

## A LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2019

### STATISTICS

1. (a) A sample consists of 15 numbers, 2,4,7,3,5,6,3,6,10,7,8,9,3,4,3. Find:  
(i) the mean. (ii) standard deviation of the sample. (iii) un biased estimate of population standard deviation

(b) The following values were recorded in a frequency table;  $\sum f(x) = 600$  ,  $\sum fx^2 = 10,000$ .  
The standard deviation was calculated as 5. Find the total number of items that were considered.

2. Study the table below;

Marks	20 - 24	25 -34	35 - 38	39 - 44	45 - 49	50 - 54
Frequency	6	10	8	9	5	2

- (a)(i) Construct a histogram using this data (ii) Use the histogram to estimate the mode.  
(b). Calculate the mean mark.

3. The table shows the distribution of heights H among spinach plants in a garden

Heights (H cm)	$\leq 10$	$\leq 20$	$\leq 30$	$\leq 40$	$\leq 50$	$\leq 60$	$\leq 70$	$\leq 80$
Number of plants	10	16	21	27	14	12	8	2

- (a) Calculate; (i) the mean (ii) the median height (iii) standard deviation  
(b). Plot an ogive for this data and use it to estimate the interval within which the central 80% of the heights lie.  
(c) Calculate the 10<sup>th</sup> and 70<sup>th</sup> percentile range.  
(d) If a sample of 80 plants is taken randomly from the garden, find the 95% confidence limit for the mean height of the total plants.
4. The Mock examination marks and the Average final examination marks of a certain school are given in the following table

Mock Marks	28	34	36	42	52	54	60
Average final examination Marks	54	62	68	70	76	66	74

- (a) (i) Plot the marks on the scatter diagram and comment on the relationship between the two marks.  
(ii) Draw a line of best fit and use it to predict the average final mark of a student whose mock mark is 50.

- (b) Calculate the rank correlation coefficient between the marks and comment on your result at 5% level of significance.

5. The table below shows the prices of four commodities and their weights in 2006 and 2007.

Commodity	Price (U shs)		Weight
	2006	2007	
Banana (1 bunch)	3000	8000	4
Meat(1kg)	2500	3000	3
Milk (1 litre)	300	400	2
Sugar (1 kg)	1500	1800	1

Taking 2006 as the base year, find:

- (i) the price relative for each commodity. (ii) Weighted price index for all the commodities

### **PROBABILITY**

6. (a) A and B are two independent events with A twice as likely to occur as B. If  $P(A) = \frac{1}{2}$  find:
- (i)  $P(A \text{ or } B \text{ but not both})$  (ii)  $p\left(\frac{A}{B'}\right)$
- (b) At a certain university, the intake policy is such that 40% and 60% of the applicants taken are women and men respectively. The corresponding percentages on government sponsorship are 50% and 40%; the rest are privately sponsored
- (i). What is the probability that a student picked at random is a man given that he is on government sponsorship
- (ii) Twenty students are picked at random, find the mean number of privately sponsored females in the group.
7. (a) X is a discrete random variable which takes all integers from 1 to 40; such that:  
 $P(X = x) = kx; \quad x = 1, 2, 3, \dots, 40.$
- (i) Find the value of the constant k; and compute the standard deviation of X.
- (ii). Find;  $P(X < 35/X > 20)$
- (b) A random variable X has a cumulative distribution function given below.
- $$F(x) = \begin{cases} 0, & x \leq 0 \\ ax, & 0 \leq x \leq 1 \\ \frac{x+b}{3}, & 1 \leq x \leq 2 \\ 1, & x \geq 2 \end{cases}$$
- Find; (i) the value of a and b (ii)  $P(X < 1.5/X > 1)$  (iii). Obtain and sketch  $f(x)$  (iv) Mean of X
8. (a) The weights of ball bearings are normally distributed with mean 25gram and standard deviation 4 grams. If a random sample of 16 ball bearings is taken, find the;
- (i) Probability that the mean of the sample is between 24.12 grams and 26.73 grams.

(ii) Interquartile range.

(b) A random sample of 120 girls taken from a normally distributed population of school girls gave a mean age of 16.5 and variance of 18. Determine the 97% confidence interval for the mean age of all the school girls.

9. (a) A box A contains 1 red, 3 green and 1 blue balls. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?
- (b) The normal distribution  $X \sim N(\mu, \delta^2)$ . Given that  $P(X > 80) = 0.0113$  and  $P(X < 30) = 0.0287$ , find the mean  $\mu$  and standard deviation  $\delta$ .

