Pure Math With Mechanics 2
0765/2

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2020 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics With Mechanics**
Paper No.	**Paper 2**
Subject Code No.	**0765**

Three hours

Full marks may be obtained for answers to ALL questions.

Mathematical Formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page

Turn Over

1. (i) Given that $(x+1)$ is a factor of $f(x)$, where $f(x) = x^3 + 6x^2 + 11x + 6$, factorise $f(x)$ completely. **(4 marks)**

(ii) Let λ be a real constant. Show that the roots of the quadratic equation

$$3x^2 + (-4 - 2\lambda)x + 2\lambda = 0$$

are always real . **(5 marks)**

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2. (i) Given that $y = \ln(4 + x^2)$, find

(a) $\frac{dy}{dx}$, **(2 marks)**

(a) the equations of the tangent and normal to the curve $y = \ln(4 + x^2)$ at the point where $x = 1$. **(4 marks)**

(ii) Solve the differential equation $\frac{dy}{dx} = xy - x$, given that $y = 2$ when $x = 0$, expressing y in terms of x. **(4 marks)**

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3. (i) Draw the truth table for each of the propositions $p \Longrightarrow q$ and $\sim p \vee q$ and show that they are identical. **(6 marks)**

(ii) Given that $\sin^{-1}(x) = \alpha$ and $\cos^{-1}(x) = \beta$ show that $\sin(\alpha + \beta) = 1$. **(4 marks)**

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4. (i) The function f is given by $f(x) = \frac{x+1}{(x-1)(x^2+1)}$.

Express $f(x)$ in partial fractions.

Hence finc $\int f(x)\,dx$ **(5 marks)**

(ii) By using the substitution $u = \sin x$, find $\int \left(\frac{\cos x}{1 + \sin^2 x}\right) dx$ **(3 marks)**

(iii) Find $\int (x+2)e^{3x}\,dx$ **(3 marks)**

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5. Given the circles $C_1 : x^2 + y^2 - 6x - 4y + 9 = 0$ and $C_2 : x^2 + y^2 - 2x - 6y + 9 = 0$,

Find,

(a) the equation of the circle C_3 which passes through the centre of C_1 and through the point of intersection of C_1 and C_2. **(4 marks)**

(b) the equations of the two tangents from the origin to C_1 and the length of each tangent. **(7marks)**

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6. (i) Determine whether the function $f: \mathbb{R} \to \mathbb{R}, f(x) = \frac{x+1}{x-3}, x \neq 3$ is surjective. **(4 marks)**

(ii) A periodic function $f: \mathbb{R} \to \mathbb{R}$, of period 4 is defined in the interval $-2 \leq x \leq 2$ by

$$f(x) = \begin{cases} x^2 + 4, 0 \leq x \leq 2, \\ -x^2 + k, -2 \leq x < 0. \end{cases}$$

(a) Find the value of k for which f is continuous and the value of $f(-5)$. **(3 marks)**

(b) Sketch the curve of $y = f(x)$ for $-2 \leq x \leq 10$. **(4 marks)**

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7. (i) Find the image of the line $2y = x$ under the transformation with matrix operator $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. **(4 marks)**

(ii) Find the inverse of the matrix **A**, where **A**= $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$.

Hence solve the equations

$$x - y + z = 7,$$
$$2x + y - 3z = -6,$$
$$x + y + z = 4.$$

(8 marks)

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8. (i) Evaluat $\sum_{r=1}^{\infty} 3\left(\frac{1}{4}\right)^r$ **(4 marks)**

(ii) The sum of the first n terms of a series is given by $S_n = 2n^2 + n$.

(a) Find an expression for the n^{th} term of the series. **(2 marks)**

(b) Show that the series is an arithmetic progession. **(2 marks)**

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9. (i) Prove by mathematical induction that for $n \in \mathbb{N}$,

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1} = \frac{n}{2n + 1}$$

(5 marks)

(ii) A relation R, is defined on $\mathbb{Z}$, the set of integers by $aRb \Leftrightarrow a - b$ is even.

Show that R is transitive. **(4 marks)**

Turn Over

10. (i) Find the range of values of x for which

(a) $\frac{2x-1}{x+2} < -1$ **(4 marks)**

(b) $|2x-4| < x+1.$ **(3 marks)**

(ii) The complex numbers z_1 and z_2 are such that $z_1 = -2+2i$ and $z_2 = -2-2i$.

Find

(a) $\arg z_1$

(b) $|z_2|^6$. **(4 marks)**