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LITTORAL MATHEMATICS TEACHERS ASSOCIATION

GENERAL CERTIFICATE OF EDUCATION REGIONAL MOCK EXAMINATION

0575 Additional Mathematics 2

MARCH 2020

ORDINARY LEVEL

CENTRE NO & NAME	
Candidate No	
Candidate Name	

Two and half hours

Answer ALL QUESTIONS IN SECTION A and ANY TWO QUESTIONS from either SECTION B or SECTION C

Section A is compulsory for all candidates

All questions carry equal marks

All necessary working must be shown. No mark will be awarded for answers without brief statements showing how the answers have been obtained.

Electronic calculators may be used

Where necessary take g as 10ms^{-2}

SECTION A (Pure mathematics)**(This section is compulsory for all candidates)**

1. (i) Given $f(x) = 2x^3 - 7x^2 + 7x - 2$
- Show that $f(1) = 0$ and hence factorize $f(x)$ **(3 marks)**
 - Solve the equation $f(x) = 0$ **(2 marks)**
- (ii) (a) In how many ways can a committee of 4 men and 3 women be chosen from 7 men and 8 women? **(2 marks)**
- (b) In how many ways can the committee be chosen if a particular man refuses to serve in the same committee as a particular woman? **(2 marks)**

2. (i) (a) Expand $\left(x - \frac{1}{2x}\right)^9$ in ascending powers of x up to and including the term in x^3 . **(3 marks)**
- (b) Also show that there is no term independent of x in the expansion **(2 marks)**
- (ii) Given the lines $\mathbf{r}_1 = \mathbf{i} - 2\mathbf{j} + \beta(2\mathbf{i} + \mathbf{j})$ and $\mathbf{r}_2 = -\mathbf{i} + 3\mathbf{j} + \mu(-2\mathbf{i} + \mathbf{j})$
- Show that the lines intersect **(2 marks)**
 - Find the position vector of this point of intersection. **(2 marks)**

3. $f(x) = 2\sin x, x \in \mathbb{R}$ and $g(x) = x + \frac{\pi}{4}, x \in \mathbb{R}$
- Find $f\left(\frac{\pi}{6}\right)$ and $g\left(\frac{\pi}{5}\right)$ **(2 marks)**
 - Find the range of f , and the range of g **(4 marks)**
 - Solve the equation $fg(x) = -1$ for $0 \leq x \leq \pi$ **(4 marks)**

4. (i) The gradient of a curve is given by $\frac{dy}{dx} = ax + b$. Given that the curve passes through $(0, 0)$, $(1, 2)$ and $(-1, 4)$ find
- the values of a and b **(2 marks)**
 - the equation of the curve **(2 marks)**
- (ii) $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}, x > 0$.
- Find $\int y dx$. **(2 marks)**
 - Hence show that $\int_1^3 y dx = A + B\sqrt{3}$ where A and B are integers to be found. **(2 marks)**

5. A sequence is defined by $10x, 9x + y, 8x + 2y, \dots, 10y$
- Show that sequence is an arithmetic progression **(2 marks)**
 - State the common difference **(1 mark)**
 - Find the sum of the sequence in terms of x and y . **(3 marks)**
 - Find the values of x and y given that the second term of the sequence is 29 and the sum of the sequence is 275. **(3 marks)**

6. A small company produces two types of armchair. The cost of labour and materials for the two types is shown in the table below.

	Labour	Materials
Standard	30.000	25.000
Deluxe	40.000	50.000

The total spent on labour must not be more than 1,150,000FCFA and the total spent on materials must not be more than 1250.000FCFA the profit on a standard chair is 70.000FCFA and the profit on deluxe chair is 100.000FCFA.

- Write down the four inequalities involving x and y **(2 marks)**
- Write down an equation for the total profit P . **(1 mark)**
- On a graph paper, shade so as to leave un-shaded, the region represented by the inequalities in (a) **(3 marks)**
- How many chairs of each type should be made to maximize the profit? **(2 marks)**
- Find the maximum profit. **(1 mark)**

7. Given that $f(x) = 2 + 4\cos 2x$.

- a. Draw a graph of $y = 2 + 4\cos 2x$ for $0 \leq \theta \leq \frac{\pi}{2}$ using the Interval of $\frac{\pi}{12}$ for x showing clearly the coordinates of the point where the curve cuts the coordinate axes. (5 marks)
- b. Using your graph, solve for x the equation of $\cos 2x = 0$. (2 marks)

8. i) The matrix M of transformation T is given by $M = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$

- a) Find the image of the point $(-1, 4)$ (2 marks)
- b) Calculate the inverse of the matrix M (2 marks)
- c) Find the image L^1 of the line L whose equation is $x - y = 0$ under T (2 marks)
- d) Calculate the tangent of the angle between L^1 and L (2 marks)
- ii) The roots of the equation $4x^2 + 5x + 1 = 0$ are α^2 and β^2 where $\alpha > 0$ and $\beta > 0$.
- a. Find without solving the equation, the values of $\alpha + \beta$ and $\alpha\beta$ (2 marks)
- b. Hence, find a quadratic equation with integral coefficients whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ (2 marks)

SECTION B: MECHANICS.

