FURTHER MATHS 3
775

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2015 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**Paper 3**
Subject Code No.	775

Two and a half hours

Answer ALL questions.

For your guidance the approximate mark allocation for parts of each question is indicated in brackets.

Mathematical formulae and tables, published by the Board, and noiseless non-programmable electronic calculators are allowed.

In calculations you are advised to show all the steps in your working giving your answer at each stage.

1. Forces $\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_n$ act at the points with position vectors $\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_n$ respectively. State the conditions under which the system reduces to a couple. **(2 marks)**

Three forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$ act at the points with position vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$ respectively where

$$\mathbf{F}_1 = (-\mathbf{i} + 2\mathbf{j} - \mathbf{k})\text{N}, \quad \mathbf{r}_1 = (2\mathbf{i} - 3\mathbf{j} - \mathbf{k})\text{m}$$
$$\mathbf{F}_2 = (4\mathbf{i} - \mathbf{j} + \mathbf{k})\text{N}, \quad \mathbf{r}_2 = (8\mathbf{i} - \mathbf{j} - \mathbf{k})\text{m}$$
$$\mathbf{F}_3 = (-3\mathbf{i} - \mathbf{j})\text{N}, \quad \mathbf{r}_3 = (\mathbf{i} - \mathbf{j} - 6\mathbf{k})\text{m}$$

(a) Find the moment of $\mathbf{F}_1$ about the origin. **(2 marks)**

(b) Show that the system is equivalent to a couple G of magnitude $\sqrt{139}$Nm. **(6 marks)**

2. A particle P describes the curve $r \equiv ke^{\theta}$ where k is a positive constant in such a way that the radial component of its acceleration is zero.

(a) Show that $\dfrac{d\theta}{dt} = \text{constant}$. **(10 marks)**

(b) Find the transverse component of the acceleration in terms of r and θ. **(2 marks)**

3. A smooth sphere S moving with velocity $2\lambda(2\mathbf{i} + \mathbf{j})\text{ms}^{-1}$, where $\lambda > 0$, collides obliquely with an identical sphere T which is at rest. After impact, sphere T starts to move in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$. The coefficient of restitution between the two spheres is ½.

(a) State the direction of the impulse exerted by sphere S on sphere T. **(1 mark)**

(b) Show that if θ is the inclination of the direction of motion of sphere S to the line of impulses then $\cos\theta = \dfrac{2}{\sqrt{5}}$. **(4 marks)**

(c) Find the velocity of sphere T immediately after impact. **(6 marks)**

4. A model boat of mass $10m$ is put in water and its engines turned on. Given that the boat experiences a resistive force of mv newtons, where $v(t)$ is the instantaneous velocity of the boat in ms^{-1} at time t seconds and that the engines exert a force of $10m$ newtons to propel the boat forward in a straight course.

(a) Show that the equation of motion of the boat can be written in the form

$$\frac{dv}{dt} = 1 - \frac{v}{10},$$ **(2 marks)**

(b) Determine the velocity $v(t)$ as a function of t. **(4 marks)**

(c) Find the terminal velocity of the boat. **(3 marks)**

(d) Show that if $x(t)$ is the distance covered by the boat, then as $t \to \infty$, $x(t) \to at + b$, and find the values of the constants a and b. **(2 marks)**

5. (i) A particle of mass 2kg is oscillating with simple harmonic motion of period π seconds. The particle passes through a point A at a distance 0.5 m from the equilibrium position while travelling with speed 3 m/s.

Find

(a) the amplitude of the simple harmonic motion. **(4 marks)**

(b) the magnitude of the force acting on the particle. **(4 marks)**

(ii) A particle P moves in a resisting medium such that its displacement x from a fixed point O is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0.$$

Show that P undergoes a damped harmonic motion and find the period of this motion. **(6 marks)**

6. Given that $\frac{dy}{dx} = x^2 + xy = 0$ and that $y = 1$ when $x = 0$,

(a) show that for values of x close to 0,

$$y = 1 - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{8}x^4 + \cdots$$

(6 marks)

(b) use the approximation

$$2h\left(\frac{dy}{dx}\right)_n \approx y_{n+1} - y_{n-1}$$

and a step length of 0.2 to find the value of y when $x = 0.4$, giving your final answer correct to 4 decimal places.

(6 marks)

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7. The random variable X has probability density function defined by

$$f(x) = \begin{cases} \lambda(1 - x^2), 0 \leq x \leq 1 \\ 0, \text{ elsewhere} \end{cases}$$

(a) Find the value of λ.
Hence determine **(2 marks)**

(b) the mean μ, and variance σ^2, of X. **(8 marks)**

(c) the cumulative distribution of X **(1 mark)**

(d) $P\left(|X - \mu| \leq \sigma\right)$ giving your answer correct to 3 significant figures. **(4 marks)**

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8. A thin uniform rod AB of mass $3m$ and length $2a$ is free to rotate in a vertical plane about a smooth horizontal axis through a point P on the rod, where $AP = \frac{3}{4}a$. The rod is released from rest when PB makes an angle β with the horizontal.

(a) Show that in the ensuing motion,

$$19a\left(\frac{d\theta}{dt}\right)^2 = 24g(\sin\theta - \sin\beta),$$

(8 marks)

where θ is the angle the rod makes with the horizontal at time t.

(b) When $\beta = 0$, find the magnitude of the reaction on the axis of rotation when $\theta = \frac{\pi}{3}$.

(7 marks)

END