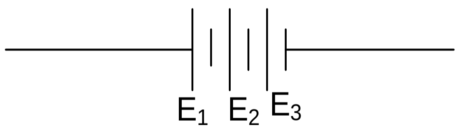


A-ammeter, V- voltmeter, R-resistor, S- switch
- Arrange the apparatus as shown above.
- Switch is closed and rheostat adjusted to a suitable value of I
- Ammeter reading I and voltmeter reading V are note and recorded

- The rheostat is varied for othe values of I and corresponding values of V are noted and recorded.
- A graph of V against I is plotted.
- The intercept on the V axis is noted which is the e.m.f of the cell.
- The slope s is obtained and the internal resistance of the cell $r = -s$

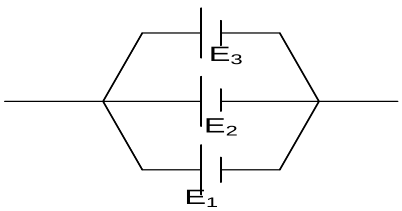
CELL ARRANGEMENTS

1. Series arrangement



Total emf $E = E_1 + E_2 + E_3$

2. Parallel arrangement

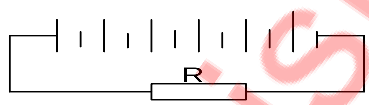


When cells of equal ***emf*** are connected in parallel
Total emf $E = E_1 = E_2 = E_3$

Example

1. Find the total emf in each of the following circuits if each cell is of emf $1.5V$

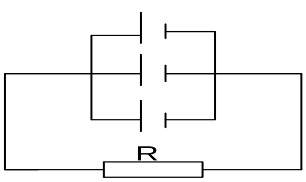
(i)



Solution

Total emf $E = 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5$
$= 9.0V$

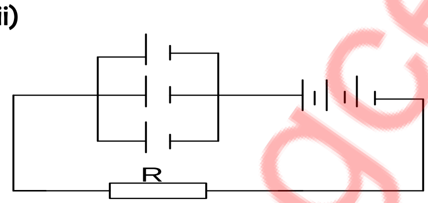
(ii)



Solution

Total emf $E = 1.5V$

(iii)



Solution

Total emf $E = 1.5 + 1.5 + 1.5 + 1.5$
$= 6.0V$

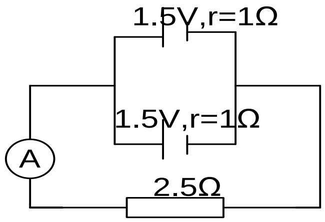
Note: If the cells are connected in parallel and have internal resistance, their resistance is calculated as resistors in parallel.

Advantages of series arrangement of cells over the paralle arrangement

In series arrangement the effective e.m.f is greater than the individual e.m.f of the cells and hence a greater current is drawn from the series com bination than in the parallel combination.
However the series aarrangement has a disadvantage of all the cells being drained at once thus the cells have a shorter life span.

Examples

1. Find the ammeter reading



Solution

$$E = I\,(R + r)$$

$$r = \frac{1x1}{1+1}$$

$$r = 0.5\Omega$$

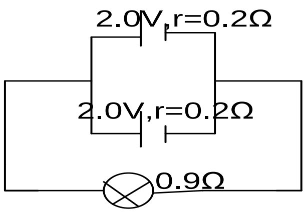
$$1.5 = I\,(2.5 + 0.5)$$

$$I = \frac{1.5}{3}$$

$$I = 0.5A$$

2. Two cells of emf 2.0V and internal resistance 0.2Ω each are connected together in parallel to form a battery. This battery is connected to a lamp of resistance 0.9Ω. Calculate the current through the lamp and voltage across the lamp.

Solution



$$E = I\,(R + r)$$

$$r = \frac{0.2x0.2}{0.2 + 0.2}$$

$$r = 0.1\Omega$$

current through the lamp

$$2 = I\,(0.9 + 0.1)$$

$$I = \frac{2}{1}$$

$$I = 2A$$

voltage across the lamp

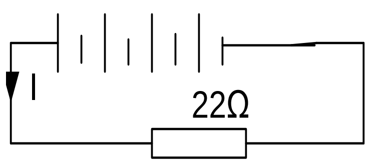
$$V = IR$$

$$V = 2x0.9$$

$$V = 1.8V$$

3. Four cells each of emf $1.5V$ and internal resistance 0.5Ω are connected in series. What current will flow through an external resistor of 22Ω

Solution



Total emf E $= 1.5x4$

$$E = 6V$$

Total internal resistance r $= 0.5x4$

$$r = 2\Omega$$

$$E = I\,(R + r)$$

$$6 = I\,(22 + 2)$$

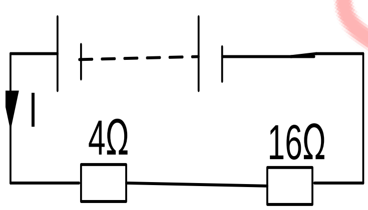
$$I = \frac{6}{24}$$

$$I = 0.25A$$

4. A battery containing 8 cells each of emf 1.5V and internal resistance 0.5Ω is connected to two other resistors of 4Ω and 16Ω . Calculate the minimum and maximum current that can flow through the battery.

Solution

For minimum current the resistors must be connected in series



Total emf E $= 1.5x8$

$$E = 12V$$

Total internal resistance r $= 0.5x8$

$$r = 4\Omega$$

Total external resistance R$= 4 + 16$

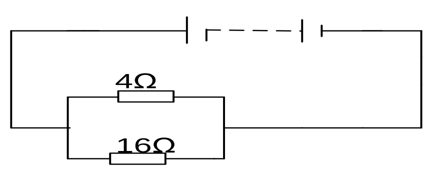
$$E = I\,(R + r)$$

$$12 = I\,(16 + 4 + 4)$$

$$I = \frac{12}{24}$$

$$I = 0.5A$$

For maximum current the resistors must be connected in parallel



Total emf E $= 1.5x8$

$$E = 12V$$

Total internal resistance r $= 0.5x8$

$r = 4\Omega$

Total external resistance R$= \frac{4x6}{4+6}$

$R = 3.2\Omega$

$E = I\,(R + r)$

$12 = I\,(3.2 + 4)$

$I = \frac{12}{7.2}$

$I = 1.67A$

WORK DONE BY AN ELECTRIC CURRENT(ELECTRICAL ENERGY)

If the $P.d, V$ is applied to the ends of a conductor and quantity of electricity, Q flows then

$work\ done = Q\,V$ but $Q = It$

$W = It\,V$

$$\boxed{W = IVt}$$

but $V = IR$

$W = I\,(IR)\,t$

$$\boxed{W = I^2\,R\,t}$$

but $I = \frac{V}{R}$

$W = \left(\frac{V}{R}\right)^2 R\,t$

$$\boxed{W = \frac{V^2\,t}{R}}$$

The work done is transferred into internal molecular energy accompanied by a rise in temperature subsequently, this energy may be given out in form of heat

ELECTRICAL POWER

This is the rate of doing work by an electric current.

$$power = \frac{work\ done}{time\ taken}$$

$$P = \frac{I\,V\,t}{t}$$

$$\boxed{P = I\,V}$$

Also

$$power = \frac{work\ done}{time\ taken}$$

$$P = \frac{I^2\,R\,t}{t}$$

$$\boxed{P = I^2\,R}$$

Also

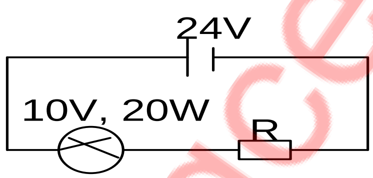
$$power = \frac{work\ done}{time\ taken}$$

$$P = \frac{V^2\,t}{R} \Big/ t$$

$$\boxed{P = \frac{V^2}{R}}$$

Examples

1. A battery of $emf\ 24V$ is connected in series with aresistance R and a lamp rated $10V, 20W$ as shown below.



if the bulb is operating normally . Find,

i) the p.d across the resistor
ii) the value of R
iii) power dissipated in the resistor

Solution

i) $p.d$ across the resistor

$= (24 - 10)V$

$= 14V$

ii) Current through the bulb

$I = \frac{P}{V}$

$I = \frac{20}{10}$

$I = 2A$

Bulb and resistor have the same current

$R = \frac{V}{I}$

$R = \frac{14}{2}$

$R = 7\Omega$

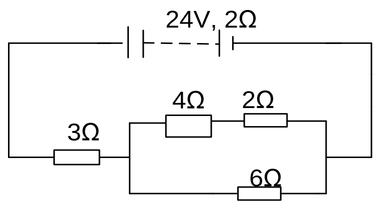
iii) power dissipated in the resistor

$P = I^2\,R$

$P = 2^2\,x\,7$

$P = 28W$

2. An accumulator of $emf\ 24V$ and internal resistance 2Ω is connected in a circuit as shown below.

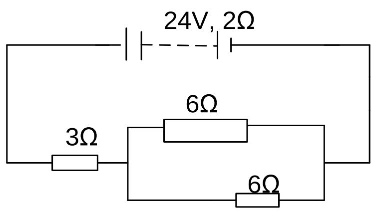


a) calculate the current through the 6Ω resistor
b) calculate the power expended in the 6Ω resistor
c) find the total power expended

Solution

a) for 4Ω and 2Ω resistors total resistance $R = (4+2)$

$R = 6\Omega$



Total resistance= $\left[3 + \left(\frac{6x6}{6+6}\right)\right]$

$= 6\Omega$

$E = I\,(R + r)$

$24 = I\,(6+2)$

$I = \frac{24}{8}$

$I = 3A$

$p.d$ through parallel combination

$V = IR$

$V = 3x\left(\frac{6x6}{6+6}\right)$

$V = 9V$

Current through the 6Ω resistor

$V = IR$

$I = \frac{V}{R}$

$I = \frac{9}{6}$

$I = 1.5A$

b) power in 6Ω resistor

$P = IV$

$P = 1.5\,x\,9$

$P = 13.5\,W$

c) total power= $I\,E$

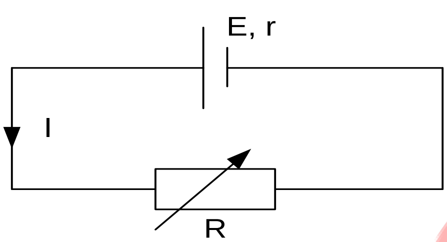
$= 3x\,24$

$= 72W$

Power out put and efficiency

Efficiency of the battery is the ratio of the useful power expended in the total external load to the power generated or supplied by the battery.

