Pure Math With Stats 3
0770/3

CAMEROON GENERAL CERTIFCATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2019 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics With Statistics**
Paper No.	**Paper 3**
Subject Code No.	**0770**

THREE HOURS

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and tables produced by the GCE Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Electronic calculators are allowed

Start each question on a fresh page.

Turn Over

1. (i) Two events A and B are such that $P(A) = 0.85, P(B) = 0.90$, and $P(A \cup B) = 0.95$.

   Calculate
   (a) $P(A \cap B)$,
   (b) $P(A \cap B^c)$.
   State, with reasons, whether or not A and B are independent.

   (ii) A bag contains 2 red and 3 green balls. Two balls are drawn from the bag at random in succession without replacement.

   (a) If the first ball drawn is red, find the probability that the second ball is green.
   (b) Find the probability that the two balls are of the same color.

(7, 2, 4 marks)

2. (i) Given that f is a probability mass function of a discrete random variable X, where $f(x_i) = p(X = x_i)$, state a formula for calculating,

   (a) the expected value, $E(X)$, of X,
   (b) the variance, $Var(X)$, of X.

   (ii) A function f defined on the set $\{1, m, 6, 8, 12\}$ is given by the table below.

x	1	m	6	8	12
$f(x)$	0.1	0.3	0.2	n	0.15

   If f is the probability mass function of the random variable X and $E(X) = 6.0$, calculate,
   (a) the values of the constants m and n,
   (b) the variance of X,
   (c) $P(X < 8)$.

(2, 11 marks)

3. (i) The probability density function, f, of a random variable, X, is given by

$$f(x) = \begin{cases} k(x+3), & -3 < x < 3, \\ 0, & \text{elsewhere.} \end{cases}$$

   Calculate
   (a) the value of the constant k,
   (b) the expected value of X,
   (c) $P(0 < X < 2)$.

   (ii) If the median of X is m, show that m satisfies the equation $m^2 + 6m - 9 = 0$. **(10, 3 marks)**

Turn Over

4. (i) The cumulative distribution function, F, of a random variable X is defined by

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{1}{4}, & 1 \le x < 2, \\ \frac{3}{4}, & 2 \le x < 3, \\ \frac{63}{64}, & 3 \le x < 4, \\ 1, & 4 \le x. \end{cases}$$

Calculate

(a) $P(X = 3)$,
(b) $P(1 < X \le 3)$.

(ii) The masses, M, of cubes of soap produced by a certain machine are normally distributed with mean 400g and standard deviation 1.6g. By using the cumulative distribution function of the standard normal distribution, find the percentage of the cubes that will weigh

(a) less than 399g,
(b) between 400.9g and 402.9g.

(iii) The machine develops a fault that affects only the mean mass of the cubes. On investigation, technicians discover that 95% of the cubes produced weigh less than 402g. What is the new mean mass?

(4, 6, 3 marks)

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5. (i) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from an infinite population with mean μ and variance σ^2.

(a) What do you understand by the sample mean $\overline{X}$?
(b) By calculating $E(\overline{X})$, show that $\overline{X}$ is an unbiased estimator of the population mean μ.
(c) If the population sampled is a normal population, state the distribution of $\overline{X}$.

(ii) The masses of cows in a certain grazing settlement are normally distributed with mean 500kg and standard deviation 20kg. Grazers from this settlement usually sell their cows in the same cattle market.

(a) A trader buys 50 cows from this market. What is the probability that the mean mass of these cows will be between 495kg and 515kg.
(b) How many cows must he buy in order to be 95% sure that the mean mass is at least 495kg?

(5, 4, 4 marks)

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6. (i) Define the following terms as used in hypothesis testing:
   (a) Null hypothesis,
   (b) Test statistic,
   (c) Critical region,
   (d) Level of significance.
   (ii) Outline the procedure of performing a test of a statistical hypothesis.
   (iii) The proprietor of Northern provision center (N.P.C) complains that of recent, the average mass of sachets of detergent supplied by a certain company is less than the stated value of 100g. In order to investigate the proprietors complaint, the company took a sample of 100 sachets and realized that their mean mass was 99.88g with a standard deviation of 0.7g.
   By using this information and the procedure outlined in (ii), determine at the 5% level of significance if the proprietors complaint is justified.

(4, 4, 5 marks)

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7. The marks obtained by 100 candidates in an examination are summarized in the distribution below.

Marks, (x)	10-19	20-29	30-39	40-49	50-59	60-69	70-79
Frequency,(f)	3	20	35	25	7	5	5

   (i) Calculate
   (a) The mean mark
   (b) The standard deviation of the marks
   (ii) Draw the cumulative frequency curve for this distribution.
   (iii) Using the curve drawn in (ii), determine
   (a) the number of candidates who will pass the exam if the pass mark is 45.
   (b) the least mark a candidate must earn to be awarded an A grade if the top 10% of the class is to earn an A grade.

(2, 3, 4, 2 marks)

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8. The scores of 10 candidates in a mock exams and the final exams are presented below:

Mock exam, M	22	41	46	53	62	65	72	80	91	92
Final exam, F	27	30	38	35	44	41	50	44	62	68

These marks are summarized as follows:

$$\sum M = 624, \; \sum M^2 = 43488, \; \sum F = 439, \; \sum F^2 = 20819, \; \sum MF = 29833.$$

(a) Calculate, to two decimal places, the Pearson product-moment correlation coefficient for this data.
(b) Find the least squares regression line of the final exam scores on the mock exam scores.
(c) If the pass mark for the final exam is 40, determine whether a candidate who scored 60 in the mock exams but missed the final exams would have passed.

(8, 2, 3 marks)