CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2020 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**Paper 2**
Subject Code No.	**0775**

THREE Hours

Answer ALL 10 questions.

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae and tables published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Turn Over

1. Use the substitution $y = vx$, where v is a function of x, to transform the differential equation

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

into a differential equation in v and x. **(3 marks)**

Hence find the solution of this differential equation given that $y = 1$ when $x = 1$. **(5 marks)**

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2. Solve for real x, the equation

$$3\cosh 2x = 3 + \sinh 2x \ .$$ **(6 marks)**

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3. Given that $$I_n = \int_0^1 x(1-x)^n \, dx,$$

where n is a positive integer,
show that
$(n+2)I_n = nI_{n-1}$, $n \geq 1$. **(5 marks)**

Hence, evaluate $\int_0^1 x(1-x)^5 \, dx$. **(3 marks)**

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4. Find the cartesian equation of the curve with polar equation
$r = \cos^2\theta$. **(3 marks)**

Hence, show that the equation of the tangent to the curve at the point $\left(\frac{1}{2}, \frac{\pi}{4}\right)$ is

$$r = \frac{1}{\sqrt{2}(3\sin\theta - \cos\theta)}.$$ **(4 marks)**

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5. Show that the equation of the normal to the hyperbola $xy = 4$ at the point $P\left(2t, \frac{2}{t}\right)$, $t \neq 0$ is

$$t^3x - ty - 2t^4 + 2 = 0.$$ **(3 marks)**

This normal cuts the hyperbola again at the point Q.

(i) Find the coordinates of Q. **(2 marks)**

(ii) Prove that the locus of the midpoint of the origin and Q is another hyperbola. **(3 marks)**

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6. Given that $z = \cos\theta + i\sin\theta$,

(i) Show that

$$z + \frac{1}{z} = 2\cos\theta .$$ **(2 marks)**

(ii) Find in a similar manner, an expression for $z^2 + \frac{1}{z^2}$. **(1 mark)**

(iii) Show that

$$z^2 - z + 2 - \frac{1}{z} + \frac{1}{z^2} = 4\cos^2\theta - 2\cos\theta .$$ **(2 marks)**

Hence, solve the equation

$$z^4 - z^3 + 2z^2 - z + 1 = 0,$$

giving the roots in the form $a + bi$. **(3 marks)**

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7. (a) Given that S is a similarity transformation(similitude) defined by

$$z' = (1+i)z - 2 + 3i \ ,$$

find

(i) the scale factor **(1 mark)**
(ii) the invariant point (centre) and **(2 marks)**
(iii) the angle of rotation of this transformation. **(2 marks)**

(b) A linear transformation, T, is defined as follows

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$
$$(x,y) \mapsto (2x+y,\ x-y).$$

Determine the kernel of T. **(3 marks)**

8. Solve in $\mathbb{N}$, the system of equations

$$3x \equiv 1(\text{mod}\,5)$$
$$5x \equiv 2(\text{mod}\,7)$$

(8 marks)

9. A sequence, (u_n), is defined recursively by

$$u_0 = 1\ ,\ u_1 = 3 \text{ and } u_{n+1} = 3u_n + 4u_{n-1}\ .$$

(i) Find u_2 . **(2 marks)**
(ii) Show that $u_{n+1} - 4u_n = (-1)^{n+1}(u_1 - 4u_0)$. **(3 marks)**
(iii) Hence express u_{n+1} in terms of u_n . **(1 mark)**

Show that

(iv) u_n is divergent. **(2 marks)**
(v) $u_n \geq 1,\ \forall n \in \mathbb{N}$. **(3 marks)**
(vi) u_n is increasing. **(2 marks)**

Two other sequences (v_n) and (w_n) are defined as

$$v_n = u_n + u_{n-1} \text{ and } w_n = u_n - 4u_{n-1}, n \geq 1.$$

(vii) Show that v_n and w_n are geometric sequences. **(3 marks)**
(viii) Express v_n and w_n each in terms of n . **(2 marks)**
Hence, or otherwise, express u_n in terms of n . **(2 marks)**

10. (a) A function, f, is defined by

$$f(x) = \sqrt{x^2 - 3x + 2}\ .$$

(i) Find the domain of f . **(3 marks)**
(ii) Show that

$$f(x) = \sqrt{\left(x - \tfrac{3}{2}\right)^2 - \tfrac{1}{4}} \text{ and that as } x \to +\infty\ ,\ f(x) \to x - \frac{3}{2}.$$

Find a similar expression for $f(x)$ as $x \to -\infty$. **(4 marks)**
(iii) Hence, state the oblique asymptotes of f . **(1 mark)**
(iv) Sketch the graph of $y = f(x)$, showing clearly the asymptotes. **(3 marks)**

(b) Given that $g(x) = \dfrac{x^2}{2x-1}$, defined on $]1,+\infty[$, and the sequence (u_n) defined recursively by

$$\begin{cases} u_0 = 2 \\ u_{n+1} = g(u_n) \end{cases}.$$

(i) Show that $\forall x > 1$, $g(x) > 1$. **(3 marks)**

Given the sequences (v_n) and (w_n) such that

$$v_n = \frac{u_n - 1}{u_n} \text{ and } w_n = \ln v_n ,$$

show that

(ii) w_n is a geometric sequence. **(4 marks)**

(iii) $v_n = 2^{-2^n}$. **(2 marks)**

END