Usually efficiency is expressed in percentage and is denoted by η



Power delivered to resistor, $P_{out} = IV$

Power supplied by cell, $P_{IN} = IE$

Eficiency,$\eta = \frac{Power\ output}{Power\ input}x100\%$

$\eta = \frac{IV}{IE}x100\%$

$\eta = \frac{V}{E}x100\%$

but $E = I(R+r)$ and $V = IR$

$\eta = \left(\frac{R}{R+r}\right)x100\%$

Example

A battery of e.m.f 18.0V and internal resistance 3.0Ω is connected to a resistor of resistance 8Ω. Calculate the

(i) power generated
(ii) efficiency
(iii)

Solution

$P_{gen} = IE = \left(\frac{E}{R+r}\right)E$

$P_{gen} = \left(\frac{18x18}{8+3}\right) = 29.45W$

$\eta = \frac{Power\ output}{Power\ input}x100\%$

$\eta = \frac{IV}{IE}x100\%$

but $E = I(R+r)$ and $V = IR$

$\eta = \left(\frac{R}{R+r}\right)x100\%$

$$\eta = \left(\frac{8}{8+3}\right) x100\%$$

$$\eta = \mathbf{72.7\%}$$

Maximum power output

Suppose the load resistance R is variable then the useful power expended in R will also vary and will be maximum when ***R = r.***

$$Power\ output = IV = \left(\frac{E}{R+r}\right) x \left(\frac{ER}{R+r}\right)$$

$$Power\ output, P_0 = \frac{E^2}{(R+r)^2}R$$

At maximum power, P_m, $\frac{dP_0}{dR} = 0$

$$\frac{dP_0}{dR} = E^2\frac{[(R+r)^2 - 2R(R+r)]}{(R+r)^4} = 0$$

$$R = r$$

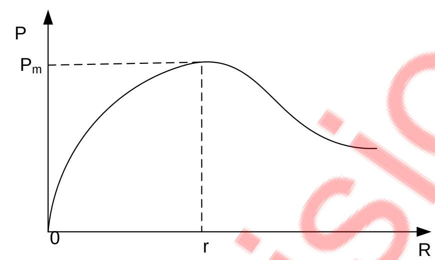
$$P_0 = \frac{E^2}{(r+r)^2}r$$

Power output, $\boxed{P_0 = \frac{E^2}{4r}}$

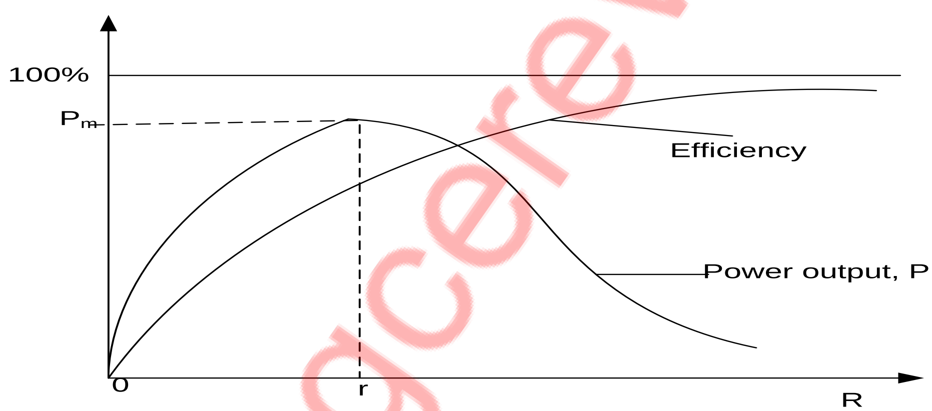
As R tends to zero, P tends to zero

As R tends to ∞, P tends to zero.

A graph of power out put P against load resistance R is shown below.



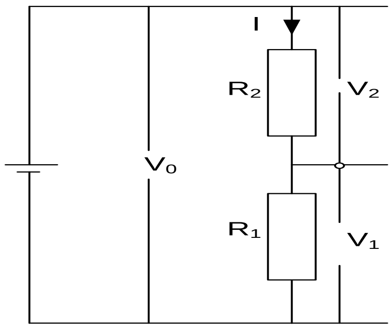
Variation of η and P with R



For values of R less than r, power output increases and attains a maximum value when R = r. For

The Potential Divider

The potential divider is used to obtain a fraction of a given p.d. The fraction can be fixed or variable.



Total resistance in the circuit, $R = R_1 + R_2$.

$$V_0 = IR$$

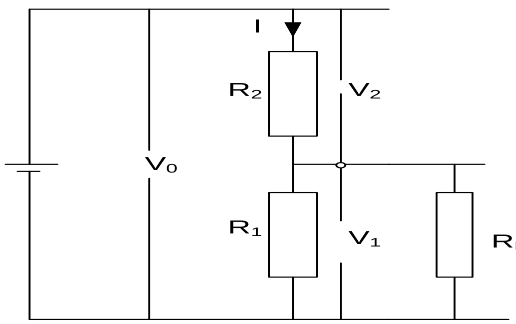
$$V_0 = I(R_1 + R_2)$$

$$I = \frac{V_0}{R_1 + R_2}$$

$$V_1 = IR_1$$

$$V_1 = \left(\frac{V_0}{R_1 + R_2}\right)R_1$$

Suppose a load of resistance R_L is connected in parallel with R_1.



Let the effective resistance of R_1 and R_L be R_P.

$$R_P = \frac{R_1 R_L}{R_1 + R_L}$$

Effective resistance in the circuit, $R = R_P + R_2$

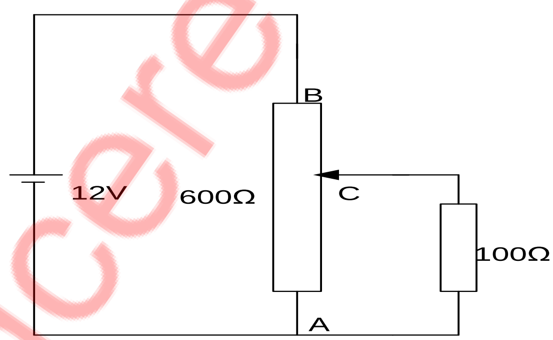
Total current in the circuit, $I = \frac{V_O}{R_P + R_2}$

Hence $V_1 = IR$

$$V_1 = \left(\frac{V_O}{R_P + R_2}\right) R_P$$

Example

1. A 12V battery is connected across a potential divider of resistance 600Ω as shown below. If the load of 100Ω is connected across the terminals A and C when the slider is half way up the divider, find:



(i) p.d across the load
(ii) p.d across a and c when the load is removed.

Solution

(i) Effective resistance

$$R = \frac{300x100}{300 + 100} + 300 = 375\Omega$$

current supplied by the battery, $I = \frac{V}{R}$

$$I = \frac{12}{375} = 0.032A$$

hence current through parallel combination of resistors $= 0.032A$

p.d across parallel combination of resistors,

$$V = IR = 0.032x\left(\frac{300x100}{300+100}\right) = 2.4V$$

Hence the p.d across the load is $2.4V$.

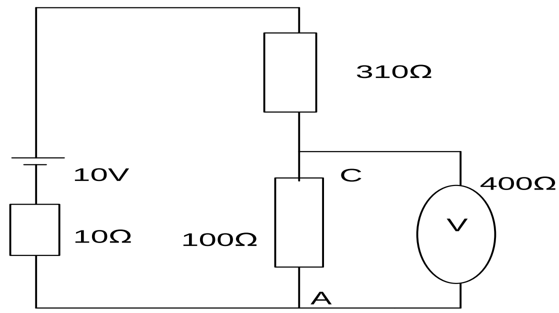
(ii) when the load is removed

$$I = \frac{12}{600} = 0.02A$$

Hence p.d across AC = $0.02x300 = 6V$

Exercise

1.



(i) Find the reading of the voltmeter

(ii) Calculate the power dissipated in the 10Ω resistor

2. A resistor of 500Ω and one of 200Ω are placed in series with a 6V supply. What will be the reading on a voltmeter of internal resistance 2000Ω when placed across

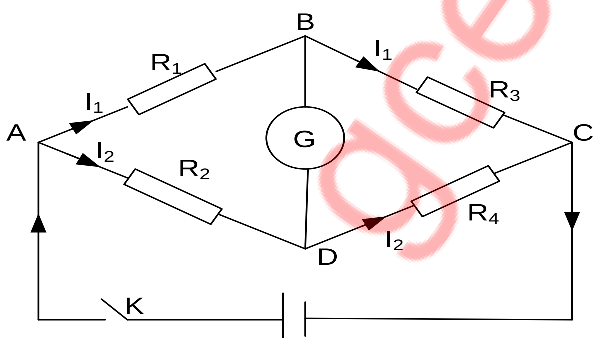
(i) the 5000Ω resistor

(ii) 2000Ω resistor.

WHEASTONE BRIDGE

It is a bridge circuit consisting of four resistances R_1, R_2, R_3, R_4 and a sensitive centre - zero galvanometer, G.

Balaance condition for a wheatstone



Switch K is closed and the reisistance R_1 , R_2 , R_3 and R_4 adjusted until the galvanometer shows no deflection.

At balance condition B and D are at the same potential.

P.d across AB, = p.d across AD,

$$I_1R_1 = I_2R_2 \dots\dots\dots\dots\dots\dots\dots..(1)$$

Current flowing through R_3 is therefore I_1 and that through R_4 is I_2

P.d across BC, = p.d across DC,

$$I_1R_3 = I_2R_4 \dots\dots\dots\dots\dots\dots\dots..(2)$$

Eqn(1) $\div (2)$

$$\frac{I_1R_1}{I_1R_3} = \frac{I_2R_2}{I_2R_4}$$

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}}$$

Equation in the box is called the balance condition of a Wheatstone bridge.

NOTE :

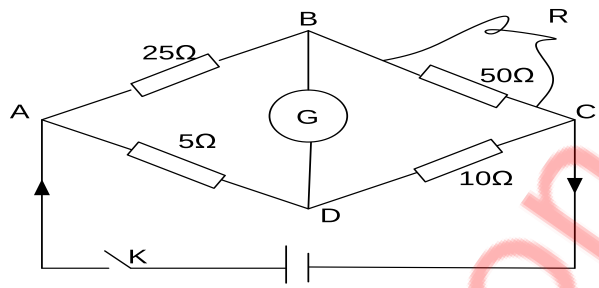
A wheastone bridge s not suitable for measureing very low and very high resistances

Explanation

- When the resistances are very low, resistances of connecting wires become comparable to the test resistances. Errors in the measurd values therefore become significant
- When the reistances are very high, the current flowing becomes very small. The galvanometer becomes less sensitive hence difficulty in determining the balance value

Examples

(g)



When the switch is closed the galvanometer shows no deflection when the 50Ω resistor is shunted with a resistance R, find the value of R

Solution

Let R_P be the total resistance of R Ω and 50Ω that are in parallel.

$$\therefore R_P = \frac{50R}{50+R}\Omega$$

Then at balance;

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \Rightarrow \frac{25}{R_P} = \frac{5}{10} \quad \therefore R_P = 12.5\Omega$$

$$\therefore 12.5 = \frac{50R}{50+R}\Omega$$

$$\therefore R = 16.67\Omega$$

(h) In a Wheatstone bridge, the ratio arms R_1 and R_2 are approximately equal. When $R_3 = 500\Omega$, the bridge is balanced. On interchanging R_1 and R_2, the value of R_3 for balancing is 505Ω. Find the value of R_4 and the ratio $R_1 : R_2$. **An(502.5Ω, 1:1.005)**

The Metre Bridge or Slide Wire

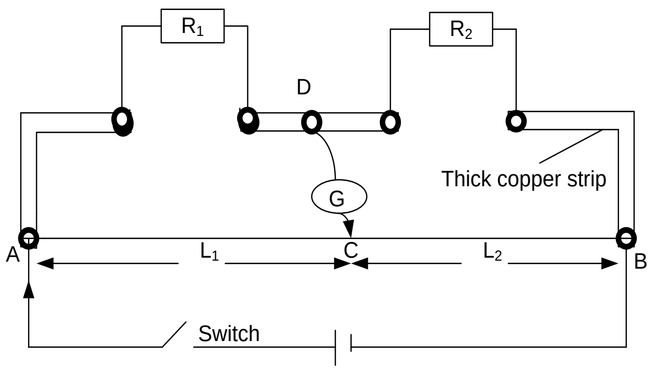
The simple metre bridge is the practical form of a Wheatstone bridge.