IF THIS SECTION IS CHOSEN THE SECTION C MAY NOT BE CHOSEN.

ANSWER ANY TWO QUESTIONS FROM THIS SECTION.

9. (i) The position vector of a particle P, moving in the plane of the coordinate axes, OX and OY, at time t seconds is given by

$$\vec{r} = \left[\frac{1}{3}t^3 \mathbf{i} - \left(\frac{1}{2}t^2 - 3t \right) \mathbf{j} \right] \text{ m. when } t = 3, \text{ find;}$$

(a) the distance of P from O.

(b) the magnitude and direction of the acceleration of P

(ii) A particle of mass 2 kg, is moving in the plane of the coordinate axes, OX and OY, with an acceleration of

$(6t\mathbf{i} - 2\mathbf{j}) \text{ ms}^{-2}$ t being the time in seconds. When $t = 0$ the velocity of the particle is $(2\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$ and the particle is at the point $(\mathbf{i} - 2\mathbf{j}) \text{ m}$. Calculate when $t = 2$

(a) the force acting on the particle.

(b) the magnitude of the impulse acting on the particle.

(c) the position vector of the particle.

10. (i) A car of mass 1000 kg is moving on a smooth level road at a steady speed of $\frac{250}{9} \text{ ms}^{-1}$ against an air resistance R . The engine works at a rate of 60 kW . Calculate the air resistance, R , when the car starts to ascend a rough plane inclined at an

angle $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal with this speed, it runs out of fuel. Assuming that the air resistance remains the same

and that the driver does not apply the brakes, find, to one decimal place, how far up the plane the car will climb before coming to rest, given that the coefficient of friction between the plane and the tyres of the car is 0.1175.

- (ii) A constant force of magnitude $F \text{ N}$ acts on a particle of mass 4 kg and changes its speed from 5 ms^{-1} to 10 ms^{-1} in 4 seconds. Find

(a) the value of F

(b) the impulse of the force on the particle,

(c) the work done on the particle in this interval.

11. A particle A of mass 3 kg , lies on a smooth plane inclined at 30° to the horizontal. A light inextensible string attached to A passes over a light smooth pulley, which is at the top of the plane, and is attached to another particle B, of mass 5 kg , which hangs freely. The string lies along the line of greatest slope of the inclined plane. The system is released from rest with the string taut.

(a) Draw a diagram to illustrate all the forces acting on the particles

(b) Find the reaction N , of the plane on the particle

(c) The acceleration, in ms^{-2} of each particle

(d) The tension, in N of the string.

(e) The resultant forces exerted on the pulley.

SECTION C: STATISTICS AND PROBABILITY

IF THIS SECTION IS CHOSEN THE SECTION B MAY NOT BE CHOSEN.

ANSWER ANY TWO QUESTIONS FROM THIS SECTION.

12. (I) Ten values of a variable y are 8.2, 8.0, 8.1, 8.2, 8.4, 7.9, 8.0, 8.3, 7.8, 8.1

- Given that $y = 8 + 0.1x$, state the corresponding values of x
- Calculate the arithmetic mean and variance of the ten values of x
- Hence deduce the arithmetic mean and standard deviation of the ten values of y .

- (ii) The table below shows the marks scored by 20 students in a mathematics test.

Marks, x	6 – 8	9 – 11	12 – 14	15 – 17	18 – 20
Number of students, f	2	4	8	6	1

Using graph papers

- draw a histogram of this distribution, and hence estimate a value of the mode of the distribution
- draw a cumulative frequency graph and from your graph estimate the median mark.
- given that 45% of the candidates were to pass the test, find the least pass mark.
- given that 10 is the pass mark, find the probability that a student chosen at random is successful.

13. (i) A bag contains n red balls and m yellow balls.

- Given that the probability of selecting a red ball at random from the bag is $\frac{3}{8}$, write down an equation connecting n and m .
- Six more red balls are added to the balls in the bag and a ball is selected at random. Given that the probability that the ball is red is now $\frac{1}{2}$, find the values of n and m .

- (ii) A girl has a total of 120 marks in x tests. In the next two tests she has 10 marks and 14 marks respectively. Find, in terms of x , her average number of marks per test for,

- the first x tests
- the $(x + 2)$ tests

Given that her average for the first x tests was the same as her average for the $(x + 2)$ tests,

- calculate the value of x

14. (i) State briefly the meaning of the statistical terms mean, mode and median. Give an example of a situation where the most appropriate average would be:

- the mode
- the median

- (ii) The distribution of marks scored by a number of candidates in mathematics in the 2013 G.C.E. was such that its 20th percentile was 25 marks, its 40th percentile was 50 marks, its 60th percentile was 64 marks, its 80th percentile was 74 marks and its 100th percentile was 100 marks. By drawing a cumulative frequency graph, or otherwise, estimate,

- the median of the distribution
- lower and upper quartile of the distribution,

- (iii) A set of 10 positive integers has a mean of 6 and variance of 14. Find

- the sum of the 10 integers.
- the sum of square of these integers.

Given that 15 is one of these integers and it is removed, find

- the mean and variance of the nine remaining integers.