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CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2020 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**Paper 3**
Subject Code No.	**0775**

GCE REVISION

Two and a half hours

Answer ALL questions.

For your guidance the approximate mark allocation for parts of each question is indicated in brackets.

Mathematical formulae and tables, published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

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Turn Over

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1. A particle P, of mass m, is projected vertically upwards from a point O with speed $\frac{2g}{k}$ ms^{-1} in a medium whose resistance to motion is of magnitude kv per unit mass, where v is the speed of P and k a positive constant.

(i) Show that

$$v\frac{dv}{dx} = -g - kv$$ **(1 mark)**

The particle P attains the highest point H of its path.

(ii) Show further that

$$\text{OH} = \frac{g}{k^2}(2 - \ln 3) \text{ metres}$$ **(5 marks)**

Given that the particle P reaches another point M with speed $\frac{g}{2k}$ ms^{-1}as it traces its path back to O, the point of projection,

(iii) Prove that

$$\text{HM} = \frac{g}{k^2}(\ln 2 - \tfrac{1}{2})$$ **(4 marks)**

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2. Given that y satisfies the differential equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0 \text{ and that } y = 1, \frac{dy}{dx} = 0 \text{ when } x = 0.$$

Using the approximations

$$h^2\left(\frac{d^2y}{dx^2}\right)_n \cong y_{n+1} - 2y_n + y_{n-1} \text{ and } 2h\left(\frac{dy}{dx}\right)_n \cong y_{n+1} - y_{n-1}$$

and a step-length of 0.2,

show that

(i) $y_{n+1} \cong \dfrac{2x_n y_n(2 - h^2) + y_{n-1}(h - 2x_n)}{h + 2x_n}$, **(2 marks)**

(ii) $y_{-1} = y_1 = 1.$ **(4 marks)**

Hence, find correct to 4 decimal places,

(iii) the value of y when $x = 0.6$. **(6 marks)**

3. A particle A moves in a resisting medium in a straight line such that its distance x from a fixed point O satisfies the equation

$$\frac{d^2x}{dt^2} + p\frac{dx}{dt} + qx = 0, \text{ where } p \text{ and } q \text{ are constants.}$$

Find the condition(s) on p and q such that the motion of A is

(i) simple harmonic. **(3 marks)**

(ii) damped harmonic. **(4 marks)**

In the case where the motion is damped harmonic, find

(iii) the damping factor. **(3 marks)**

(iv) the period of the motion. **(2 marks)**

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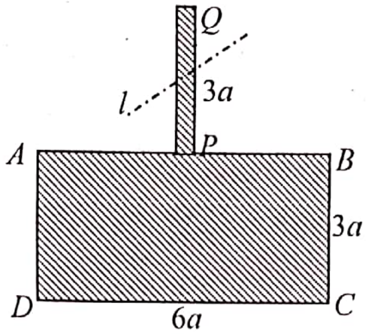
4. A plane composite object is made by attaching a thin uniform rod PQ of mass M, at right angles to the midpoint AB of a uniform rectangular lamina $ABCD$ of mass m. Given that

$$AB = 6a, \ BC = 3a = PQ,$$

as shown in the diagram below:



(i) Show that the moment of inertia I of the object about a smooth horizontal axis l, through the midpoint of PQ and perpendicular to the plane of the lamina, is given by

$$I = \tfrac{3}{4}(17m + M)a^2$$ **(3 marks)**

The object is slightly disturbed through small angles θ from its equilibrium position to swing below Q. After time t,

(ii) show that

$$I\frac{d^2\theta}{dt^2} = -3mga\sin\theta.$$ **(4 marks)**

(iii) find an expression for the period of small oscillations of the object about l. **(3 marks)**

(iv) show further that if $m = M$, then the component of the force perpendicular to the rod exerted by the axis of rotation is $\frac{8}{3}mg\sin\theta$. **(4 marks)**

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5. A system of forces $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ acting through points with position vectors $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{r}_3$ respectively, where

$$\mathbf{F}_1 = (3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})N, \qquad \mathbf{r}_1 = (\mathbf{i} + \mathbf{j} + \mathbf{k})m$$

$$\mathbf{F}_2 = (3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})N, \qquad \mathbf{r}_2 = (3\mathbf{i} + 2\mathbf{j} + \mathbf{k})m$$

$$\mathbf{F}_3 = (-4\mathbf{i} - 2\mathbf{j} + \lambda\mathbf{k})N, \qquad \mathbf{r}_3 = (2\mathbf{i} - \mathbf{j})m$$

reduces to a couple $\mathbf{G}$ and a single force $\mathbf{F} = (2\mathbf{i} - \mathbf{j} + 8\mathbf{k})N$ acting through the point with position vector

$$\mathbf{r}_0 = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

Find

(i) the value of λ, **(2 marks)**

(ii) the vector moment of $\mathbf{F}_2$ about the point with position vector $\mathbf{r}_0$, **(3 marks)**

(iii) the Cartesian equation of the line of action of $\mathbf{F}$, **(4 marks)**

(iv) the magnitude of the couple $\mathbf{G}$. **(4 marks)**

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6. A particle P moves on the curve with polar equation

$$r = \frac{1}{2 - \sin\theta}.$$

Given that at any instant t, during the motion,

$$r^2 \frac{d\theta}{dt} = 4,$$

(i) write an expression for $r\frac{d\theta}{dt}$ in terms of θ. **(2 marks)**

(ii) show that

$$\frac{dr}{dt} = 4\cos\theta \text{ and } \tfrac{1}{3} \le r \le 1.$$ **(3 marks)**

(iii) Find the speed of P when $\theta = 0$. **(4 marks)**

(iv) Prove that the force acting on P is directed towards the pole. **(4 marks)**

7. Three vectors

$$\mathbf{a} = (3\mathbf{i} + 4\mathbf{j}),\ \mathbf{b} = (-4\mathbf{i} + 3\mathbf{j}),\ \mathbf{c} = (-3\mathbf{i} + 4\mathbf{j})$$

lie on a horizontal plane, Π.

(i) Determine which of the vectors **b** or **c** is perpendicular to **a**. **(2 marks)**

Two smooth spheres, S and T, each of mass m lie on the plane Π. Sphere S is projected along the plane towards sphere T with velocity $5u(\mathbf{i} + 2\mathbf{j})$ so that it collides obliquely with T. At the instant of collision, the line of impact is parallel to the vector **a**. Given that the coefficient of restitution between the two spheres is $\frac{1}{2}$.

Show that

(ii) the component of the initial velocity of S parallel to **a** is $11u$. **(2 marks)**

(iii) the component of the final velocity of S parallel to **a** is $\frac{11}{4}u$. **(4 marks)**

(iv) the magnitude of the impulse exerted by sphere T on sphere S is $\frac{33}{4}mu$. **(4 marks)**

8. The lengths of rods from a certain factory are normally distributed with mean μ cm and standard deviation 6cm. It is known that 4.78% of the rods have lengths greater than 82cm.

Find to the nearest integer,

(i) the mean μ. **(5 marks)**

(ii) the range, α (symmetrical about the mean) within which 75% of the lengths of the rods lie. **(6 marks)**

Find also, to 4 decimal places,

(iii) the probability that a rod chosen at random has a length between 62cm and 72cm. **(3 marks)**

END