It consists of a uniform resistance wire 1 m long lying alongside or on a metre rule scale.

Thick copper or brass strips of low resistance connect various parts of the metre bridge.

Since the wire is uniform its resistance per cm is constant. The resistance of wire between any two points on the wire is proportional to the length separating them.

$$R_{AC} = kl_1$$



Balance condition

On closing switch K, the is jockey is tapped along AB until a point is found when the galvanometer shows no deflection. At balance point, D and C are at the same potential.

p.d across R_1 =p.d across l_1

p.d across R_2 =p.d across l_2

but current through R_1 = current through $R_1 = I_1$ and current through AB= I_2

$I_1R_1 = I_2rl_1$............................... 1

$I_1R_2 = I_2rl_2$...............................2

Where r is resistance per cm of wire AB.

Divide equation 1 by 2

$\frac{I_1R_1}{I_1R_2} = \frac{I_2kl_1}{I_2kl_2}$...................................3

$$\boxed{\frac{R_1}{R_2} = \frac{l_1}{l_2}}$$

End – errors

R_2 should be chosen such that the balance point, C is fairly near the centre of the wire (between 30 and 70 cm). This minimizes errors in the result and gives a more accurate value because the end –errors from both ends will be evenly distributed. A better result can be obtained by interchanging R_1 and R_2 and obtaining a second pair of values of l_1 and l_2. An average value of R_1 can then be taken.

If either l_1 or l_2 is very small, the resistance of the end connections is not negligible and must be added to R_{AC} or R_{CB}.

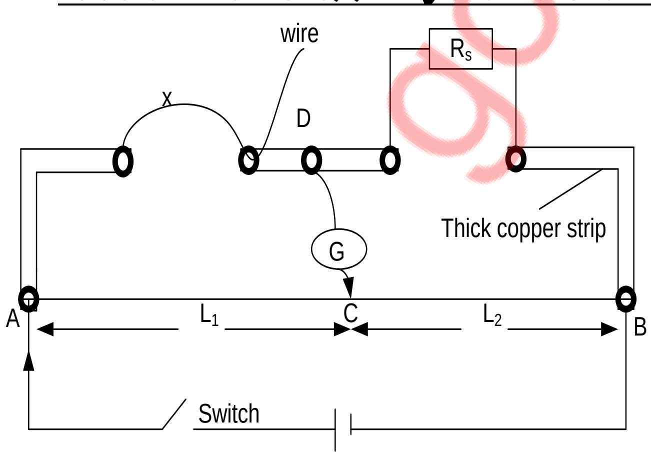
Let the end connection errors be equivalent to lengths e_1 and e_2 from A and B respectively.

Equation 3 then becomes; $\boxed{\frac{R_1}{R_2} = \frac{l_1+e_1}{l_2+e_2}}$

Note:

The metre bridge is therefore unsuitable for very low resistances because the contact resistances become comparable to the test resistances. It is equally not suitable for very high resistances because the galvanometer becomes insensitive

To determine the resistivity of a material inform of a wire



- With a micrometer screw gauge measure the diameter *d*, of the wire and the cross-sectional area ***A***, of the wire determined from $A = \frac{\pi d^2}{4}$.
- Connect a length *x* of the wire across the left hand gap and a standard resistance R_s in the right hand gap as shown above.
- Close switch K and tap the jockey along AB until you locate a point for which the galvanometer shows no deflection.
- Measure and record the balance lengths l_1 and l_2
- Determine the resistance R of the wire from

$$R_x = R_s \frac{l_1}{l_2}$$

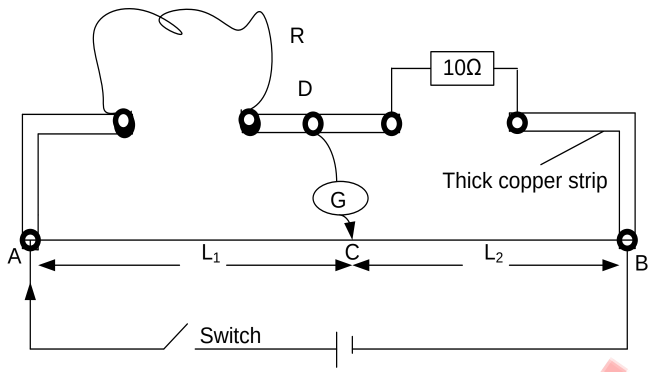
❖ Repeat the procedure for different values of x and tabulate the results in a suitable table.

❖ Plot a graph of R_x against x and determine the slope s of the graph.

❖ Determine the resistivity of the wire from $\rho = SA$

Examples

1. A 110cm length of wire of diameter 0.85mm is placed in the left hand gap of a metre bridge and standard 10Ω coil being placed in the right hand gap.



Balance length obtained was 46.7cm from end of the bridge. Calculate the resistivity of the wire.

Solution

At balance; $\frac{R}{46.7} = \frac{10}{53.6}$

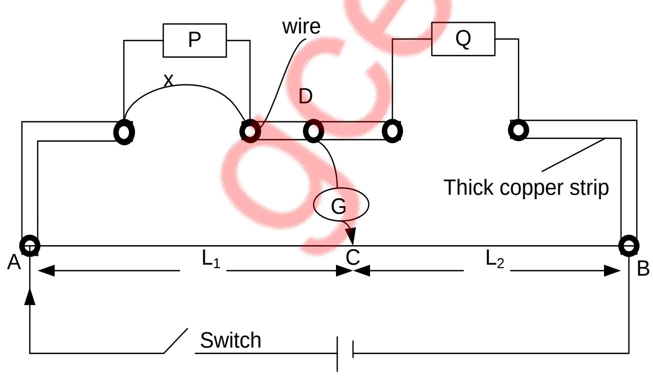
$\therefore R = 8.713\Omega$

$$\therefore \rho = \frac{RA}{L} = \frac{R\pi d^2}{L\,x4}$$

$$\rho = \frac{8.713 \times 3.14 \times (0.85 \times 10^{-3})^2}{1.1x4}$$

$$\rho = 4.5 \times 10^{-6}\Omega m$$

2.



In the diagram below, resistors P and Q are 5 Ω and 2Ω respectively. A wire X of length 60.0 cm and diameter 0.02 mm is connected across P so that the balance point is 66.7 cm from A. Calculate the resistivity of the wire.

Solution:

Let the effective resistance of P and X be R

When G shows no deflection, there is balance, hence

$$\frac{R}{Q} = \frac{l_1}{l_2}$$

$$\frac{R}{Q} = \frac{66.7}{33.3}$$

$$\frac{R}{2} = \frac{66.7}{33.3}$$

$$R = 4\,\Omega$$

But $R = \frac{PX}{P+X}$

$$\therefore 4 = \frac{5X}{5+X}$$

$$X = 20\,\Omega$$

Length, *l* of X = 60 cm = 0.60 m and diameter, *d* = 0.02 mm

Cross sectional area of X, $A = \frac{\pi d^2}{4}$

$A = \frac{\pi}{4} \times (0.02 \times 10^{-3})^2 = 3.14 \times 10^{-10}$ m²

From $R = \frac{\rho l}{A}$, it follows that $\rho = \frac{RA}{l}$

$$\rho = \frac{20 \times 3.14 \times 10^{-10}}{0.6}$$

$\rho = 1.05 \times 10^{-8}\ \Omega$ m

3. A material of a wire of length 120cm and crossectional area $0.04cm^2$ has a resistance as 0.5Ω at 0°C. Find the resistivity of metal at 300°C, given temperature coefficient of resistance $as\ 7.5x\ 10^{-3}K^{-1}$.

Solution

$$R_\theta = R_0(1 + a\theta)$$

$$R_{300} = 0.5(1 + 7.5 \times 10^{-3} \times 300)$$

$$= 1.625\Omega$$

$$\therefore \rho = \frac{RA}{L} = \frac{1.625 \times 0.04 \times 10^{-4}}{1.2}$$

$\rho = 5.42 \times 10^{-6}\Omega$ m

4. Find length of a wire of diameter 1.5mm and resistivity $2x10^{-6}$ at 30°C needed to make a coil of resistance 4Ω at 125°C, if temperature coefficient of resistance is $2.5 \times 10^{-3}\ K^{-1}$.

Solution:

$$R_\theta = R_0(1 + a\theta)$$

Resistance at 30°C:

$$R_{30} = R_0(1 + 2.5 \times 10^{-3} \times 30).$$

$R_{30} = 1.075R_O$..................(i)

Resistance at 125°C:

$$R_{125} = R_0(1 + 2.5 \times 10^{-3} \times 125)$$

$R_{125} = 1.3125R_0$.............(ii)

But $R_{125} = 4\Omega$

$$4 = 1.3125R_0$$

$$R_0 = 3.048\Omega$$

$R_{30} = 1.075R_O$

$R_{30} = 1.075x3.048 = 3.28\Omega$

$$R = \frac{\rho L}{A}$$

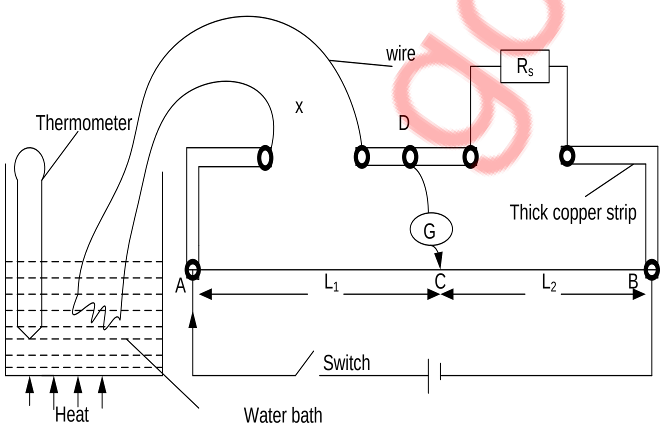
We are required to find length at 30°C

$L = \frac{RA}{\rho} = \frac{R_{30}A}{\rho_{30}}$ and $A = \frac{\pi d^2}{4}$

$$L = \frac{3.28 \times \pi(1.5 \times 10^{-3})^2}{2 \times 10^{-6}x\,4}$$

$$L = 2.9m$$

To Determine the Temperature Coefficient of Resistance of a Material



- A specimen fine wire is made into a coil and immersed in a water bath placed.
- The ens of the coil are connected in the left hand gap of the meter bridge and a standard resistance R_S in the right hand gap.
- The water bath is heated to a suitable temperature θ and stirred to ensure uniform temperature.
- Switch K is closed and the jockey tapped at different points on a uniform wire AB until a point is found where the galvanometer shows no deflection.
- The balance lengths l_1 and l_2 are measured and recorded.

- The resistance R_θ of the coil at temperature θ is determined from

$$R_\theta = R_s \frac{l_1}{l_2}$$

- The experiment is repeated for different values of temperature, θ and the results tabulated.
- A graph of R_θ against θ is plotted.
- The intercept R_0 on the R_θ - axis is read and the slope, S of the graph is determined.
- The temperture coeffiecnt of ressitance of the material is calculated from $\alpha = \frac{S}{R_o}$.