### NUMERICAL METHODS

10. Given that  $A = |a||b|\sin\theta$ ;

(a) Show that the maximum possible relative error in A is given by  $\left|\frac{\Delta a}{a}\right| + \left|\frac{\Delta b}{b}\right| + |\Delta\theta|\cos\theta$ .

where  $\Delta a$ ,  $\Delta b$  and  $\Delta\theta$  are small numbers compared to  $a$ ,  $b$  and  $\theta$  respectively.

(b) Write down the possible errors in each measurement where  $a = 2.5\text{cm}$ ,  $b = 3.4\text{cm}$  and  $\theta = 30^\circ$ .

(c) Value within which the area is enclosed.

(d) Maximum percentage relative error in the area.

11. Show that the equation  $\pi \sin x - x = 0$  has a root between  $\frac{\pi}{2}$  and  $\pi$ . Hence use linear interpolation only once to find the root correct to three significant figures.

(b) Using the trapezium rule with 7 ordinates, estimate  $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2-\cos x}} dx$ , correct to 3 decimal places.

(c) Calculate the percentage error in your estimation in (a) above.

(d) State how the above error can be reduced.

12. (a) Using the graphical method, show that the equation  $3x^3 + x - 4 = 0$  has a root between 0 and 2.

(b) Use linear interpolation once to find the initial approximation of the root.

(c) Hence using Newton Raphson's method, find the root correct to three decimal places.

13. The correct fares a special car hire company charges are proportional to the squares of the distances travelled. The only correct amounts the drivers of a company declare are shs 5,000 and shs 9,800 for distances of 10km and 14 km travelled respectively, and they expect to cheat the travelers by charging the rest of the distances by linear interpolation or extrapolation. Find;

(a) The amount by which one who travels 13km is cheated.

(b) What distance, on the wisdom of the drivers, one travels for free.

14. (a) Show that the Newton Raphson's formula for approximating the natural logarithm of the  $K^{\text{th}}$  root of a number N is given by  $x_{n+1} = \frac{1}{K}(Kx_n - 1 + Ne^{-Kx_n})$ .

- (b) Draw a flow chart that;
- (i) reads the number  $N$ ,  $K$  and the initial approximation  $x_0$ .
- (ii) Computes and prints the root, correct to 3dps.
- (c) Perform a dry run for  $K = 4, N = 20$  and  $x_0 = 0.65$ .

### MECHANICS

15. (a) A particle of mass 5 Kg resting at point (1, -4, 4) is acted upon by three forces.  $F_1 = 3i + 3j$ ,  $F_2 = 2j + 4k$ ,  $F_3 = 2i + 6k$ . Find the position and momentum of the particle after 4 seconds.
- (b) A particle of mass 4kg moves with a velocity of  $e^t i + 2e^{2t} j - \sin t k$ . Find the power developed after 4 seconds.
16. (a) A body of mass 5 kg slides a distance of 8 m down a rough plane inclined at an angle of  $\sin^{-1} \left( \frac{4}{5} \right)$  to the horizontal. If the coefficient of friction is 0.4, find the velocity attained by the body.
- (b) A particle of mass 50 kg is suspended by two light inelastic strings of lengths 9 m and 12 m attached to two points distant 15 m apart. Calculate the tensions in the strings.
17. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:
- (a) acceleration of the system.                      (b) tension in the string.
- (c) distance moved by the moveable pulley in 1.5 s.
18. (a) A particle is executing simple harmonic motion with amplitude 2metres and period 12 seconds. Calculate the maximum speed of the particle.
- (b) If initially, the particle was moving at a maximum speed, find the;
- (i) distance moved by the particle until its speed is half the maximum value
- (ii) time taken by the particle to travel this distance.
19. (a) A, B, C, D, is a square. Forces of magnitudes 9N, 5N, and  $3\sqrt{2}$  N act along  $\overrightarrow{AB}, \overrightarrow{BC}$  and  $\overrightarrow{BD}$  respectively. Find the magnitude and direction of their resultant force.
- (b) A non-uniform ladder AB whose centre of gravity is 2m from end A is of length 6m and weight,  $W$ . The ladder is inclined at an angle  $\theta$  to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is  $\mu$ . If the ladder is about to slip when a man of weight  $5W$  ascends two-thirds of the way up the ladder, show that  $\tan\theta = \frac{11-7\mu^2}{18\mu}$

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