Examples:

1. A resistance coil is connected across the left hand gap of a metre bridge. When a 5.0 Ω standard resistor is connected across the right hand gap and the coil is immersed in an ice – water mixture, the balance point is at a point 45 .0 cm from the left hand end. When the coil is immersed in a steam bath at $100\ °C$, the balance point shifts to a point 52.8 cm from the left hand end of the bridge. Find the temperature coefficient of the material of the coil.

Solution:

At 0°C ; Using the balance condition

$$\frac{R_o}{R_s} = \frac{l_1}{l_2}$$

$$R_o = \frac{45}{55} \times 5 = 4.09\Omega$$

At 100°C: resistance at 100°C be R_{100}

$$\frac{R_{100}}{R_s} = \frac{l_1}{l_2}$$

$$R_{100} = \frac{52.8}{47.8} \times 5 = 5.59\Omega$$

$$\alpha = \frac{R_{100} - R_o}{\Delta\theta R_o}$$

$$\alpha = \frac{5.59 - 4.09}{4.09\ x\ (100-0)}$$

$$\alpha = 3.6 \times 10^{-3}\text{K}^{-1}$$

2. A resistance coil consists of a nichrome wire of diameter $4 \times 10^{-4}m$ and length $\frac{\pi}{2}$. The coil is connected across the left hand gap of a metre bridge. When a 10 Ω standard resistor is connected across the right hand gap and the coil is immersed in an ice – water mixture, the balance point is at a point 60 .0 cm from the left hand end.
   (i) Find the resistivity of nichrome wire and its resistance at 0°C
   (ii) What would the balance length be when the coil is immersed in a steam bath at $100\ °C$ (temperature coefficient of the nichrome wire between 0°C and 100°C is $1.7x10^{-4}\text{K}^{-1}$)

Solution:

(i) At 0°C ;

Using the balance condition we have;

$$\frac{R_o}{R_s} = \frac{l_1}{l_2}$$

$$R_o = \frac{60}{40} \times 10 = 15\Omega$$

$$\rho = \frac{RA}{L} \text{ and } A = \frac{\pi d^2}{4}$$

$$\rho_0 = \frac{15 \times \pi(4 \times 10^{-4})^2}{4x\ ^{\pi}/_2}$$

$$\rho_0 = 1.2 \times 10^{-6}\Omega\text{ m}$$

(ii)

$$R_{100} = R_0(1 + 100\alpha)$$

$$R_{100} = 15(1 + 1.7x10^{-4}x100)$$

$$R_{100} = 15.26\Omega$$

At 100°C

$$\frac{R_{100}}{R_s} = \frac{l_1}{l_2}$$

$$\frac{15.26}{10} = \frac{l}{100 - l}$$

$$l = 60.4cm$$

3. A nickle wire and a 10 Ω standard resistor are connected across the gaps of a meter bridge. When the nickel wire was at 0°C, balance point was found 40 .0 cm from the end of the bridge wire adjacent to the nickel wire. When the nickel wire was at $100\ °C$, the balance point shifts to a point 50.0 cm. Find the temperature of the nickel wire when the balance point was at 42.0cm and the resistivity of nickel at this temperature. (length of the wire is 150cm and cross-sectional area is $2.5 \times 10^{-4}cm^2$)

Solution:

At 0°C ; Using the balance condition

$$\frac{R_o}{R_s} = \frac{l_1}{l_2}$$

$$R_o = \frac{40}{60} \times 10 = 6.67\Omega$$

At 100°C: resistance at 100°C be R_{100}

l_1 = 50 cm, l_2 =100 – 50 = 50 cm

$$\frac{R_{100}}{R_s} = \frac{l_1}{l_2}$$

$$R_{100} = \frac{50}{50} \times 10 = 10\Omega$$

$$\alpha = \frac{R_{100} - R_o}{\Delta\theta R_o}$$

$$\alpha = \frac{10-6.67}{6.67x\,(100-0)}$$

$$\alpha = 5.0 \times 10^{-3}\mathrm{K}^{-1}$$

when the balance length is 42cm let the resistance be R_θ

$$R_\theta = \frac{42}{58} \times 10 = 7.24\Omega$$

$$R_\theta = R_0(1 + \theta\alpha)$$

$$7.24 = 6.67(1 + 5.0 \times 10^{-3}\theta)$$

$$\boldsymbol{\theta = 17.24}\text{°C}$$

$$\rho = \frac{RA}{L}$$

$$\rho_\theta = \frac{7.24 \times 2.5 \times 10^{-4} x 10^{-4}}{1.5}$$

$$\rho_\theta = 1.21 \times 10^{-7}\Omega\ \mathrm{m}$$

4. When a coil x is connected across the Left hand gap of a metre bridge and heated to a temperature of 30°C, the balance point is found to be 51.5°C from the left hand side of the slide wire. when the temperature is raised to 100°C, the balance point is 54.6cm from the left hand side. Find the temperature coefficient of resistance of x.

Solution

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

$$R_{30} = \frac{51.5}{100 - 51.5}R_x = 1.06R_x$$

$$R_{100} = \frac{54.6}{100 - 54.6}R_x = 1.203R_x$$

$$R_{30} = R_0(1 + 30\alpha) \ldots\ldots\ldots\ldots.(i)$$

$$R_{100} = R_0(1 + 100\alpha) \ldots\ldots\ldots\ldots.(i)$$

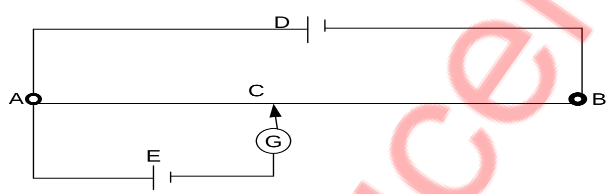
$$\frac{R_{30}}{R_{100}} = \frac{R_0(1 + 30\alpha)}{R_0(1 + 30\alpha)} = \frac{1.06R_x}{1.203R_x}$$

$$\alpha = 2.01x10^{-3}K^{-1}$$

Exercise

1. Two resistance coils P and Q are placed in the gaps of a metre bridge. A balance point is found when the movable contact touches the bridge wire at a distance of 35.5cm from the end joined to end P. When the coil Q is shunted with a resistance of 10Ω, the balance point is moved through a distance of 15.5cm. Find the values of the resistances P and Q.
2. In a metre bridge when a resistance in left gap is 2Ω and unknown resistance in right gap, the balance point is obtained from the zero end at 40cm on the bridge wire. On shunting the unknown resistance with 2Ω, find the shift of the balance point on the bridge wire. **An (22.5cm)**
3. With a certain resistance in the left gap of a slide wire, the balancing point is obtained when a resistance of 10Ω is taken out from the resistance box. On increasing the resistance from the resistance box by 12.5Ω, the balancing point shifts by 20cm. Find the value of unknown resistance. **An(15Ω)**

PRINCIPLE OF POTENTIOMETER SLIDE- WIRE



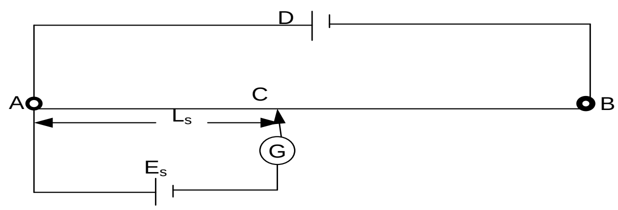
- A driver cell **D**, maintains a constant current through the slide wire.

- The wire is uniform hence has a constant resistance per cm and therefore the p.d per cm is also constant.
- Knowing the p.d per cm of the slide wire any p.d can be determined by balancing it against a known length of the wire
- If p.d per cm is k and balance elenght is l then the required p.d is $V = kl$

NB: The slider or jockey must not be scrapped on the potentiometer wire otherwise the wire will become non-uniform when scrapped.

Standardization (calibration) of a potentiometer wire

This refers to determining the p.d per cm of a potentiometer wire using a standard cell so that it can be used to measure p.ds.



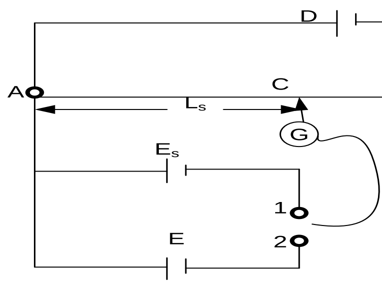
- Connect a standard cell of e.m.f E_s as shown above.
- The slidding contact is moved along the uniform wire AB until a point is found where the galvanometer G shows no deflection.
- The balance length l_s is measured.
- At balance point, E_s = p.d across AC.

$$\therefore E_s = kl_s$$

$$\Rightarrow \ k = \frac{E_s}{l_s}$$ where k =calibration constant

Applications of the potentiometer

To Measure e.m.f of a cell by comparison



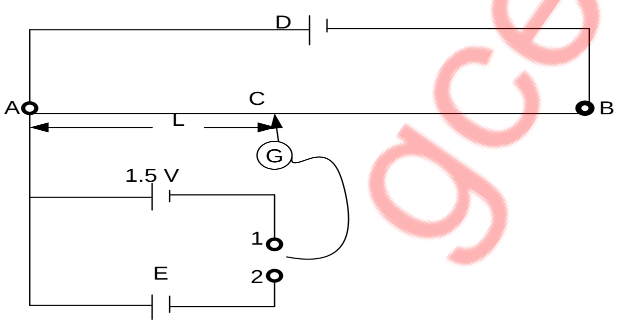
- Connect a standard cell of e.m.f E_s and the cell of unknown e.m.f E as shown above.
- With galvanometer connected to position 1, the jockey is tapped at different points along wire AB until a point is found where the galvanometer G shows no deflection.
- Measure the balance length l_s.
- With galvanometer connected to position 1, the jockey is tapped at different points along wire AB until a point is found where the galvanometer G shows no deflection.
- Measure the balance length l.
- The e.m.f of the test cell is got from

$$E = \left(\frac{l}{l_s}\right) E_s$$

Example:

A standard cell of e.m.f $1.5\ V$ is balanced on a potentiometer wire by a length of 60.0 cm. Another cell of unknown e.m.f, E is balanced on the same potentiometer wire by a length of 75.0 cm. Calculate the value of E.

Solution:



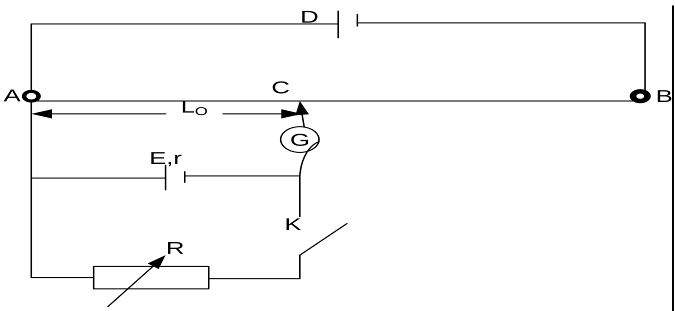
When G is at 1, the standard cell is balanced. At balance, p.d across AC = e.m.f of the standard cell

$$E = \left(\frac{l}{l_s}\right) E_s$$

$$E = \left(\frac{75}{60}\right) x1.5$$

$$E = 1.88V$$

To measure the internal resistance, *r* of a cell.



- Connect the cell and a resistance box *R*, as shown above.
- With switch K open, tap the jockey at different positions along the slide wire AB until you locate a point at which the galvanometer shows no deflection.
- Measure the balance length l_O
- Set the resistance box to a suitable value ***R*** and then close switch K.
- Tap the jockey at different positions along AB until a point C at which the galvanometer shows no deflection is located.
- Measure and record the balance length AC $= l$.
- Internal resistance of cell, $r = R\left(\frac{l_0}{l} - 1\right)$

Theory of experiment

At balance when K is open, the cell is on an open circuit and p.d across AC is equal to e.m.f of cell.

$E = klo$ 1

When K is closed the cell supplies current to R and is now on a closed circuit.

At balance p.d across R = p.d across AC

$V = kl$ 2

Divide equation 1 by 2

$\frac{E}{V} = \frac{l_0}{l}$...............................3

$E = I(R + r)$ and $V = IR$

Substitute for *E* and *V* in equation 3

$$\frac{I(R + r)}{IR} = \frac{l_0}{l}$$

$$\boxed{r = R\left(\frac{l_0}{l} - 1\right)}$$

Example:

1. A dry cell gives a balance length of 84.8 cm on a potentiometer wire. When a resistor of resistance 15Ω is connected across the terminals of the cell, a balance length of 75.0 cm is obtained. Find the internal resistance of the cell.

Solution

$$\frac{E}{V} = \frac{l_0}{l}$$

$E = I(R + r)$ and $V = IR$

$$\frac{I(R + r)}{IR} = \frac{l_0}{l}$$

$$r = R\left(\frac{l_0}{l} - 1\right)$$

$$r = 15\left(\frac{84.8}{75} - 1\right)$$

$r = 1.96\Omega$

2. The e.m.f of a battery A is balanced by a length of 75.0cm on a potentiometer wire. The e.m.f of a standard cell of 1.02V is balanced by a length of 50.0cm. Find;
   (i) E.m.f of battery A
   (ii) The new balance length if A has internal resistance of 2Ω and a resistor of 8Ω is connected across its terminals

Solution

$$E = \left(\frac{l}{l_s}\right)E_s$$

$$E = \left(\frac{75}{50}\right)x1.02$$

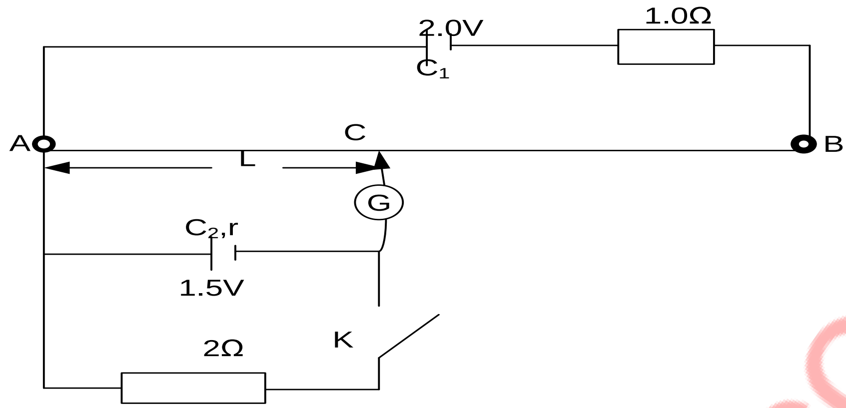
$E = 1.53V$

$$r = R\left(\frac{l_0}{l} - 1\right)$$

$$2 = 8\left(\frac{75}{l} - 1\right)$$

$l = 60cm$

3.



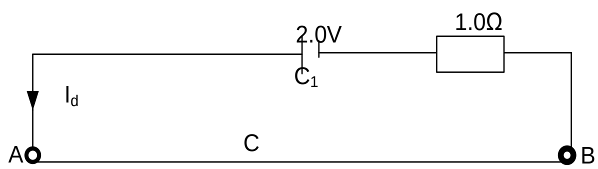
In the circuit above, AB is a uniform wire of length 1 m and resistance 4.0 Ω. C_1 is an acummulator of e.m.f 2 V and negligible internal resistance. C_2 is a cell of e.m.f 1.5 V.

(a) Find the balance length AC when the switch is open

(b) If the balance length is 75.0 cm when the switch is closed, find the internal resistance of C_2.

Solution:

(a) Consider the driver cell circuit only.



$$2 = I_d(R_{AB} + 1)$$
$$2 = I_d(4 + 1)$$
$$I_d = \frac{2}{5} = 0.4\ A$$

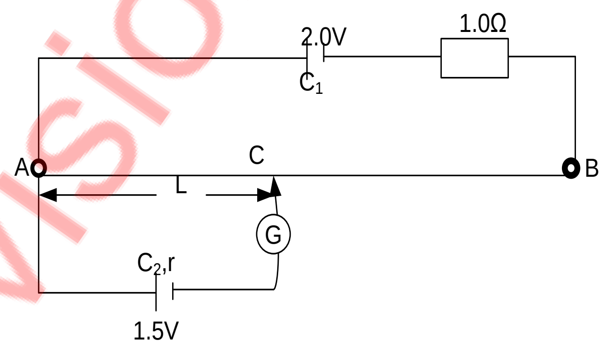
P.d across the whole wire, $V_{AB} = I_d R_{AB}$ where R_{AB} is the resistance of wire AB.

$$V_{AB} = 0.4 \times 4 = 1.6\ V$$

P.d per cm, k of AB is given by:

$$k = \frac{V_{AB}}{AB} = \frac{1.6}{100} = 0.016\ \text{V cm}^{-1}$$

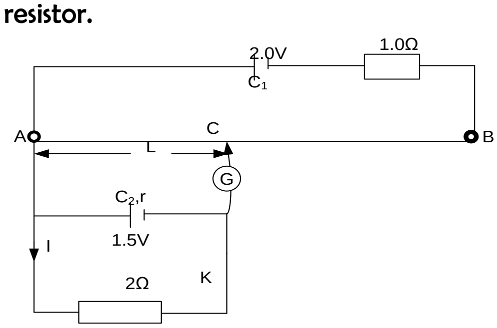
With S open cell C_2 is now on an open circuit and at balance; p.d across AC = e.m.f of C_2.

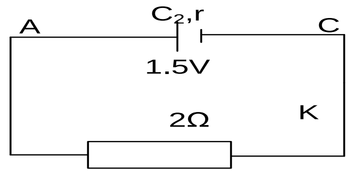


$$V_{AD} = kl = 1.5$$

$$l = \frac{1.5}{k} = \frac{1.5}{0.016} = 93.75\ \text{cm}$$

(b) With S closed, current is drawn from C_2 and the cell is now on a closed circuit and supplies current I to the 2.0 Ω resistor.





At balnce p.d across AC = p.d across the 2.0 Ω resistor, V

$V = k.l$ where l = 75.0 cm

$$V = 0.016 \times 75 = 1.2 \text{ V}$$

But V = IR, hence $I = \frac{V}{R} = \frac{1.2}{2} = 0.6 \text{ A}$

Since at balance the current through G is zero, we can now only consider the lower circuit shown above.

From $E = I(R + r)$ we have;

$$1.5 = 0.6(2 + r)$$

$$1.5 = 1.2 + 0.6r$$

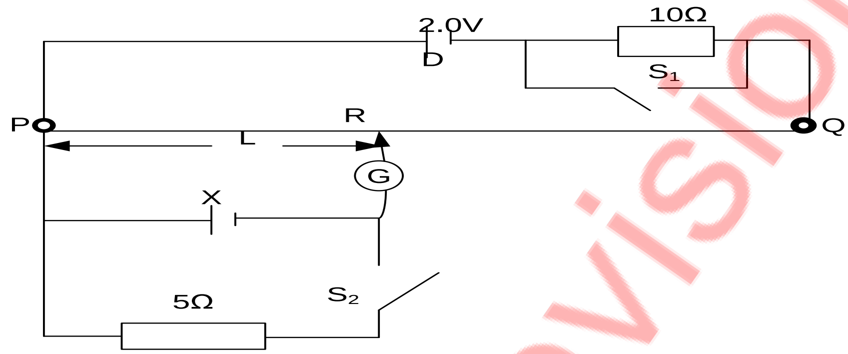
$r = \frac{0.3}{06}$ = 0.5 Ω

Or

$$r = R\left(\frac{l_0}{l} - 1\right)$$

$$r = 2\left(\frac{93.7}{75} - 1\right) = 0.5 \ \Omega$$

4. The figure below shows a cell D of negligible internal resistance with e.m.f $2V$. PQ is a uniform slide wire of length 1.00 m and resistance 50Ω.



With both switches S_1 and S_2 open, the balance length PR is 0.90 m. When S_2 is closed and S_1 left open, the balance length changes to 0.75 m. Determine the

(i) e.m.f of cell X.

(ii) internal resistance, r, of X

(iii) balance length when both S_1 and S_2 are closed

solution

(i) with both switches open: $2 = I_d(R_{AB} + 10)$

$$2 = I_d(50 + 10)$$

$$I_d = \frac{1}{30} A$$

P.d across the wire AB: $V_{AB} = \frac{1}{30} \times 50 = \frac{5}{3} V$

P.d per cm, k of AB is given by:

$$k = \frac{5}{300} \text{ V cm}^{-1}$$

e.m.f of cell x = kl

$$E = \frac{5}{300} x 90 = 1.5V$$

(ii) With S_2 closed:

p.d across x;

$$V = \frac{5}{300} x75 = 1.25V$$

current supplied by x:

$$I = \frac{V}{R} = \frac{1.25}{5} = 0.25A$$

But $E = I(r + R)$

$$1.5 = 0.25(r + 5)$$

$$r = 1\Omega$$

Or

$$\frac{E}{V} = \frac{l_0}{l}$$

$E = I(R + r)$ and $V = IR$

$$\frac{I(R + r)}{IR} = \frac{l_0}{l}$$

$$r \;= 5\left(\frac{90}{75} - 1\right) = \;1.0\;\Omega$$

(iii) When both switches are closed

10Ω is out of the circuit, current passes through the switch

$$2 = I_d(R_{AB})$$
$$2 = I_d(50)$$
$$I_d = \frac{1}{25}A$$

P.d across the wire AB: $V_{AB} = \frac{1}{25} \times 50 = 2\;V$

P.d per cm, k of AB is given by:

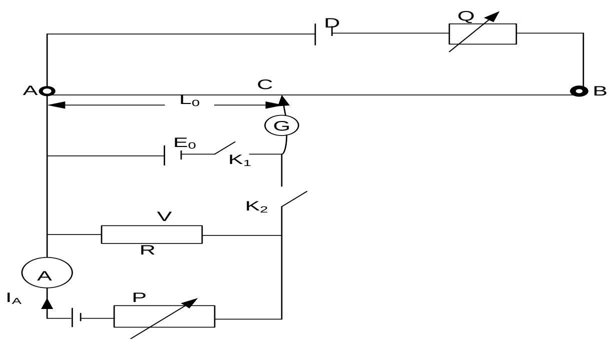
$$k = \frac{2}{100}\;\text{V cm}^{-1}$$

e.m.f of cell x $= kl$

$$1.25 = \frac{2}{100}xl$$
$$l = 62.5cm$$

Calibration of an ammeter and Current Measurement

Current can be measured on a potentiometer by measuring the p.d *V* it sets up across a standard resistance *R*, and then using $I = \frac{V}{R}$.



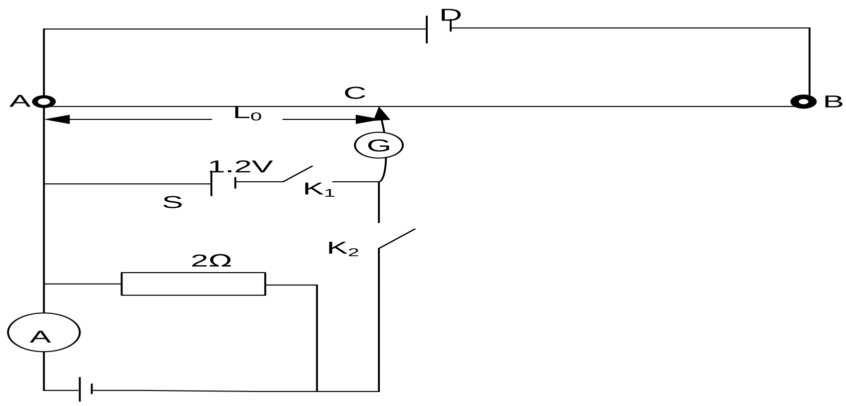
- Connect a standard cell of e.m.f E_o and a standard low resistance *R* as shown above.
- With switches K_1 closed and keep K_2 open, tap the jockey at different positions along AB until the galvanometer shows no deflection.
- Measure and record the balance length l_o
- P is adjusted so that the ammeter records the smallest current I_r. With switch K_1 open and K_2 closed, the balance length *l* is obtained and recorded.
- Determine the actual current I_A, $\left(I_A = \frac{E_0}{l_0}\,x\,\frac{l}{R}\right)$ and error *e*, in the ammeter reading, $e = I_A - I_r$ and
- The experiment is repeated for different adjustments of P and hence for different readings of the ammeter I_r.
- Tabulate the results including values of I_r and *e*.
- Plot a calibration graph of e against I_r

NB: The percentage error in the ammeter reading$= \frac{e}{I_a} \times 100\%$.

By calculating I_a the potentiometer is being used to measure current.

The nature of the graph shows that the errors in I_r occur randomly.

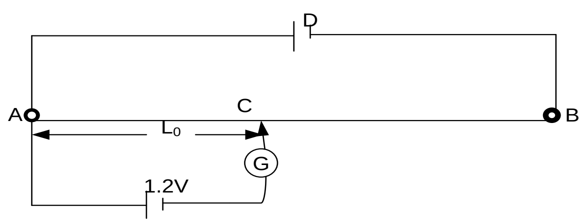
Example:



In the circuit above, S is a standard cell of e.m.f $1.2\ V$. When switch K_1 is closed and K_2 is open, a balance length AC= $30.2\ cm$ is obtained. When K_1 is opened and K_2 is closed, the balance length is $26.8\ cm$ and the ammeter, A reads $0.4\ A$. Calculate the percentage error in the ammeter reading.

Solution

K_1 is closed and K_2 is open;

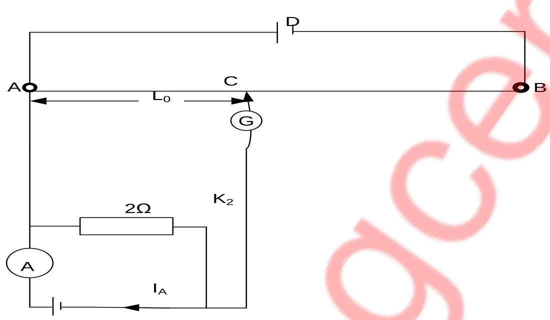


At balance, p.d across AC = 1.2

$30.2 \times k = 1.2$

$k = \frac{1.2}{30.2} = 0.0397$ V cm^{-1}

K_1 is opened and K_2 is closed;



Calibration of Voltmeter

At balance, p.d acrossAC= p.d across 2 Ω

$V = AC \times k$

$V = 26.8 \times 0.0397 = 1.064$ V

But $V = I_a R$

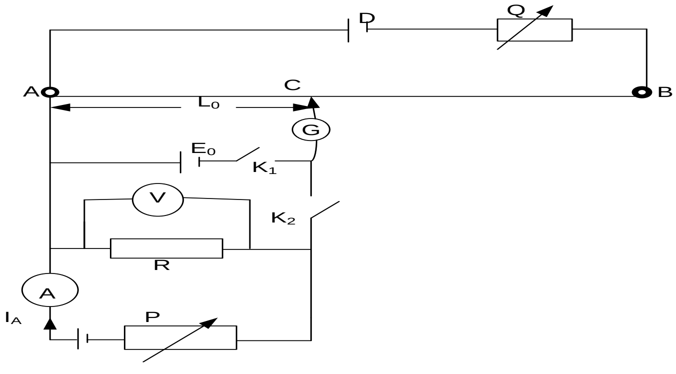
$I_a = \frac{V}{R} = \frac{1.064}{2} = 0.532$ A

Error in the ammeter reading, $e = I_a - I_r$

$e = 0.532 - 0.4 = 0.132$ A

$$percentage\ error = \frac{e}{I_a} \times 100\%$$

$$= \frac{0.132}{0.532} \times 100 = 24.8\%$$

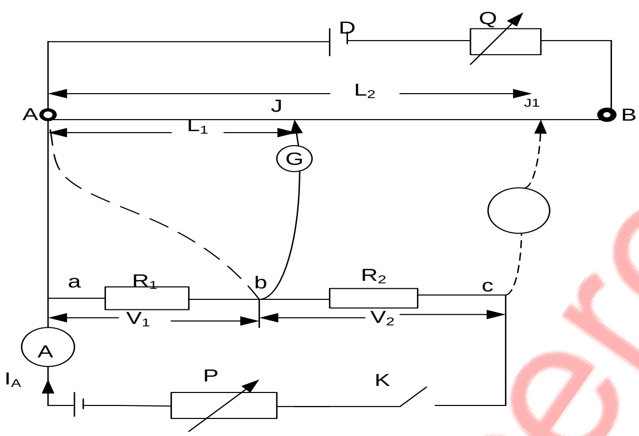


- A standard cell of e.m.f E_o and a rheostat P are connected as shown above.
- Switch K_1 is closed while K_2 is left open. A balance point is located where the galvanometer indicates zero deflection. The balance length l_o is measured and recorded..
- The p.d per cm, k of the slide wire is calculated from

$$k = \frac{E_0}{l_0} \text{ .... } 1$$

- P is adjusted so that the voltmeter records the smallest p.d it possible to read. K_1 is then closed and a point is located where the galvanometer register zero current.
- The balance length l is obtained and recorded.
- The experiment is repeated for different adjustments of P and hence for different readings, V_r of the voltmeter. Balance length l is determined.
- The results are tabulated including values of $V_a = kl$ and the error $e = (V_a - V_r)$.
- A graph of e against V_r is plotted and constitutes the calibration curve for the voltmeter.

Comparison of two Resistances (measurement of resistance)



- Connect the two resistances R_1 and R_2 in series with an ammeter A and rheostat P as shown above.
- Close switch K. With the galvanometer at ***a*** and ***b***, the balance length AJ = l_1 is measured and recorded. Hence $IR_1 = kl_1 \ldots\ldots\ldots\ldots (i)$
- Connections at ***a*** and ***b*** are removed and replaced by those at ***b*** and ***c***(dotted lines).
- The balance length AJ' = l_2 is measured and recorded. Hence $IR_2 = kl_2 \ldots\ldots\ldots\ldots (ii)$
  Equation (i) divide by (ii)

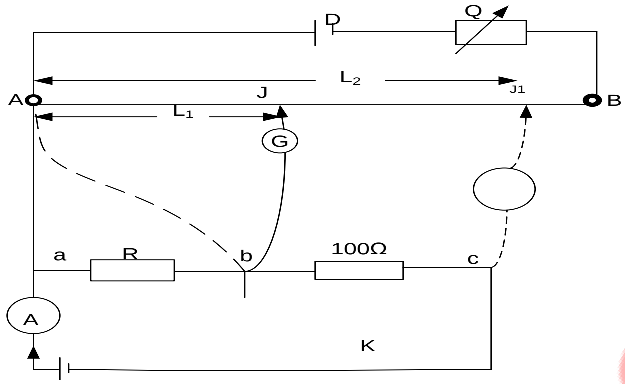
$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

Practical points to note

1. If the balance point cannot be obtained with both resistances, then either I is too large or the p.d V_{AB} across AB is very small. Balance can be achieved by adjusting rheostats P and Q.
2. If balance can be obtained with R_1 and not with R_2, then R_2 is much greater than R_1. The method is thus suitable for resistances that do not differ much in magnitude.
3. If the balance lengths are very small, an end – error (correction) due to the resistance of the contact at the zero end must be added to the balance lengths.

Example

1. The circuit below is used to compare the resistance R of an unknown resistor with a standard 100 Ω resistor.



The distances l from one end of the slide wire of the potentiometer to the balance point are 40.0 cm and 58.8 cm, respectively when G is connected to b and then to c respectively. If the slide wire is 1.00 m long, find the value of R.

Solution

Let the current in the lower circuit be I

With G connected at b,

the p.d across $100\ \Omega$ = p.d across *l*

$$100I = kl_1$$

where k is the p.d per cm and l_1 = 40.0 cm

$$100I = 40k \text{ ------------------- } 1$$

With G at c p.d 100 and R = p.d across the new balance length,

$$(100 + R)I = kl_2$$

$$(100 + R)I = 58.8k \text{ ------------------------ } 2$$

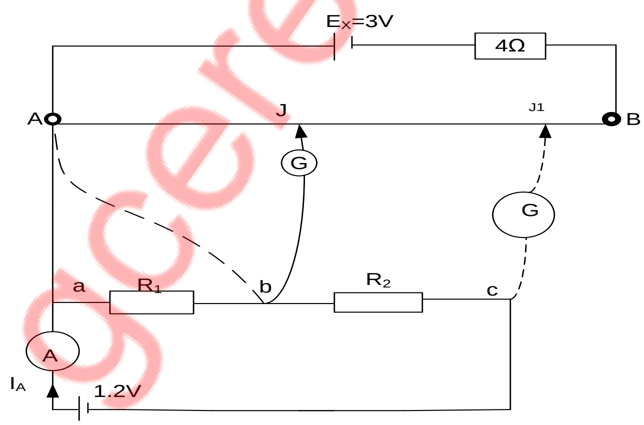
Divide equation 2 by equation 1

$$\frac{100 + R}{100} = \frac{58.8}{40.0}$$

$$100 + R = 147$$

$$R = 47\ \Omega$$

2. An accumulator of e.m.f 3V with negligible internal resistance is connected in series to a 4 Ω resistor and a potentiometer wire AB of length 1.0m as shown below.



The accumulator supplies a steady current of 0.25A through the wire AB. With the galvanometer connected at b, the balance length $AJ = 46cm$ and when the galvanometer is at c, the balance length $AJ^1 = 75cm$. Find;

(i) The value of R_1 and R_2

(ii) The reading of the ammeter A

Solution

(i) The current along AB $I = 0.25A$

p.d across 4 $\Omega = 0.25x4 = 1.0V$

P.d across AB $V_{AB} = 3 - 1 = 2V$

P.d per cm along AB, $k = \frac{2}{100} = 0.02\ Vcm^{-1}$

With G connected at b,

the p.d across R_1 = p.d across l

$$R_1 I = kl_1$$

where k is the p.d per cm and l_1 = 46.0 cm

$$R_1 x0.25 = 0.02x46$$

$$R_1 = 3.68\Omega$$

With G at c, p.d R_1 and R_2 = p.d across the new balance length,

$$(R_1 + R_2)I = kl_2$$

$$(3.68 + R_2)x0.25 = 0.02x75$$

$$R_2 = 2.32\Omega$$

(ii)

$$(R_1 + R_2)I_A = 1.2$$

$$\frac{1.2}{3.68 + 2.32} = I_A$$

$$I_A = 0.2A$$

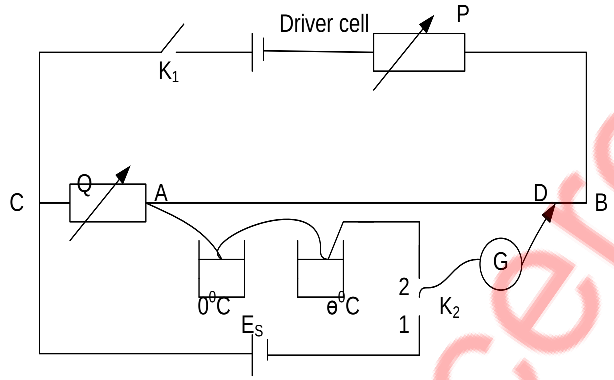
MEASUREMENT OF THERMOELECTRIC E.M.F

The e.m.fs of thermo junctions (thermocouples) are very small; of the order of a millivolt.

If such an e.m.f is measured on a simple potentiometer the balance – point is found to be very near end A and results in the end –error being serious, thus affecting the value obtained.

How to modify the simple potentiometer to measure small e.m.fs or p.ds

To obtain measurable balance lengths a suitable high resistance R is connected in series with the slide wire so that the driver cell sets up a small p.d across AB. This helps in producing a small p.d per cm.



- The standard cell of emf E_s is connected across Q and the slide wire.
- K₁ is closed and K₂ is connected to position 1. Tap the jockey at different positions along wire AB until the galvanometer shows no deflection.
- The balance length l_s is measured and recorded.
- While K₁ is closed, K₂ is connecteds to position 2 and the point on AB when the galvanometer registers zero current is found. The balance length l is measured.
- E is found using the formula $E = \frac{E_s r\, l}{Q + r\, l_s}$ where r is the resistance per cm of the slide wire.

Theory of experiment

Current through the wire AB $i_p = \frac{V_O}{P+Q+r\,l}$

where L is the length of the wire.

Hence p.d per cm, $k = i_p r = \frac{V_O\, r}{P+Q+r\,l}$

When K₂ is in position 1, $E_s = i_p Q + k\, l_s$

$$E_s = \frac{V_O}{P + Q + r\,l}(Q + r\,l_s) \dots\dots\dots\dots\dots (i)$$

When K₂ is in position 2, $E = kl$

Where E is the emf of the thermocouple.

$$E = \frac{V_O\, r\, l}{P + Q + r\,l} \dots\dots\dots\dots\dots (ii)$$

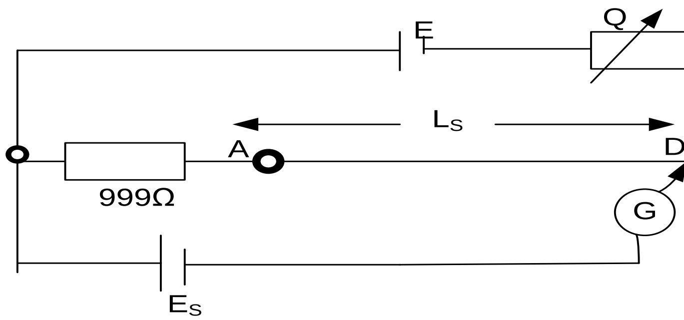
equation (ii) divide by (i)

$$\frac{E}{E_s} = \frac{r\,l}{Q + r\,l_s}$$

Hence $E = \frac{E_s r\, l}{Q + r\, l_s}$

Examples:

1.



In the figure above, E is a driver cell of e.m.f 2 V and negligible internal resistance. Es is a standard cell of e.m.f 1.00 V and AB is a uniform wire of resistance 10 Ω and length 100 cm. the galvanometer G shows no deflection when $ls = 10.0\ cm$. Find

(i) The current in the driver circuit

(ii) The resistance of the rheostat

(iii) The e.m.f of a thermocouple which is balanced by a length of 60 cm of the slide wire AB.

Solution:

(i) Let the resistance per cm of AB be β

$$\beta = \frac{R_{AB}}{100} = \frac{10}{100} = 0.1\ \Omega\ cm^{-1}$$

$$R_{AD} = \beta l_s$$

$$R_{AD} = 0.1 \times 10 = 1\ \Omega$$

But, $E_s = V_{999} + V_{AD}$

$$E_s = I_d(R + R_{AD})$$

$$I_d = \frac{E_s}{R+R_{AD}}$$

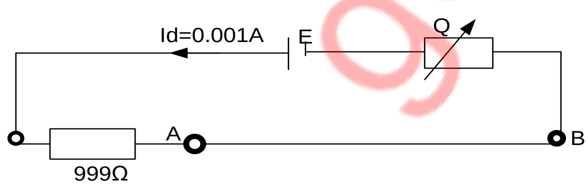
$$I_d = \frac{1}{999+1} = \frac{1}{1000} = 0.001\ \text{A}$$

(ii) $E = I_d(999 + 10 + R)$

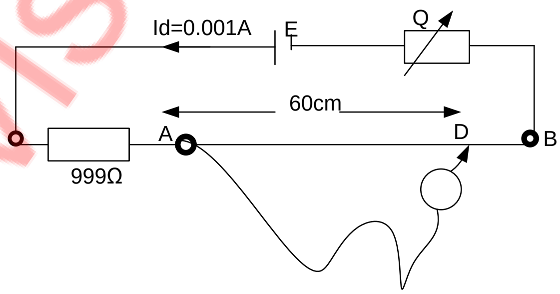
$$2 = 0.001(1009 + R)$$

$$= 1.009 + 0.001R$$

$$R = 991\ \Omega$$



(iii)



$$V_{AB} = I_d R_{AB}$$

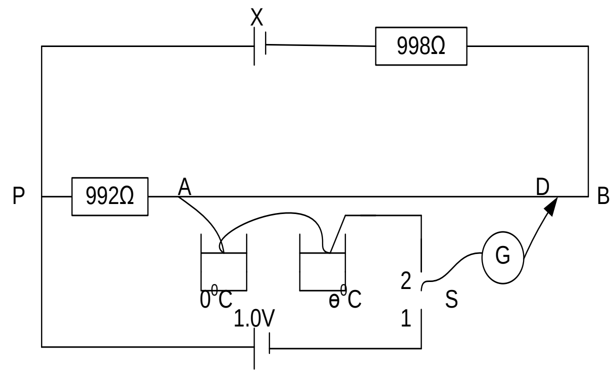
$$V_{AB} = 0.001 \times 10 = 0.01\ \text{V}$$

$$p.d\ per\ cm, k = \frac{V_{AB}}{100} = \frac{0.01}{100}$$

$k = 0.0001$ V cm^{-1}

$$E = kl = 0.0001 \times 60 = 0.006\ \text{V}$$

2.



In the figure above X is an accumulator of negligible internal resistance. AB is a uniform wire of length 1.0m and diameter $3.57x10^{-4}m$ and resisitivity $1.0x10^{-6}\ \Omega m$. G is a galvnomeer connected to a sliding contact D. When s is in position 1, G shows no deflection when AD is 80cm. When s is in position 2, G shows no deflection when AD is 40cm. find;

(i) The resistance of AB
(ii) E.m.f of the thermocouple
(iii) The e.m.f of the accumulator x

Solution

(i) $R = \rho\frac{l}{A}$

$$R_{AB} = 1.0x10^{-6}x\frac{1}{\pi\left(\frac{3.57x10^{-4}}{2}\right)^2}$$

$$R_{AB} = 10\Omega$$

(ii) When s is in position 1, the 1.0V cell is being balanced

For the driver cell; Let the resistance per cm of AB be β

$$\beta = \frac{R_{AB}}{100} = \frac{10}{100} = 0.1\ \Omega\ cm^{-1}$$

$$R_{AD} = \beta l_s$$

$$R_{AD} = 0.1 \times 80 = 8\ \Omega$$

At balance: $1.0 = V_{PA} + V_{AD}$

$$1.0 = I_d(R + R_{AD})$$

$$I_d = \frac{1}{992+8} = \frac{1}{1000} = 0.001\text{ A}$$

$$V_{AB} = I_d R_{AB}$$

$$V_{AB} = 0.001x\ 10$$

$$V_{AB} = 0.01V$$

p.d per cm along AB; $k = \frac{0.01}{100} = 1x10^{-4}Vcm^{-1}$

When s is in position 2, the thermocouple is being balanced

$$E_T = kl$$

$$E_T = 1x10^{-4}x40$$

$$E_T = 4mV$$

(iii) $E_x = I_d(998 + 992 + 10)$

$$E_x = 0.001(998 + 992 + 10)$$

$$E_x = 2.0V$$

Advantages of a potentiometer

1. It does not draw any current from the p.d being measured and therefore gives accurate results. Resistance of the connecting wires and the galvanometer does not affect the results.
2. It can measure a wide range of p.ds since the length of the slide wire can be adjusted.
3. It gives accurate results since they depend only on measurements of lengths, standard resistances and standard e.m.fs.

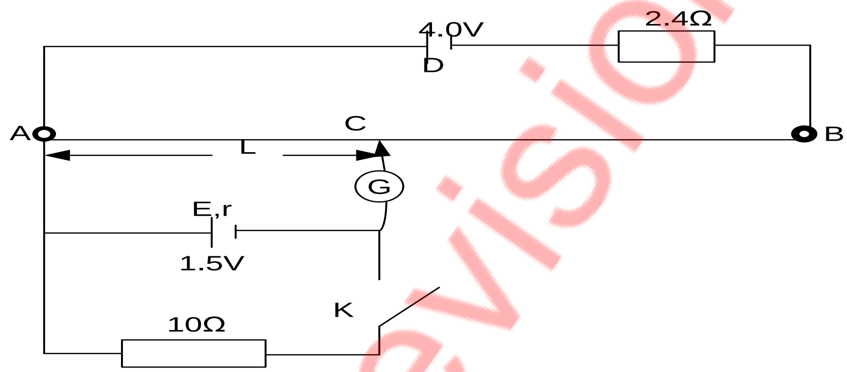
Disadvantages

1. It does not give direct results and is therefore time consuming;
2. It requires technical skills to operate;
3. It is cumbersome.

Exercise

1. A dry cell gives a balance length of 85.0cm on a potentiometer wire. When a resistor of resistance 16Ω is connected across the terminals of the cell, a balance length of 76.0cm is obtained. Find the internal resistance of the cell. **An(1.89Ω)**
2. A dry cell gives a balance length of 0.75m on a potentiometer wire. When the cell is shunted by a resistance of 14Ω, the balance length of 0.70m is required. Find the internal resistance of the cell. **An(1.0Ω)**
3. A 1Ω resistor is in series with an ammeter m in a circuit. The p.d across the resistor is balanced by a length of $60cm$ on a potentiometer wire. A standard cell of emf $1.02V$ is balanced by a length of $50cm$. If m reads $1.1A$, what is the error in the reading? $\boldsymbol{An}(\mathbf{0.124A})$
4. A potentiometer wire of length $1m$ and resistance 1Ω is used to measure an emf of the order mV. A battery of emf $2V$ and negligible internal resistance is used as a driver cell. Calculate the resistance to be in series with potentiometer so as to obtain a potential drop of $5mV$ across the wire. $\boldsymbol{An}(\mathbf{399\Omega})$
5. In a potentiometer, a cell of emf **x** gave a balance length of **a** cm and another cell of emf **y** gave a balance length of **b** cm. When the cells are connected in series, a balance length of **c** cm was obtained.

   It was also discovered that **a + b** ≠ **c.** Show that the true ratio $\frac{x}{y} = \frac{c-b}{c-a}$.
6.

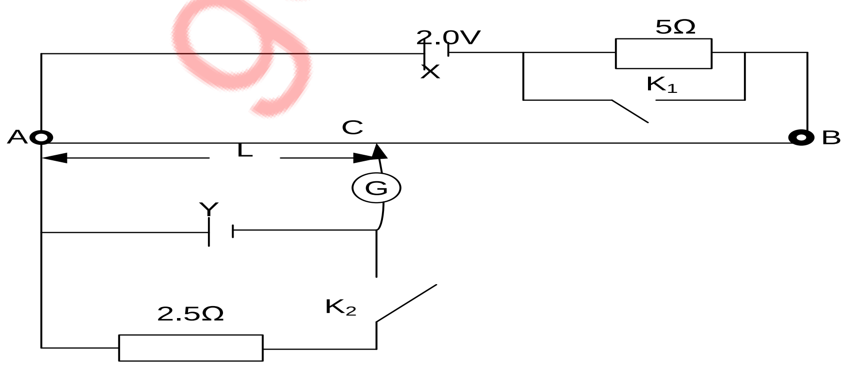


In the circuit above, AB is a uniform wire of length 1 m and resistance 2.0 Ω. D is a cell of e.m.f 4.0 V and negligible internal resistance. E is a cell of e.m.f 1.5 V.

(i) Find the balance length AC when the switch is open. **An(82.5cm)**

(ii) If the balance length is 71.5 cm when the switch is closed, find the internal resistance of E. **An(1.54Ω)**

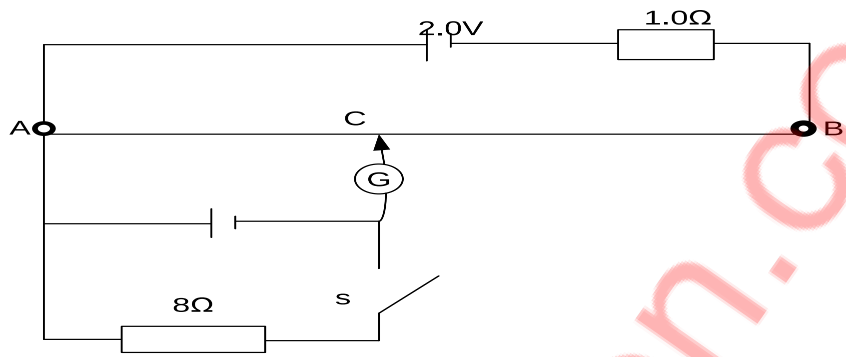
7.



In the circuit above, x has negligible internal resistance and length AB is 100cm and resistance of AB is 20Ω. When K_1 and K_2 is open, the balance length $AC = 80cm$. When K_2 is closed and K_1 open, the balance length $AC = 65cm$. Find

(i) the emf of cell y
(ii) internal resistance of cell y.
(iii) the balance length when K_1 and K_2 are closed.

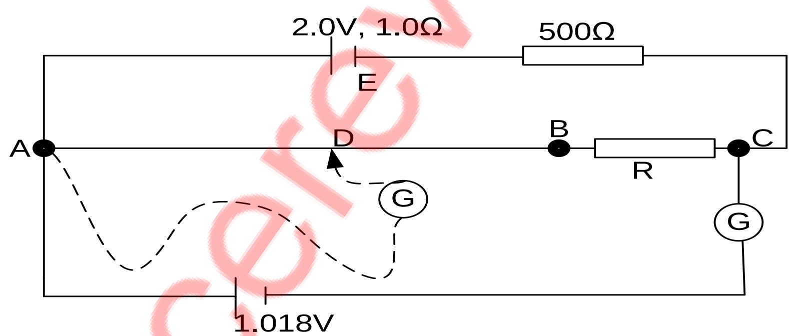
8.



In the figure above the slide wire AB is 1 m long and has a resistance of 4Ω . When switch S is:

(i) open, the balance length AC is 88.8 cm. Find the value of the e.m.f of the cell
(ii) closed, the balance length is found to be 82.5 cm. Calculate the internal resistance of this cell

9. In the figure below, AB is a uniform resistance wire of length 1.00 m and resistance 10.0Ω E. is an accumulator of e.m.f 2.0 V and internal resistance 1.0 Ω

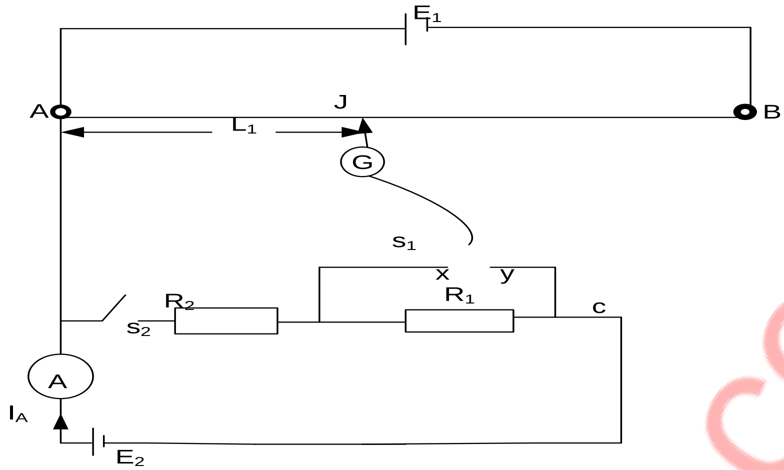


When a standard cell of e.m.f 1.018 V is connected in series with a galvanometer, G across AC, the galvanometer shows no deflection. When the standard cell is removed and a thermocouple connected via the galvanometer, G, as shown by the dotted line, G shows no deflection when AD = 41.0 cm.

Calculate the:

(i) value of R,
(ii) e.m.f of the thermocouple. **An(509.3Ω, 8.04mV)**
(iv)

10.



The circuit above shows a uniform slide wire AB of length 100 cm. The e.m.f. of cell E_1 is 2.0 V and its internal resistance is negligible. E_2 is a cell of e.m.f 1.1 V and its internal resistance is 1.0 Ω, R_1 = 1.0 Ω and R_2 = 2.0 Ω. The switch S_1 enables the galvanometer G to be connected to X or Y. Calculate the balance length for each position of S_1 when the switch S_2 is

(a) open and

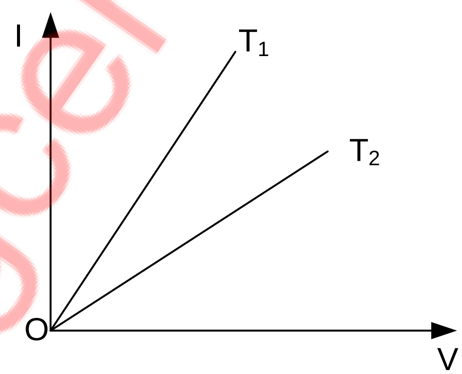
(b) Closed.

Uneb 2016

(a) **(i)** Define **electrical resistivity** (01mark)

(ii) Explain how length and temperature of a conductor affect its resistance. (04marks)

(iii) Figure below shows the current- voltage graphs for a metallic wire at two different tempeatures T_1 and T_2

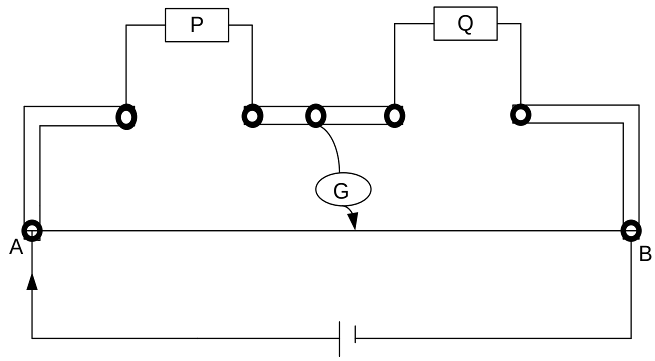


State which of the two temperatue is greater and explain your answer. (03marks)

(b) (i) Derive the balance condition when using a meter bridge to measure resistance. (04marks)

(ii) State two precautions taken to achieve an accurate measurement. (02marks)

(c) Figure below show two resistors P and Q of resistance 5Ω and 2Ω repectively connected in the two gaps of the meter bridge.



A resistance X of cross-sectional area $1mm^2$ is connected across P so that the balance point is 66.7 cm from A. If the resistivity of the wire X is $1.0x10^{-5}\Omega m$ and the resistance wire AB of the meter bridge is 100cm long, calculate the length of X. (04marks)

(d) Explain how electrons attain a steady drift velocity when current flows through a conductor. (02marks)

Uneb 2015

(a) **(i)** Define **temperature coefficient** of resistance (01mark)

(ii) Explain the origin of the heating effect of electric current in a metal conductor. (03marks)

(iii) Describe with the aid of an I-V sketch the variation of current with p.d a cross a semiconductor. (02marks)

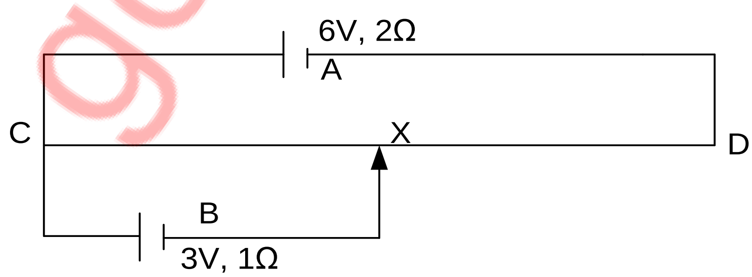
(b) A cell, a bulb, a swtich and a thermistor with negative temperature coefficient of resistance are connected as shown below

(i) Explain what would happen when in figure (i) switch K is closed (04marks)

(ii) If the bulb in figure (i) is removed and circuit connected as shown in figure (ii), explain what happens when switch K is closed. (03marks)

(c) State the **law of conservation of current at a junction**. (01mark)

(d) Two cells A, of e.m.f 6V and internal resistance 2Ω and B of e.m.f 3V and internal resistance 1Ω respectively are connected across a uniform resistance wire CD of resistance 8Ω as shown below



If X is exactly in the middle of the wire CD, calculate the;

(i) Power dissipated in CX (04marks)

(ii) P.d across the terminals of cell A (02marks)